

MT 453 Elements Day 29

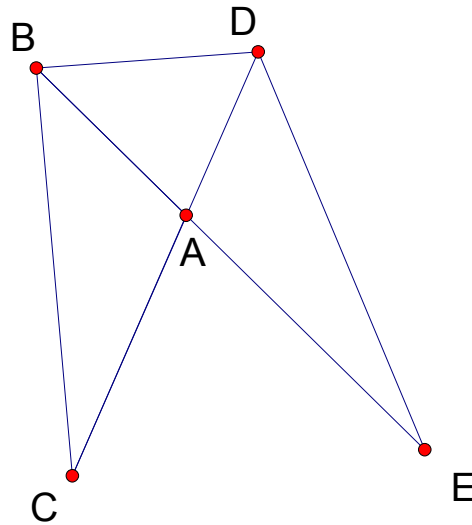
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Proposition VI.15

- a. In equal triangles, and by equal we mean equal in area, with one angle equal to one angle, then the sides about the equal angles are reciprocally proportional.*
- b. If two triangles have one angle equal to one angle and the sides about the angle are reciprocally proportional, then the triangles are equal.*

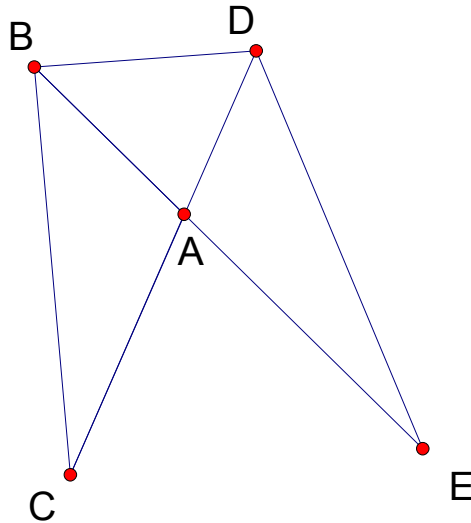


Proof:

Suppose $\triangle ABC = \triangle ADE$ and that $\angle BAC = \angle DAE$.

Claim: $a:b = c:d$.

Place a in a straight line with b .



Then c will be in a straight line with d . (Prop. I.14)

Draw BD . (Post. 1)

We know that $\triangle ABC:\triangle BAD = \triangle ADE:\triangle BAD$. (Prop. V.7)

But we also know that $\triangle BAC$ and $\triangle BAD$ have the same height, and thus $\triangle ABC:\triangle BAD = a:b$. (Prop. VI.1)

Similarly, we can say that $\triangle EAD$ and $\triangle BAD$ have the same height, and thus $\triangle EAD:\triangle BAD = c:d$. (Prop. VI.1)

But, we know that $\triangle ABC:\triangle BAD = \triangle ADE:\triangle BAD$, so $a:b = c:d$.

Therefore, we have proved the first part of this proposition.

The second part of this proposition follows from the proof of the first part.

Thus we have shown that in equal triangles with one angle equal to one angle, then the sides about the equal angles are reciprocally proportional, and also if two triangles have one angle equal to one angle and the sides about the angle are reciprocally proportional, then the triangles are equal.

Q.E.D.