Notes 3,4,5 EXTRA PROBLEMS

1. Compute the double integrals $\iint_R f\ dR$, where R is the square $0 \le x \le \pi,\ 0 \le y \le \pi$, for the following functions $f(x,y)$

a) $f(x,y) = xy$

b) $f(x,y) = x^3 - 3xy^2e^y$.

c) $f(x,y) = \sinh(x)\cos(y)$

2. Let R be the disk of radius a centered at (x_0, y_0). Compute $\iint_R f\ dR$ for the following functions $f(x,y)$.

a) $f(x,y) = x^2$.

b) $f(x,y) = xy$

c) $f(x,y) = x^4 - 6x^2y^2$ (Use the Jacobian and exercise 3.5 in Note 4.)

3. Compute

$$\iint_R e^{-x^2-y^2}\ dR$$

where R is the disk of radius a centered at $(0,0)$.

4. Compute the average of $x^{2n}y^{2m}$ over the unit circle. (This is very easy, if you think about the double integral over the unit disk, which you have already done.)

5. Compute the integral of $f(x,y) = x + 2y - 3$ over the square R with vertices $(1,1),\ (2,2),\ (1,3),\ (0,2)$, in THREE ways:

i) Find the area and center of mass.

ii) Find P and Q so that $Q_x - P_y = x + 2y - 3$, and use Green's theorem.

iii) Find a map $R(u,v)$ from the nice square $0 \le x, y \le 1$ to R and use the Jacobian.

6. Compute the integral of x^{100} over the unit disk centered at $(0,0)$.

7. Find the measures of the spheres S^{n-1} for $1 \le n \le 10$.

8. Calculate the derivative with respect to a of the measure of the ball $B^n(a)$. Then set $a = 1$. What do you get? (If you want, just do it for $1 \le n \le 10$.)

9. Use Green's Theorem to compute the line integral $\oint_{\mathbf{c}} y\ dx + x\ dy$, where $\mathbf{c}$ is the counterclockwise hexagon whose six vertices are $(\cos\frac{2k\pi}{6}, \sin\frac{2k\pi}{6})$, $k = 0,1,2,3,4,5$. (You'll need to find the area of the hexagon. A hexagon is made of triangles. We know the area of a triangle (extra problems for Note 2).)

10. Use line integrals to find the center of mass of the triangle with vertices $(0,0),\ (a,b),\ (c,d)$.

11. Here is a new mapping to think about.

$$R(u,v) = (u\cosh v, u\sinh v).$$

It is the hyperbolic analogue of polar coordinates. Find the Jacobian of $R(u,v)$. Draw the images under $R(u,v)$ of horizontal and vertical lines in the u,v plane. Just consider $0 \le u \le 1,\ 0 \le v \le \infty$.