MT 453 Elements Day 25

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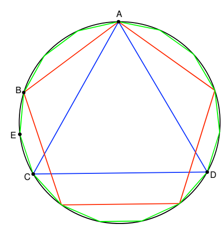
March 20, 2009

Proposition IV.16

In a given circle to inscribe a regular pentadecagon.



Let circle $ABCD$ be a circle.
Let AC be the side of an equilateral triangle in $ABCD$.
Let AB be the side of the regular pentagon.
A regular pentagon divides the circle into arcs equal to one fifth of the circle.
arc AB = $\frac{1}{5}$ circle $ABCD$
An equilateral triangle divides the circle into arcs equal to one third of the circle.
arc AC = $\frac{1}{3}$ circle $ABCD$
arc ABC - arc AB = arc BC
$\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$ circle $ABCD$
Bisect are BC at E (III.30), so arc BE equals $\frac{1}{15}$ of the circumference of the circle.
Claim: We can draw a 15 sided figure that is equiangular and equilateral. First step is to connect BE and EC, then we repeat this same process to each vertex on the inscribed pentagon.
Here Euclid skips the steps he has used in previous proofs and uses the notion that equal sides implies equal angles to claim we have inscribed a regular pentadecagon.

Q.E.D

Corollary: We can draw the tangents of the circle at the points of division, and there will be a circumscribed regular pentadecagon about the circle. Furthermore, by proofs similar to those in this case of the pentagon, we can also inscribe a circle in the given regular pentadecagon and circumscribe a circle about it.