Kidney Exchange

Tayfun Sönmez

16th Jerusalem Summer School in Economic Theory
"Matching, Auctions, and Market Design"

Based on:

- Alvin Roth, Tayfun Sönmez & Utku Ünver, "Kidney Exchange," *Quarterly Journal of Economics* 119,2: 457-488, May 2004.

- Alvin Roth, Tayfun Sönmez & Utku Ünver, "Pairwise Kidney Exchange," *Journal of Economic Theory*, in press.

- Alvin Roth, Tayfun Sönmez & Utku Ünver, "Efficient Kidney Exchange: Coincidence of Wants in a Structured Market," NBER working paper.

Practical Market/Mechanism Design

- In the past decade economists became increasingly more involved in the design of markets/practical mechanisms such as:
  - labor market clearinghouses (Roth *Econometrica* 2002),
  - power markets (Wilson *Econometrica* 2002),
  - auctions (Milgrom *Cambridge U. Press* 2004), and
  - school choice (Abdulkadiroğlu & Sönmez *AER* 2003).
- Designs which are fortunate to be adopted often involve incremental changes to existing practices, because
  - it is easier to get incremental changes adopted rather than radical departures from existing practices, and
  - there may be lots of *hidden* institutional constraints.

Kidney Transplants

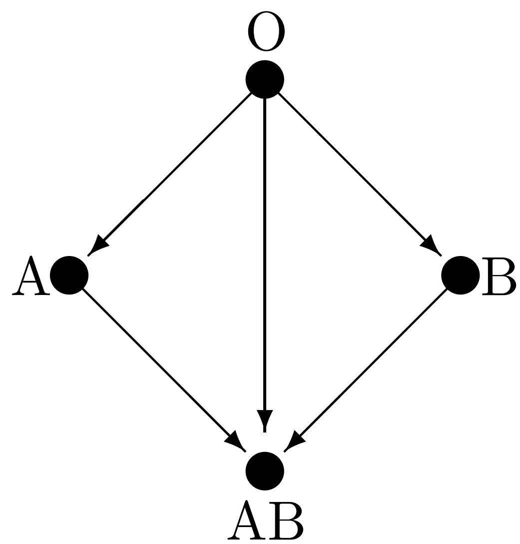
- There are over 60,000 patients on the waiting list for cadaver kidneys in the U.S.
- In 2003 there were over 8,600 transplants of cadaver kidneys performed in the U.S.
- In the same year, about 3,400 patients died while on the waiting list.
- In 2003 there were also over 6,400 transplants of kidneys from *living* donors, a number that has been increasing steadily from year to year.

Institutional Constraint: No Money

- The shortage of kidney increases by about 3,500 kidneys each year in the U.S.
- The 1984 National Organ Transplant Act (and in many states the Uniform Anatomical Gift Act) makes paying for an organ for transplantation a felony.
- There is a rich literature on whether the ban on buying and selling of kidneys be repealed (ex: Becker & Elias 2002).

Medical Constraint: Blood Type Compatibility

- There are four blood types: A, B, AB and O.
- In the absence of other complications:



  * Type O kidneys can be transplanted into any patient;
  * type A kidneys can be transplanted into type A or type AB patients;
  * type B kidneys can be transplanted into type B or type AB patients; and
  * type AB kidneys can only be transplanted into type AB patients.

Medical Constraint: Tissue Type Compatibility

- Tissue type or HLA type: Combination of six proteins.
- Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against donor HLA. If present (*positive crossmatch*), the transplantation cannot be carried out.

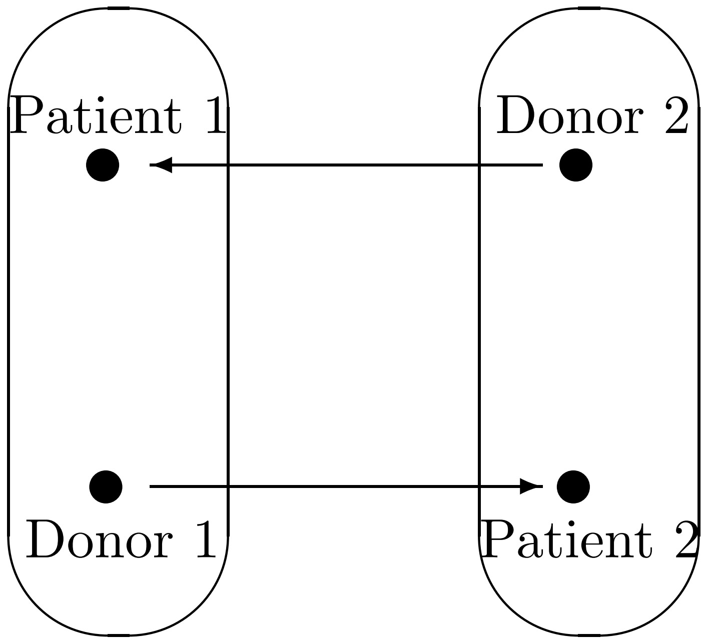
Cadaver Kidneys

- U.S. Congress views cadaveric kidneys offered for transplantation as a national resource, and the National Organ Transplant Act of 1984 established the Organ Procurement and Transplantation Network (OPTN).
- Run by the United Network for Organ Sharing (UNOS), it has developed a centralized priority mechanism for the allocation of cadaveric kidneys.

