MT 453 Elements

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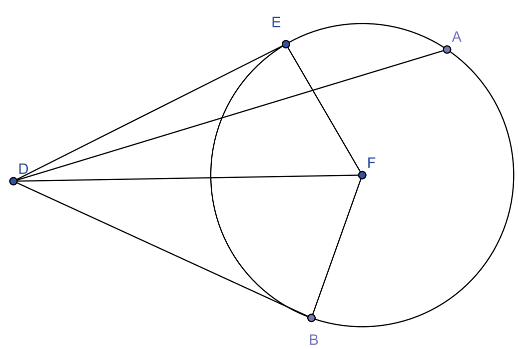
Proposition III.37

If a point D falls outside a circle ABC and $(DC)(DA) = (DB)^2$, then DB is tangent to the circle.



Given the circle ABC and the point D outside the circle, draw the secant line DCA, where C is the first intersection of the secant and the circle.
Connect DB, and assume that $(DC)(DA) = (DB)^2$.
From point D, draw DE tangent to circle ABC. [Prop. III.17]
Then $(DE)^2 = (DC)(DA)$ [Prop. III.36],
So $(DE)^2 = (DB)^2$ and $DE = DB$ [C.N.1]
Find the center, F, of circle ABC [Prop. III.1], and connect FB, FD, and FE.
$FE = FB$, since they are both radii, FD is common, and $DB = DE$.

Therefore $\triangle DFE \cong \triangle\ DFB$, so $\angle DEF = \angle DBF$. [Prop. I.8]
But $\angle DEF = \lrcorner$ [Prop. III.18], so $\angle DBF = \lrcorner$.
Therefore DB must be a tangent line to circle ABC. [Prop. III.16, Por.]

Q.E.D.

Comments:
Euclid also makes the point that this same proof works whether or not AC is a diameter of circle ABC.