MT 453 Elements Day 29

Speaker: Professor Keane
Scribe: Tracy Maciolek
Artist: Kevin O'Neil

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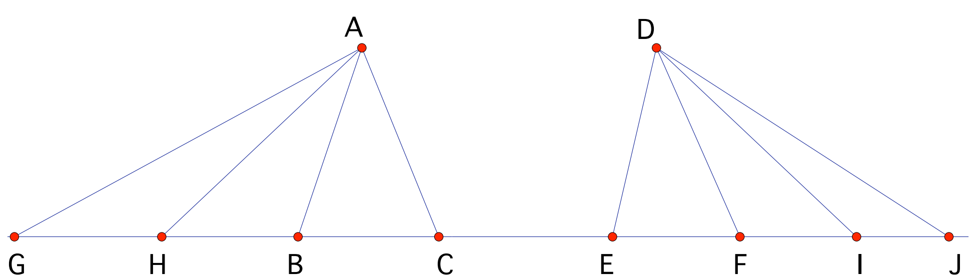
Proposition VI.1

Triangles (and parallelograms) of same height are to one another as their bases.

Proof:

Part 1:

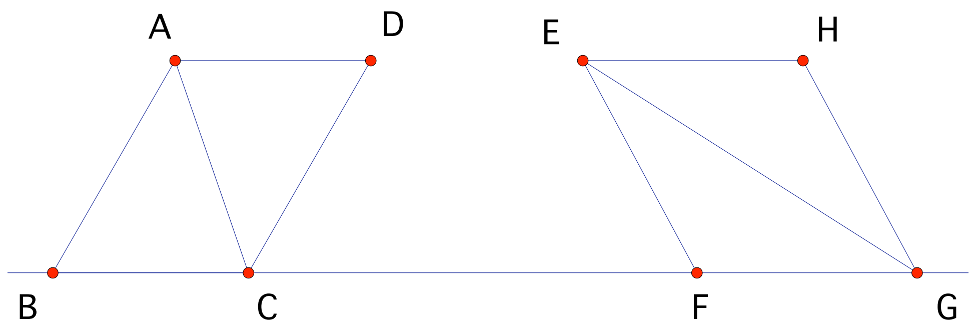


We want to show $\triangle ABC : \triangle DEF = BC : EF$.
Mark off on BF points G, H, I, K with $GH = HB = BC$ and $EF = FI = IJ$.
Draw AG, AH, DI, DJ.
Then, $\triangle AGH = \triangle AHB = \triangle ABC$
and $\triangle DEF = \triangle DFI = \triangle DIJ$.

Notice, $\triangle AGC \overset{>}{\underset{<}{=}} \triangle DEJ \leftrightarrow GC \overset{>}{\underset{<}{=}} EJ$. (Prop. I.38)

But we have shown that equimultiples of BC and $EF \overset{>}{\underset{<}{=}}$ equimultiples of $\triangle ABC, \triangle DEF$.
By Definition, V.5, $BC : EF = \triangle ABC : \triangle DEF$.

Part 2:



For parallelograms, draw BD and FH.
Then $\Diamond ABCD = 2\triangle BCD$ and $\Diamond EFGH = 2\triangle FGH$. (Prop. I.34)
But parts are in the same ratio as multiples of parts. (Prop. V.15)
So $\Diamond ABCD : \Diamond EFGH = \triangle BCD : \triangle FGH$.
But $\triangle BCD : \triangle FGH = BC : FG$. (Prop. VI.1, Part 1)
So $\Diamond ABCD : \Diamond EFGH = BC : FG$. (Prop. V.11)
Q.E.D.

Comments:

1. The picture in *Euclid's Elements* implies that we need a common side to complete the proof because the triangles are next to each other, but a common side is never used, so it is unnecessary. The picture used in this version of the proof shows that we can prove the proposition without the triangles sharing a common side.

2. When we use Proposition I.38, we say that the triangles and sides are greater than, equal to, or less than one another. While Proposition I.38 only proves that they are equal, we can justify the greater than and less than parts as well. If you extend the base of the triangle and draw a line from the point A to a point on the extended base, the original triangle is a part of the new triangle, so it is smaller than the new triangle, i.e. less than. If you shorten the base, the original triangle is now larger than the new, i.e. greater than.

3. Throughout the proof, when we say two triangles are equal, we are saying that the areas are equal. We make no claims about the congruence of triangles.