Live Donor Transplants: Much Less Organized

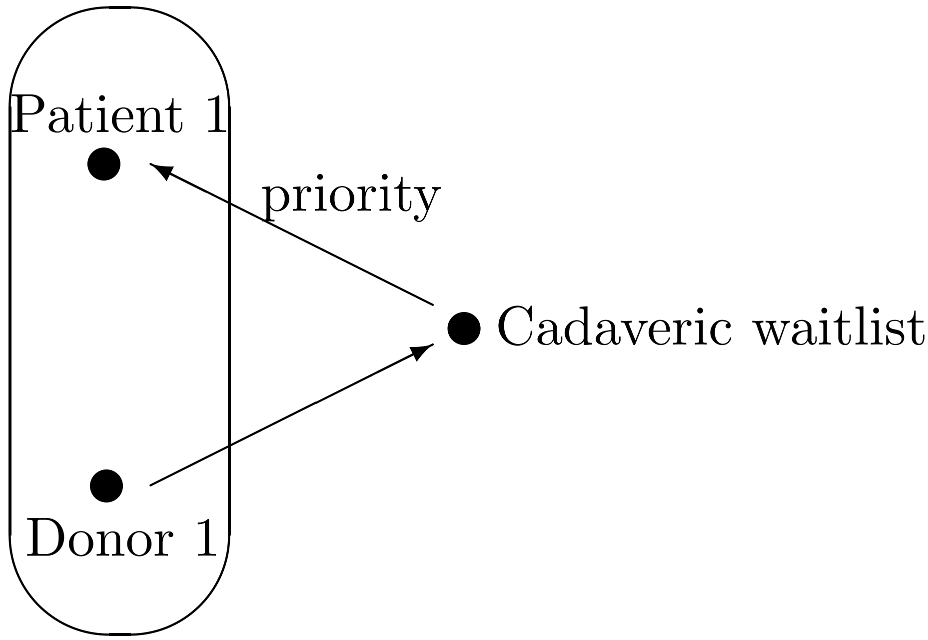
- A patient identifies a willing donor and, if the transplant is feasible, it is carried out.
- Otherwise, the patient remains on the queue for a cadaver kidney, while the donor returns home.
- Recently, however, in a small number of cases, additional possibilities have been utilized:
  - *Paired exchanges*: Exchanges between two incompatible couples.
  - *Indirect exchanges*: An exchange between an incompatible couple and the cadaver queue.

Paired Exchange



- Still very rare: Since 2000 five paired exchanges in New England.
- In 2000 the transplantation community issued a *consensus statement* indicating paired exchange to be "ethically acceptable."
- *Incentives Constraint*: All four operations shall be carried out simultaneously!

Indirect Exchange

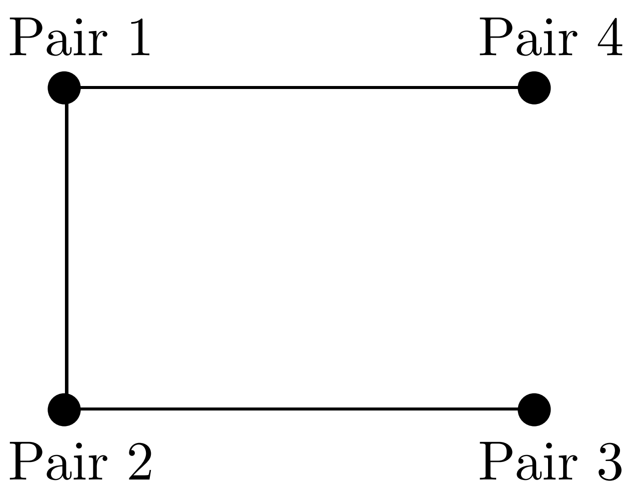


- Widespread concern in transplantation community: Indirect exchanges can harm type O patients with no living donors.
- Nevertheless, many transplant centers have started pilot indirect exchange programs since 2000 (ex: Johns Hopkins Comprehensive Transplant Center, New England Medical Center.)

Our Focus: Design and Potential Benefits of a More Comprehensive Mechanism

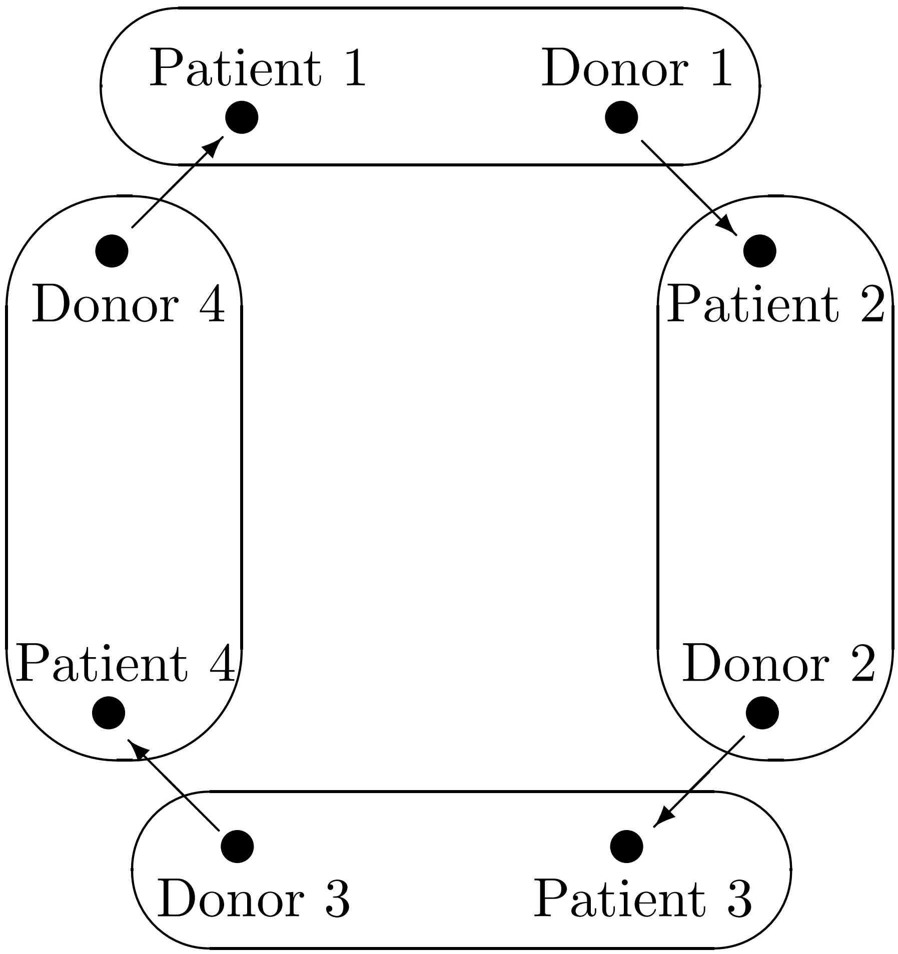
- Building on the existing practices in kidney transplantation, we consider how an efficient and incentive-compatible system of exchanges might be organized, and what its welfare implications might be.

