MT 453 Elements

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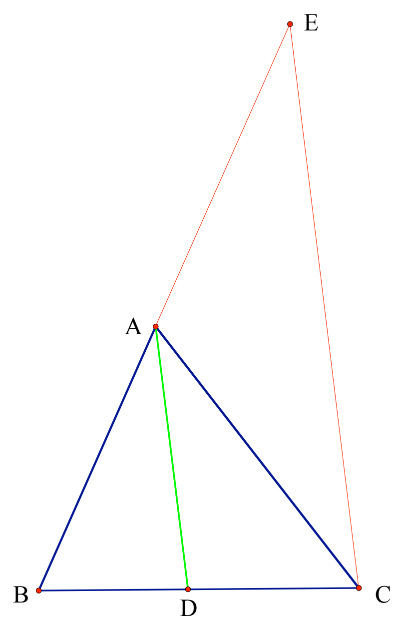
Proposition VI.3

If an angle in a triangle is bisected the segments of the base are in the same ratio as the remaining sides, and the converse of this statement.
Draw triangle ABC and bisect $\angle BAC$ with line AD so that point D lies on BC.



Given that $\angle BAD = \angle DAC$, prove $BD : DC = BA : CA$.
Draw a line parallel to AD through point C [I.31], and extend BA until it hits that line at point E [Post 2].
$\angle DAC = \angle ACE$ [I.29]
$\angle BAD = \angle BEC$ [I.29]
So $\angle ACE = \angle BEC$ [c.n.1]
Thus $AC = AE$ [I.6]
So $BD : BC = BA : AE$ [VI.2]
Since $AC = AE$, $BD : BC = BA : AC$ [c.n.1]

The Converse:
Given $BD : DC = BA : AC$, prove $\angle BAD = \angle DAC$.
$\angle DAC = \angle ACE$ and $\angle BAD = \angle AEC$ [I.29]
$BD : DC = BA : AE$ [VI.2]
Using the given information, $BA : AC = BA : AE$ [V.11]
So $AC = AE$ [V.9]
So $\angle ACE = \angle AEC$ [I.5]
Thus $\angle BAD = \angle DAC$ [c.n.1]. Therefore AD bisects $\angle BAC$.
QED