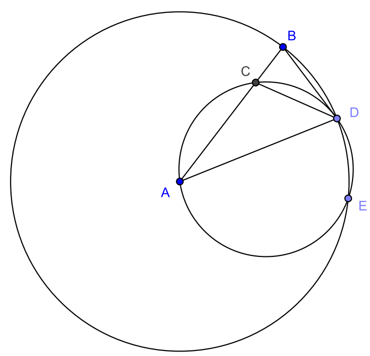
MT 453 Elements Day 23

Speakers: Jacob
Scribes: Sarah, Andrew

March 16, 2009

Proposition IV.10

To construct an isosceles triangle such that the base angles are each twice the remaining angle.



Let AB be a given line segment and choose C such that $(AB)(BC) = (AC^2)$. (Prop. II.11)
Construct a circle centered at A and has radius AB.
Choose D on circle BDE such that $BD = AC$. (Prop. IV.1)
Connect CD, AD and BD and circumscribe circle ACD around $\triangle ACD$.
Then $(AB)(BC) = (BD^2)$.
And BD is tangent to ACD. (Prop. III.37)
And $\alpha = \gamma$. (Prop. III.32)

So $\alpha + \beta = \gamma + \beta = \delta$.
We also know $\phi = \alpha + \beta$. (Prop. I.16)
Therefore, $\phi = \delta$.
And we know that $\theta = \delta$. (Prop. I.5)
Therefore, $\delta = \theta = \phi$.
Then $BD = CD$. (Prop. I.6)
But $BD = AC$ so $AC = CD$.
Therefore, $\alpha = \beta$. (Prop. I.5)
And therefore, $\alpha + \beta = 2\alpha$.
But since $\phi = \alpha + \beta$, $\phi = 2\alpha$.
And we know $\phi = \delta$, so $\delta = 2\alpha$.
But since $\delta = \theta$, $\theta = 2\alpha$.
Therefore, $\triangle ABD$ is isosceles with each base angle at DB being twice the remaining angle.
QED

Comments:
1. AD will not be a diameter.
2. $\triangle BDC$ is isosceles.
3. Point E is not really used in the proof, Euclid just uses it to name the circle.