Value-Added of a Structured Exchange?



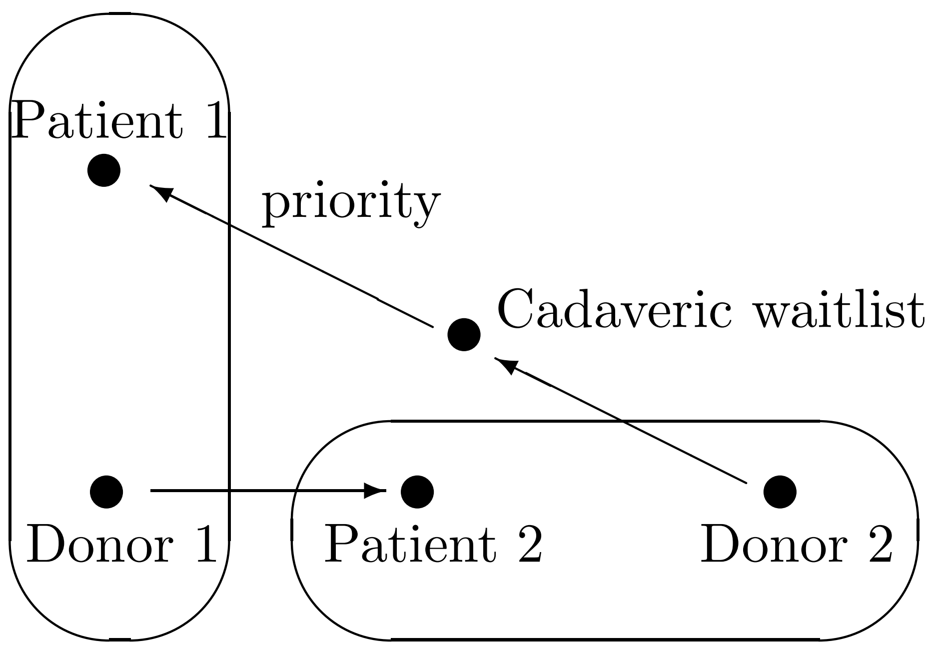
- Even in the absence of more elaborate exchanges, merely organizing the paired-exchanges may result in increased efficiency.

Value-Added of a Structured Exchange?



- Additional live-donor transplants may be possible through three-way, four-way, . . . , exchanges.

Value-Added of a Structured Exchange?



- Additional benefits from more elaborate indirect exchanges.

Modeling Kidney Exchange

- Three assumptions are important in studying kidney exchange:
  1. Patient preferences over compatible kidneys.
     (a) The "European" view: The graft survival rate increases as the tissue type mismatch decreases (Opelz *Transplantation* 1997).
     (b) The "American" view: The graft survival rate is the same for all compatible kidneys (Gjertson & Cecka *Kidney International* 2000, Delmonico *NEJM* 2004).
  2. The number of simultaneous transplants.
  3. Feasibility of indirect exchanges.

Model 1: Roth, Sönmez & Ünver, "Kidney Exchange," *QJE* 2004.

- *Assumption 1*. The graft survival rate increases as the tissue type mismatch decreases (i.e. the European view).
- *Assumption 2*. There is no constraint on the number of transplants that can be simultaneously carried out.
- *Assumption 3*. Indirect exchanges are feasible.

Related Literature:
House Allocation

- Shapley & Scarf (*J. Math. Econ* 1974)
- Roth & Postlewaite (*J. Math. Econ* 1977)
- Roth (*Economics Letters* 1982)
- Abdulkadiroğlu & Sönmez (*JET* 1999)

Kidney Exchange Problem

(k_i, t_i): A donor-patient pair.

K_i: Living donor kidneys compatible with patient t_i.

w: Priority in the waitlist in exchange for a live kidney.

P_i: Strict preferences over $K_i \bigcup \{k_i, w\}$.

