MT 453 Elements

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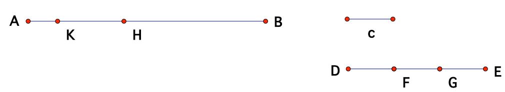
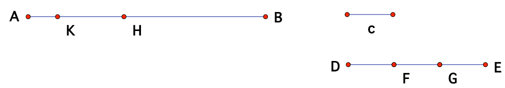
Proposition X.1

Let $\epsilon > 0$. Continually removing more than half of a magnitude leaves a magnitude less that ϵ.



Let AB and C be our magnitudes with $AB > C$. C is out ϵ.
By **(def. V.4)** , $n(C) > AB$.
Let this be DE.
Let n = 3 so DF = FG = GE = C.
Cut off HB on AB so that HB is more than half of AB.
Repeat - cut off HK on AH so that HK is more than half of AH.
Do this n times.
Claim: $AK < C$
Since DE = 3(C) and we know C = DE, we can say $GE < \frac{1}{2}DE$.
$HB > \frac{1}{2}AB$ (by the way we cut it)
$DE > AB$ so $DG > AH$ which we get by taking away HB and GE.
Therefore, $\frac{1}{2}DE > \frac{1}{2}AB$ since we took away less than half of DE and more than half of AB.
FG = $\frac{1}{2}DG$ and $HK > \frac{1}{2}AH$
Therefore, $DF > AK$.



We know $DF = C$, so $C > AK$.

Since C is our ϵ, we can say that $\epsilon > AK$.

Q.E.F.

Comment:
This works for any n.

Porism:
This works for removing exactly half as well.