MT 453 Elements Day 17

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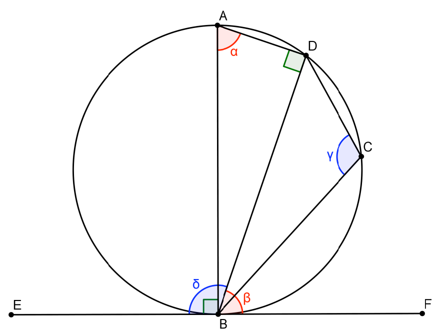
Propsition.III.32

A straight line touches a circle and from the point of contact there be drawn across , in a circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.



Claim: $\alpha = \beta, \delta = \gamma$
Draw AB perpendicular to EF at point B.
BA is a diameter of circle $ABCD$ (III.19).
$\angle ADB = \lrcorner$ (III.31); and $\alpha + \angle ABD = \lrcorner$ (I.32).
$\angle ABF = \lrcorner$; $\alpha + \angle ABD = \angle ABF$ (c.n.1)
$\alpha = \angle ABF - \angle ABD$; $\angle ABF - \angle ABD = \beta$ (c.n.3)
$\therefore \alpha = \beta$ (c.n.1)
$ABCD$ is a quadrilateral in a circle, so opposite angles equal $\lrcorner\lrcorner$ (III.22).
$\beta + \delta = \lrcorner\lrcorner$ (I.13).
$\alpha + \gamma = \lrcorner\lrcorner$ (III.22).
$\beta + \delta = \alpha + \gamma$ (c.n.1).
$\therefore \delta = \gamma$ (c.n.3.)

Q.E.D

Notes: BD can't be a diameter. For showing BD is a diameter, then the triangle is a right triangle, and the angle between the tangent and diameter is a right angle. But in this cases, $\beta \neq \alpha$. Therefore, BD must be a cord.