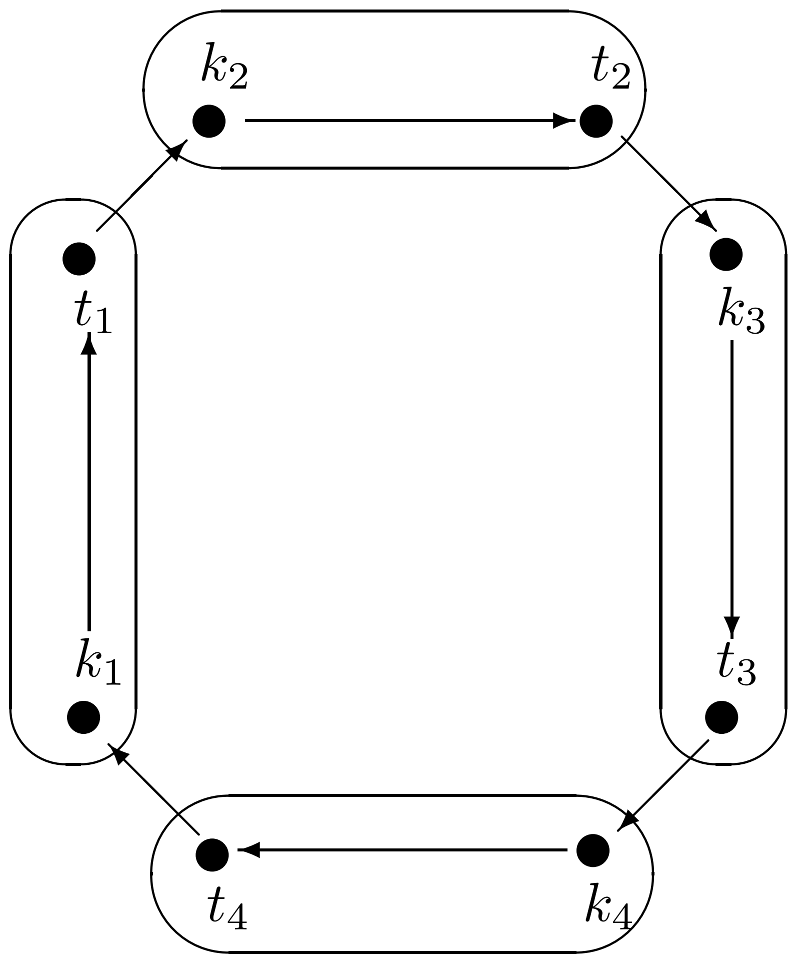
The Matching

- The outcome: *Matching* of kidneys/waitlist option to patients such that:
  1. each patient is either assigned a compatible kidney, or her donor's kidney, or the waitlist option, and
  2. no kidney can be assigned to more than one patient although the waitlist option w can be assigned to several patients.

TTCC Mechanism

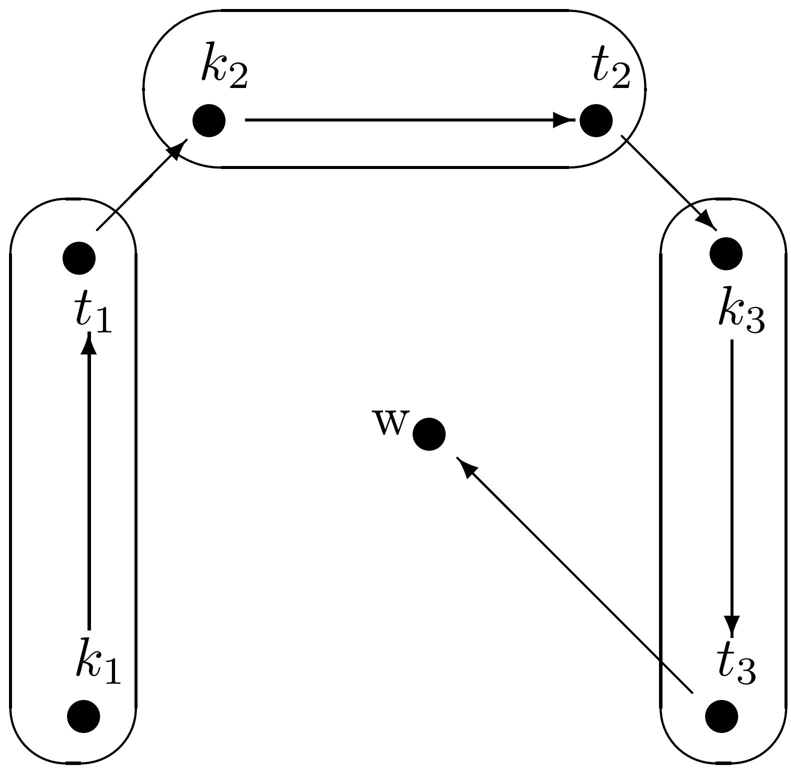
- A *kidney exchange mechanism* is a systematic procedure to select a matching for each kidney exchange problem.
- *Top Trading Cycles and Chains* mechanism relies on an algorithm consisting of several rounds. In each round:
  - * each patient t_i points either towards a kidney in $K_i \cup \{k_i\}$ or towards w, and
  - * each kidney k_i points to its paired recipient t_i.

