Proposition X.2
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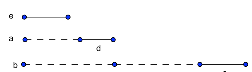
Prop X.2: If, when the less of two unequal magnitudes is continually subtracted from the greater, that which is left never measures the one before it, then the two magnitudes are incommensurable



Background: This proposition introduces the idea of two magnitudes being commensurable and incommensurable. In simplest terms, two magnitudes are commensurable when there exists some other magnitude which will evenly measure both original magnitudes. If no such magnitude exists, then the magnitudes are said to be incommensurable. A more modern way of thinking about this idea is through the idea of a greatest common divisor. In that process, for incommensurable magnitudes, we will never obtain a GCD, and the one before will never measure the greater.

Proof:

-Let a, b be two magnitudes, and let $a < b$

-Suppose the hypothesis of X.2 holds

-Claim: a and b are incommensurable

-Suppose not, then there exists an e which divides evenly (measures) both a and b

-Subtract some multiple of a from b, and call the remainder c, with $c < a$
-Subtract some multiple of c from a, and call the remainder d, where $d < c$
-Repeat this process until the remainder is less than e. In this case, suppose $d < e$. (X.1)
-Therefore, e measures a, so e also must measure na (na being the multiples of a in b)
-Thus, e also measures b, so e must also measure c
-Then, e measures mc (mc being the multiples of c in a)
-So, therefore, e must measure d.
-But, this is impossible, as $d < e$ so there is a contradiction.
-Thus, the assumption was false, there exists no e such as was assumed, and a and b are incommensurable.
-QED