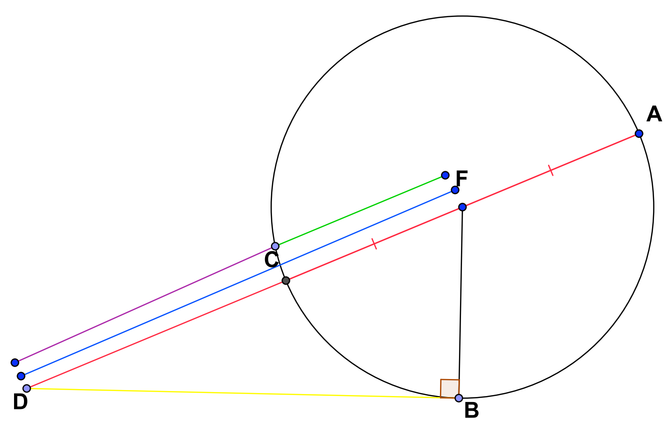
MT 453 Elements Day 18

Speaker: Sean Johnson
Scribes: Rebecca Wentzel, Kerry Fitzmaurice

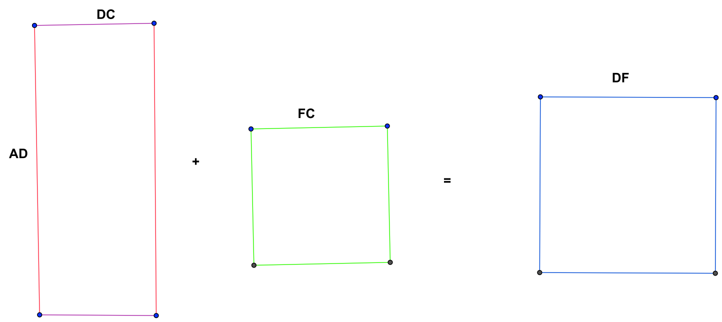
February 27/March 9, 2009

Proposition III.36

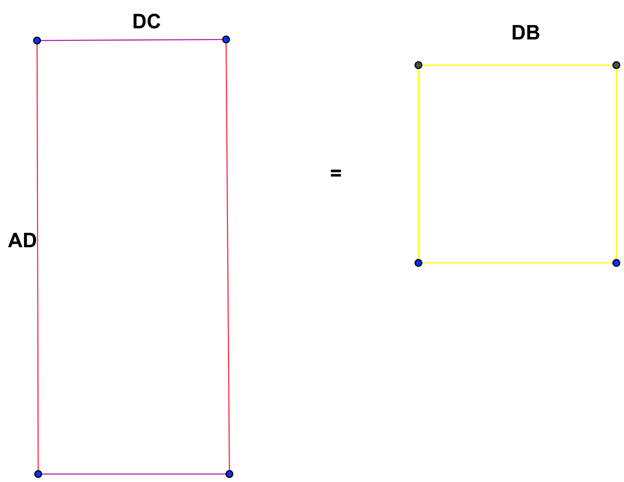
If from a point outside the circle is drawn a tangent and secant line, then the square on the tangent is equal to the rectangle formed by the secant and the distance from the point to the circle.



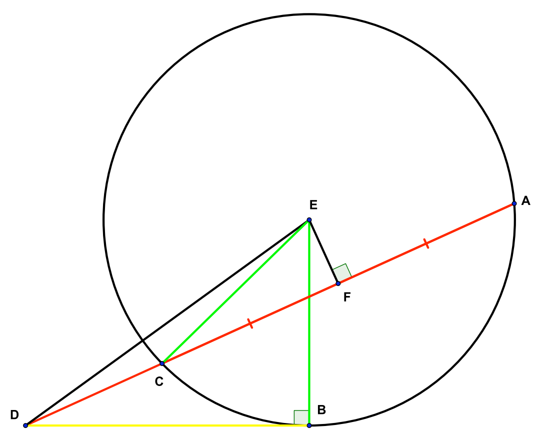
Let D be the point outside a circle, and DB tangent at B.
Case I: the secant drawn passes through the center F.
Draw BF. Then $\angle FBD = \lrcorner$ (prop. III.8)
Since F bisects AC, $(AD)(DC) + FC^2 = DF^2$ (prop II.6).
$FC = BC$, because they are both radii, so $FC^2 = FB^2$. Then $(AD)(DC) + FB^2 = DF^2$.



By prop. I.47, $FB^2 + DB^2 = DF^2$.
Then $(AD)(DC) + FB^2 = FB^2 + DB^2$, so $(AD)(DC) = DB^2$.



Case II: The secant does not pass through the center.
Let E be the center of the circle (prop III.1). Draw $EF \perp DA$ (prop I.12).
Draw EB. Then $\angle EBD = \lrcorner$ (prop. III.8). In addition, $AF = CF$ (prop. III.3).
By prop. II.6, $(DA)(CD) + CF^2 = DF^2$. Then $(DA)(CD) + CF^2 + EF^2 = DF^2 + EF^2$ (c.n. 2)
This can be rewritten as $(DA)(CD) + CE^2 = DE^2$ (prop. I.47).
$CE = BE$ since they are radii, so $(DA)(CD) + BE^2 = DE^2$.
Then $(DA)(CD) + BE^2 = DB^2 + BE^2$ (prop. I.47), so $(DA)(CD) = DB^2$ (c.n. 3) **QED**



Comment:
This is another example of something that doesn't change-the rectange $(AD)(DC)$ remains the same as the secant line pivots around the fixed point D. When A and C converge to B, $AD = CD = BD$, so $(AD)(DC) = BD^2$.