Cycles



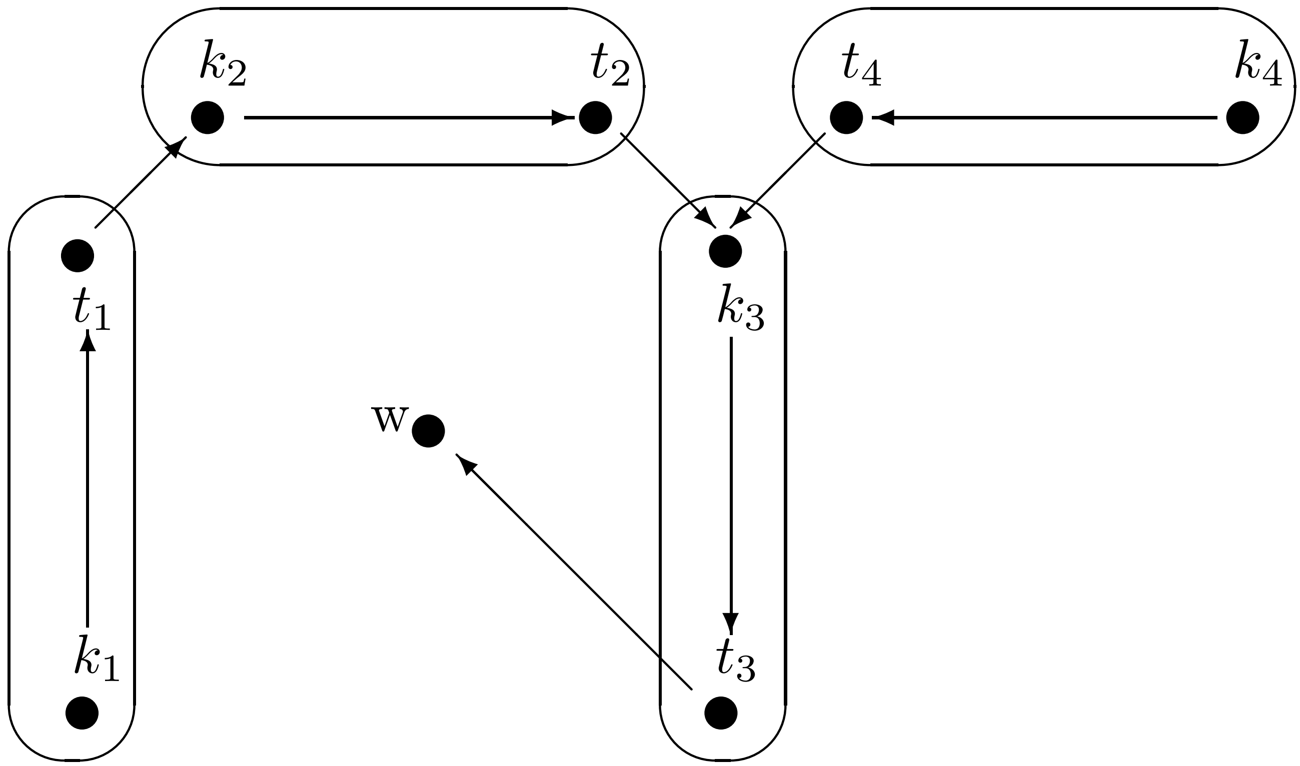
- Cycles are associated with direct exchanges.
- No two cycles can intersect.

w-chains



- w-chains are associated with indirect exchanges.

w-chains can intersect!



- Unlike in a cycle, a kidney-patient pair can be part of several w-chains.
- **Practical Possibility**: Choosing among w-chains with a *chain selection rule*.
- **Remark**: Choice of the chain selection rule has efficiency and incentive-compatibility implications.

- *Lemma 1*. Consider a graph in which both the patient and the kidney of each pair are distinct nodes as is the waitlist option w. Suppose each patient points either towards a kidney or w, and each kidney points to its paired recipient. Then either there exists a cycle or each pair initiates a w-chain.

The Exchange

Fix a chain selection rule. The TTCC mechanism determines the exchanges as follows:

1. Initially all kidneys are available and all agents are active. At each stage
   * each remaining *active* patient t_i points to the best remaining unassigned kidney or to the waitlist option w, whichever is more preferred,
   * each remaining passive patient continues to point to his assignment, and
   * each remaining kidney k_i points to its paired recipient t_i.

2. *By Lemma 1, there is either a cycle, or a w-chain, or both.*
   (a) Proceed to Step 3 if there are no cycles. Otherwise locate each cycle and carry out the corresponding exchange. Remove all patients in a cycle together with their assignments.
   (b) Each remaining patient points to its top choice among remaining choices and each kidney points to its paired recipient. Proceed to Step 3 if there are no cycles. Otherwise locate all cycles, carry out the corresponding exchanges, and remove them.

   Repeat this step until no cycle exists.

3. If there are no pairs left, then we are done. Otherwise by Lemma 1, *each* remaining pair initiates a w-chain.

   Select *only one* of the chains with the *chain selection rule*. The assignment is *final* for the patients in the selected w-chain. In addition to selecting a w-chain, the chain selection rule also determines

   (a) whether the selected w-chain is removed, or

   (b) the selected w-chain remains in the procedure although each patient in it is passive henceforth.

4. Each time a w-chain is selected, a new series of cycles may form. Repeat Steps 2 and 3 with the remaining active patients and unassigned kidneys until no patient is left.

Plausible Chain Selection Rules

a. Choose the longest w-chain and remove it.

b. Choose the longest w-chain and keep it.

c. Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and remove it.

d. Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair and keep it.

Efficiency

- *Theorem 1.* Consider a chain selection rule where any w-chain selected at a non-terminal round remains in the procedure and thus the kidney at its tail remains available for the next round. The TTCC mechanism, implemented with any such chain selection rule, is efficient.
- Two examples:
  1. the rule that chooses the longest w-chain and keeps it, and
  2. the priority based rule that selects the w-chain starting with the highest priority pair and keeps it.

Incentives

- *Theorem 2*. Consider the priority based chain selection rules c and d. The TTCC mechanism, implemented with either of these chain selection rules is strategy-proof.
- *Corollary*. The TTCC mechanism, implemented chain selection rule d is efficient and strategy-proof.

