MT 453 Elements

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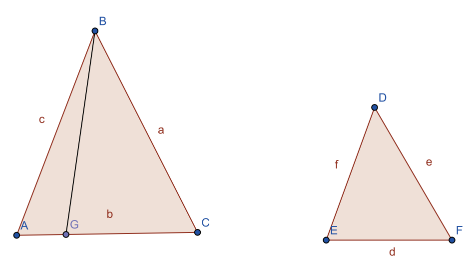
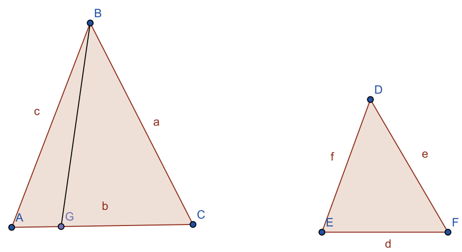
Proposition VI.19

Similar triangles are to one another in the duplicate ratio of their corresponding sides.



Let $\triangle ABC$ and $\triangle EDF$ be similar triangles.
Label the following side lengths: $BC = a$, $AC = b$, $AB = c$, $EF = d$, $DF = e$, and $DE = f$.
Locate point G on AC so $b : d = d : g$ [Prop. VI.11], where g is the length AG.
Connect BG.
Since $\triangle ABC \sim \triangle EDF$, $c : b = f : d$. [Prop. V.16]
Then $c : f = b : d$ [Prop. V.11], so $c : f = d : g$.
Again because $\triangle ABC \sim \triangle EDF$, $\angle A = \angle E$.



Since $c : f = d : g$ and $\angle A = \angle E$, $\triangle ABG = \triangle EDF$. [Prop. VI.15]
By Proposition VI.1, we have that $b : g = \triangle ABC : \triangle ABG$.
$\frac{b}{d} \cdot \frac{d}{g} = \frac{b}{g}$, but $\frac{b}{d} = \frac{d}{g}$, so $\frac{b}{g} = \frac{b}{d} \cdot \frac{b}{d} = \left(\frac{b}{d}\right)^2$, the duplicate ratio. [Def. V.9]
$b : g = \triangle ABC : \triangle ABG$ so $(b : d)^2 = \triangle ABC : \triangle ABG$.

Q.E.D.

Corollary:

$$\frac{a}{b} = \frac{b}{c} \quad \Rightarrow \quad \frac{a}{c} = \left(\frac{a}{b}\right)^2$$

This can also be extended to

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} \quad \Rightarrow \quad \frac{a}{d} = \left(\frac{a}{b}\right)^3,$$

etc.