Proposition V.11
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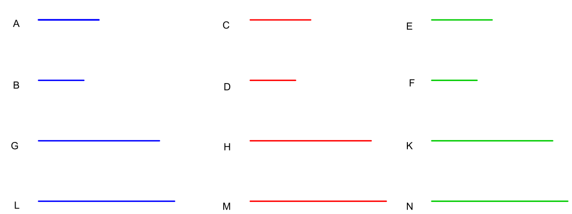
1. Ratios which are the same with the same ratio are also the same with one another.



notes: This proposition essentially proves the transitivity of ratios
Ratios are pairs of equivalence relations. In V.11, we check three things:
1. check that the first is related to the first. So in $A : B = A : B$, $A = A$
2. Check that the second is related to the second. So in $A : B = A : B$, $B = B$
3. Check that the pairs are related to eachother. So $A : B = A : B$
Now, on to the proof...
We claim that
IF $A : B = C : D$ and $C : D = E : F$
THEN $A : B = E : F$
Take equimultiples G, H, and K of A, C, and E respectively (so, if n is any number, $G = nA$, $H = nC$ and $K = nE$
Take equimultiples L, M, and N of B, D, and F respectively
If $A : B = C : D$, then:
$G > L$ implies $H > M$
$G = L$ implies $H = M$

$G < L$ implies $H < M$
If $C : D = E : F$, then:
$H > M$ implies $K > N$
$H = M$ implies $K = N$
$H < M$ implies $K < N$
Since these relationships go both ways (i.e. the stuff on the right implies the stuff on the left),
$K > N$ implies $G > L$
$K = N$ implies $G = L$
$K < N$ implies $G < L$
But K and G are equimultiples of E and A
Also N and L are equimultiples of F and B
Therefore $E : F = A : B$
Endnotes:
Why prove this when we didn't prove the transitivity of other things? Ratios are new, and so we're proving it to show that we CAN compare ratios. Euclid has to show that his new notion of equality (equality of ratios) satisifes his common notion 1. The proof itself uses definition V.5 exclusively at every step of the proof.