Model 2: Roth, Sönmez & Ünver, "Pairwise Kidney Exchange," *JET*, in press.

- *Assumption 1*. The graft survival rate is the same for all compatible kidneys (i.e. the American view).
- *Assumption 2*. No more than two transplants can be carried out simultaneously.
- *Assumption 3*. Indirect exchanges are not allowed.

Related Literature

Operations Research

- Gallai (*MTAMKIK* 1963, 1964)
- Edmonds (*Can. J. of Math.* 1965)

Economics

- Bogomolnaia & Moulin (*Econometrica* 2004)

Pairwise Kidney Exchange Problem

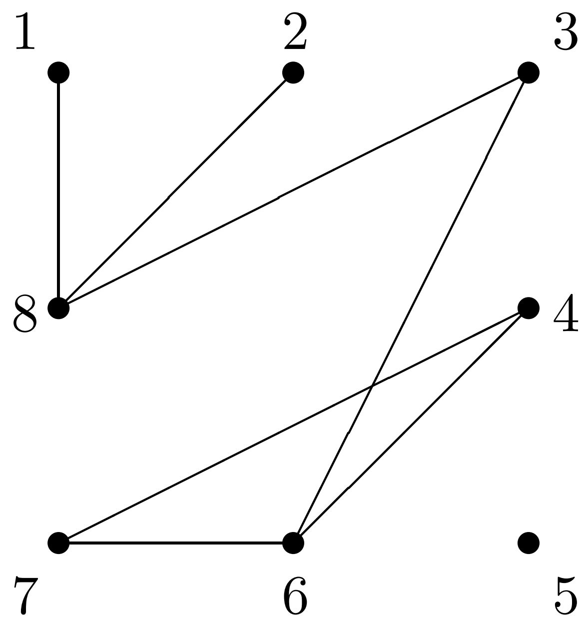
N: Set of patients (each with one or more incompatible donors).

$r_{i,j}$: Indicates mutual compatibility between patients i and j ($r_{i,j} = 1$ if compatible, $r_{i,j} = 0$ otherwise).

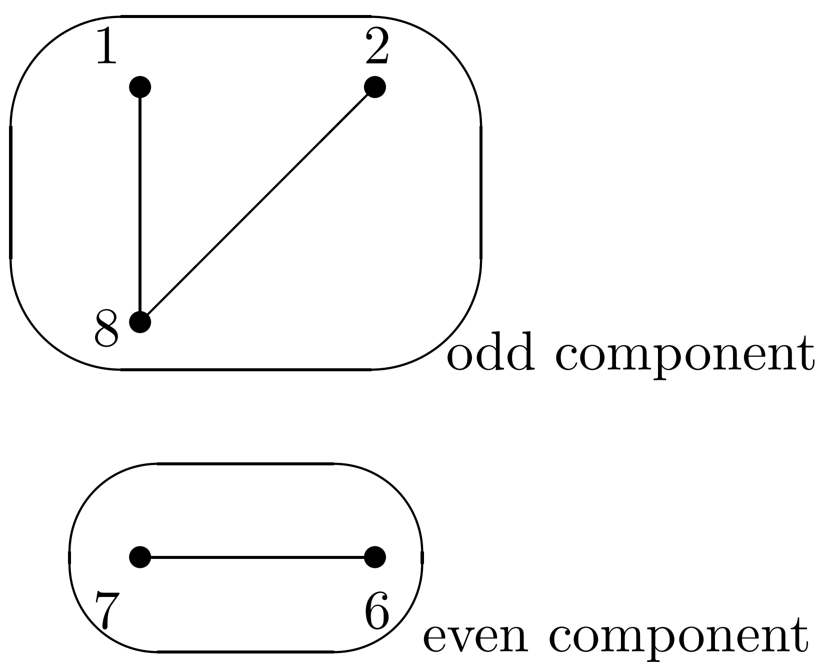
R: Mutual compatibility matrix for all patient pairs.

Representation with Graphs

Problem:



Subproblem for {1,2,6,7,8}:



The (Pairwise) Matching

- The deterministic outcome: A function $\mu : N \to N$ such that
  1. if $\mu(i) = j$ then $\mu(j) = i$ (i.e. only pairwise exchanges are possible), and
  2. if $\mu(i) = j$ then $r_{i,j} = 1$ unless $i = j$ (i.e. only mutually beneficial exchanges are possible).

Lotteries

- The stochastic outcome: A *lottery* λ among matchings.
- $a_{i,j}(\lambda)$: The probability that patients i and j are matched with each other under λ.

  $u_i(\lambda)$: Utility of patient i under λ. ($u_i(\lambda) = \sum_{j \in N \setminus \{i\}} a_{i,j}(\lambda)$ specifies the odds for a transplant.)

Efficiency

- A matching is *Pareto efficient* if there is no other matching that makes every patient weakly better off and some patient strictly better off.
- A lottery is *ex-post efficient* if it gives positive weight to only Pareto efficient matchings.
- A lottery is *ex-ante efficient* if there is no other lottery that makes every patient weakly better off and some patient strictly better off.

Efficient Pairwise Exchange

- *Lemma 2*. The same number of patients are matched at each Pareto efficient matching.
- **Remark**: Lemma 2 would not hold if exchange was possible among three or more patients.

