MT 453 Elements Day 34

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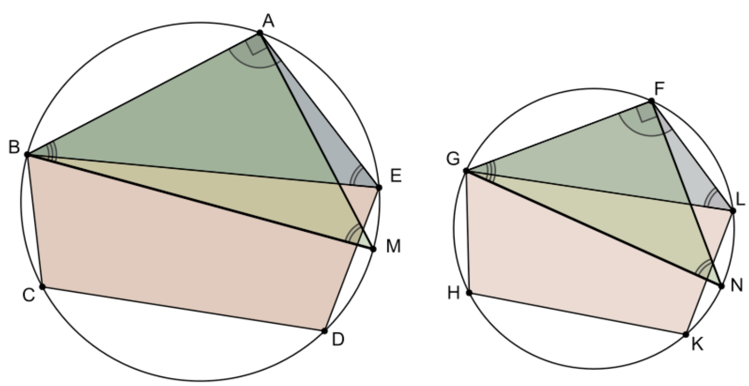
Proposition XII.2

Similar polygons inscribed in circles are to one another as are the squares on their diameters.

Proof: We wish to show that $ABCDE : FGHKL = BM^2 : GN^2$.



Draw BE, AM, GL, FN. (Post. 1)
Since the polygons are similar, we know that $\angle A = \angle F$ and $AB : AE = FG : FL$. (Def. VI.1)
Thus $\triangle ABE \sim \triangle FGL$ (VI.6).
Therefore $\angle AEB = \angle FLG$.
We also have $\angle AEB = \angle AMB$ and $\angle FLG = \angle FNG$ (III.27), so $\angle AMB = \angle FNG$ (c.n.1).

Now $\angle BAM = \lrcorner$ and $\angle GFN = \lrcorner$ (III.31), so $\angle ABM = \angle FGN$ (I.31).
Thus $\triangle ABM \sim \triangle FGN$, so $BM : GN = BA : GF$ (VI.4).
But $BM^2 : GN^2 = (BM : GN)^2$, and $ABCDE : FGHKL = (BA : GF)^2$ (VI.20).
We conclude that $ABCDE : FGHKL = BM^2 : GN^2$ (c.n.1).

Q.E.D.