

MT 453 Elements Day 22

Speaker: Kerry Fitzmaurice

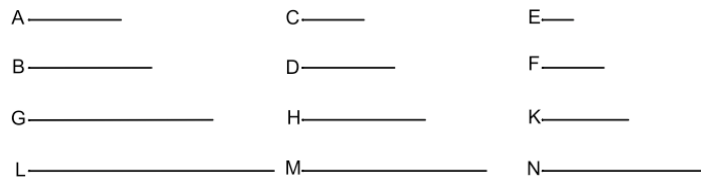
Scribes: Bill Keane, Tracy Maciolek

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Proposition V.12

If $A : B = C : D = E : F$, then $A + C + E : B + D + F$ is the same ratio.

Proof: Take equimultiples G, H, K of A, C, E and equimultiples L, M, N of B, D, F .



If $G \geq L$, then $H \geq M$ and $K \geq N$ (Def. V.5), so also

$$G + H + K \geq L + M + N.$$

We conclude that G and $G + H + K$ are equimultiples of A and $A + C + E$ (Prop. V.1). Similarly, L and $L + M + N$ are equimultiples of B and $B + C + F$.

Therefore, $A : B = A + C + E : B + D + F$. (Def. V.5).

Q.E.D.

Comment:

Euclid states the proposition for an arbitrary number of equal ratios. He proves it for 3, suggesting that his argument easily extends.