Priority Mechanisms

For a given priority ordering of $|N|$ patients, the induced *priority mechanism* selects a matching in the set $\mathcal{E}^{|N|}$ where $\mathcal{E}^{|N|}$ is constructed in $|N|+1$ steps as follows:

- $\mathcal{E}^0$ is the set of all matchings.
- $\mathcal{E}^1 = \mathcal{E}^0$ if there is no matching that matches the highest priority patient, and it is the set of all matchings which matches the highest priority patient otherwise.

For each $k \leq |N|$,

- $\mathcal{E}^k = \mathcal{E}^{k-1}$ if there is no matching in $\mathcal{E}^{k-1}$ that matches the k^{th} priority patient, and it is the set of all matchings in $\mathcal{E}^{k-1}$ which matches the k^{th} priority patient otherwise.

Theorem 3. The priority mechanism is not only Pareto efficient but also it makes it a dominant strategy for a patient to reveal both

a. her full set of compatible kidneys, and

b. her full set of available donors.

Ex-ante and Ex-post Efficiency

- In general

$$\text{Ex-ante Efficiency} \;\Rightarrow \text{Ex-post Efficiency}$$

- But here

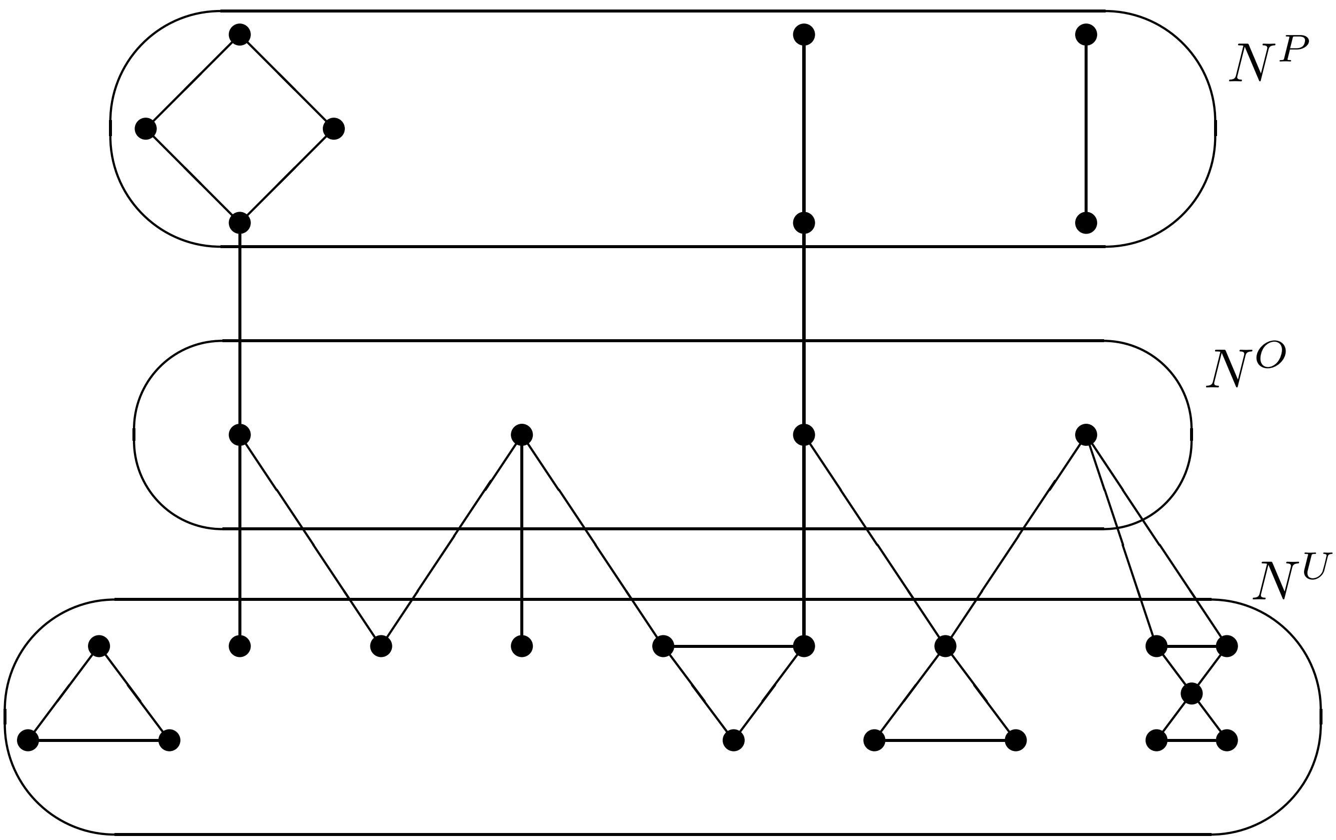
$$\text{Ex-ante Efficiency} \quad \Longleftrightarrow \quad \text{Ex-post Efficiency}$$

Underdemanded Patients N^U
Overdemanded Patients N^O
Perfectly-matched Patients N^P

N^U: Patients unmatched at least at *some* efficient matching.

N^O: "Neighbors" of N^U.

N^P: Others.


