Practice Problems for Note 2

1. Compute the line integral $\int_{\mathbf{c}} x\, dy$ over the counterclockwise triangle with vertices $(0,0),\ (a,b),\ (c,d)$.

Answer: $(1/2)(ad - bc)$.

2. Compute $\int_{\mathbf{c}} x\, dy$ over the counter-clockwise ellipse with equation

$$\frac{(x-c)^2}{a^2} + \frac{(y-d)^2}{b^2} = 1.$$

3. Determine which of the following vector fields is conservative, and find a potential function for those which are conservative.

a) $\mathbf{F} = (x^2, y^2)$
b) $\mathbf{F} = (y^2, x^2)$
c) $\mathbf{F} = (x + 2y + 1, x - 2y + 1)$
d) $\mathbf{F} = (x + 2y + 1, 2x - y + 1)$

4. Calculate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{c}$ for

a) $\mathbf{F} = (x + 2y + 1, 2x - y + 1), \quad \mathbf{c}(t) = (t, t^2),\ 0 \le t \le 1.$
b) $\mathbf{F} = (\cos x, \sin y), \quad \mathbf{c}(t) = (\cos t, \sin t),\ 0 \le t \le 2\pi.$
c) $\mathbf{F} = (y^2, x^2), \quad \mathbf{c}(t)$ is the line segment from $(1,0)$ to $(0,2)$.
d) $\mathbf{F} = (ye^x, xe^y), \quad \mathbf{c}(t)$ is the line segment from $(0,0)$ to $(1,2)$.
e) $\mathbf{F} = (x + 2y + 1, 2x - y + 1), \quad \mathbf{c}(t)$ is a squiggly path from $(0,0)$ to $(1,1)$.

5. Calculate $\int_{\mathbf{c}} \mathbf{F} \cdot \mathbf{T}\, ds$ and $\int_{\mathbf{c}} \mathbf{F} \cdot \mathbf{N}\, ds$, where $F = (-y, x)$, and $\mathbf{c}$ is the line segment from (a,b) to (c,d).

(Answers: $ad - bc$ and $\frac{1}{2}(c^2 + d^2 - a^2 - b^2)$.)

6. Calculate $\int_{\mathbf{c}} \mathbf{F} \cdot \mathbf{T}\, ds$ and $\int_{\mathbf{c}} \mathbf{F} \cdot \mathbf{N}\, ds$, where $F = (x^4 - 6x^2y^2 + y^4, 4xy^3 - 4x^3y)$, and $\mathbf{c}$ is a circle centered at (a,b). (Hint: Compute $Q_x - P_y$ and $P_x + Q_y$.)

7. Draw the curve $\mathbf{c}(t) = (e^{-t}\cos t, e^{-t}\sin t)$ for $0 \le t \le \infty$ and compute its length. (An infinite curve can have finite length.) Then compute the average of the function $f(x,y) = x$ over the part of $\mathbf{c}(t)$ given by $0 \le t \le 2\pi$.

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