MT 453 Elements

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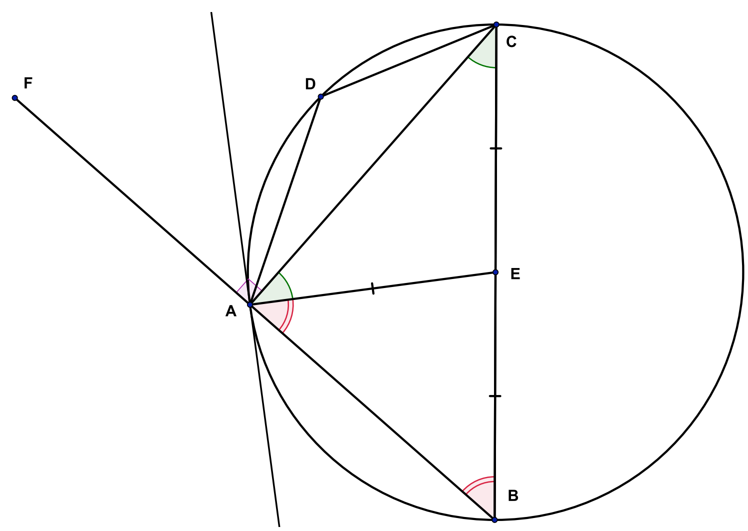
Proposition III.31

1. The angle IN a semicircle is right.
2. The angle IN a segment > a semicircle is < A Right Angle.
3. The angle IN a segment < a semicircle is > A Right Angle.
4. The angle OF a segment > a semicircle is > A Right Angle.
5. The angle OF a segment < a semicircle is < A Right Angle.



Draw circle with diameter BC, center E.
Draw $\angle BAC$ in the semicircle.
Pick D between A and C.

Connect AD, CD, and AE.
Extend BA to F.
$BE = AE$, since both are radii.
So $\angle ABE = \angle BAE$. (Prop. I.5)
$AE = CE$, since both are radii.
So $\angle CAE = \angle ACE$. (Prop I.5)
$\angle BAC = \angle ABE + \angle ACE$.
But $\angle FAC = \angle ABE + \angle ACE$. (Prop I.32)
So $\angle BAC = \angle FAC$. (C.N.1)
They are also adjacent, so they are both right angles. (Def. I.10), which proves the first statement.

Now consider segment ABC.
$\angle BAC + \angle ABC <$ Two Right Angles. (Prop. I.17)
$\angle BAC =$ A Right Angle.
So $\angle ABC <$ A Right Angle, which proves the second statement.

Consider Quadrilateral $ADCB$.
$\angle ABC + \angle ADC =$ Two Right Angles. (Prop. III.22)
$\angle ABC <$ A Right Angle
So $< ADC >$ A Right Angle, which proves the third statement.

The angle OF segment ABC contains $\angle BAC$.
Since $\angle BAC$ is a right angle, the angle of segment BAC music be greater than a right angle.
This proves the fourth statement.

The angle OF segment ADC is a part of $\angle FAC$.
But $\angle FAC$ is a right angle.
So the angle of ADC is a part of a right angle.
Therefore it is less than a right angle, proving the fifth and final statement.
Q.E.D.