N^P
N^O
N^U

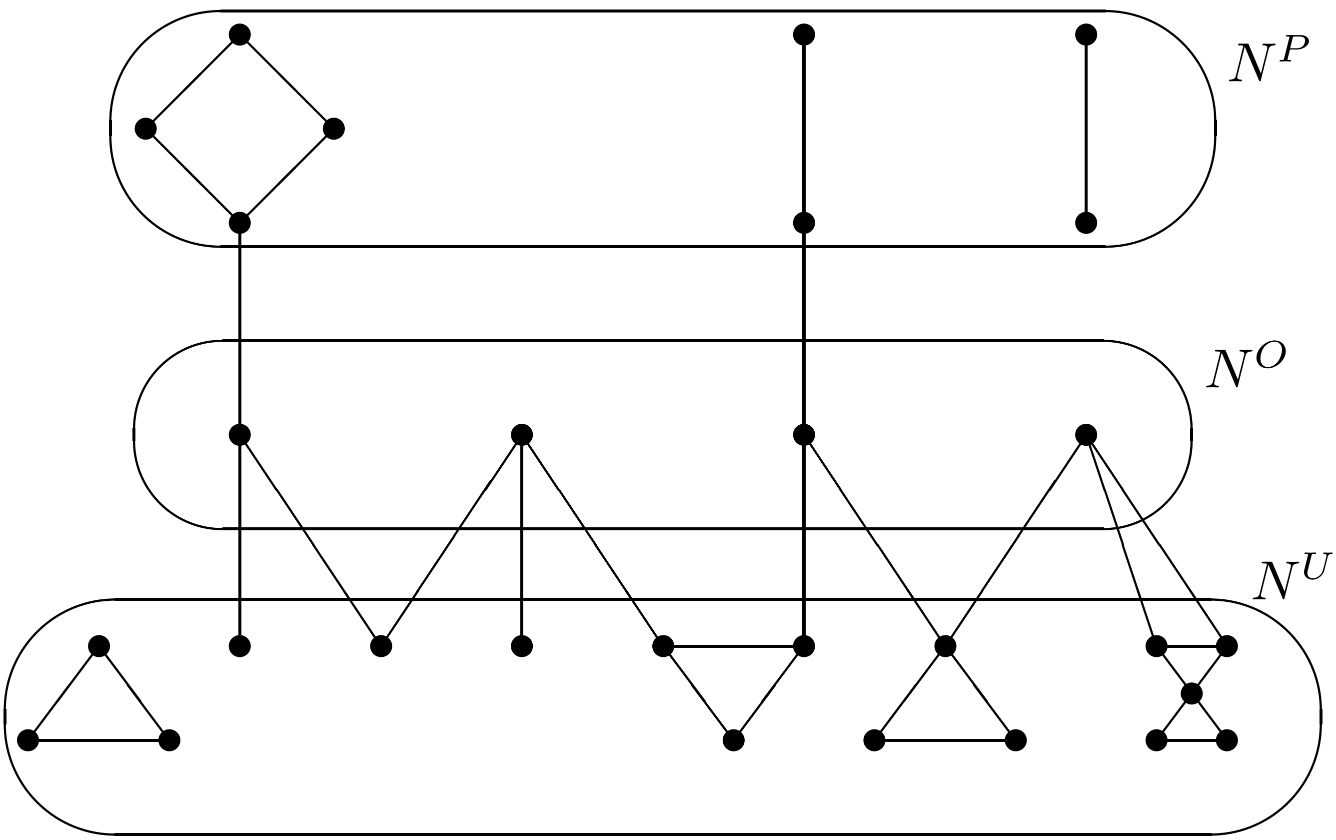
Gallai-Edmonds Decomposition Lemma

Let μ be *any* Pareto efficient matching for the original problem (N, R) and (I, R_I) be the subproblem for $I = N \setminus N^O$.

1. Any overdemanded patient is matched with an underdemanded patient under μ.
2. $J \subseteq N^P$ for any even component J of the subproblem (I, R_I) and all patients in J are matched with each other under μ.
3. $J \subseteq N^U$ for any odd component J of the subproblem (I, R_I) and for any patient $i \in J$, it is possible to match all remaining patients with each other under μ. Moreover under μ
   (a) either one patient in J is matched with an overdemanded patient and all others are matched with each other,
   (b) or one patient in J remains unmatched while the others are matched with each other.

Competition Among Odd Components

- $\mathcal{D} = \{D_1, \ldots, D_p\}$: Set of odd components.
- Based on GED Lemma, Pareto efficient matchings each leave unmatched $|\mathcal{D}| - |N^O|$ patients, each one in a distinct odd component.
- Competition at two levels:
  1. Competition among odd components for overdemanded patients.
  2. Competition among members of each odd component that does not secure an overdemanded patient.



N^P
N^O
N^U

Equity

- Utility: The probability of receiving a transplant.
- In this context equilizing utilities as much as possible may be considered very plausible from an equity perspective.

Useful Intellectual Exercise

- Let
  * $\mathcal{J} \subseteq \mathcal{D}$ be an arbitrary set of odd components,
  * $I \subseteq N^O$ be an arbitrary set of overdemanded patients, and
  * $C(\mathcal{J}, I)$ denote the "neighbors" of $\mathcal{J}$ among I.

  Qn: Suppose only overdemanded patients in I are available to be matched with underdemanded patients in $\left|\bigcup_{J \in \mathcal{J}} J\right|$. What is the upper-bound of the utility that can be received by the *least fortunate* patient in $\left|\bigcup_{J \in \mathcal{J}} J\right|$?

- Answer:

$$f(\mathcal{J}, I) = \frac{\left|\bigcup_{J \in \mathcal{J}} J\right| - (|\mathcal{J}| - |C(\mathcal{J}, I)|)}{\left|\bigcup_{J \in \mathcal{J}} J\right|}$$

The Egalitarian Mechanism

- This upper-bound can be received only if:
  1. all underdemanded patients in $\left|\bigcup_{J\in\mathcal{J}} J\right|$ receive the same utility, and
  2. all overdemanded patients in $C(\mathcal{J}, I)$ are committed for patients in $\left|\bigcup_{J\in\mathcal{J}} J\right|$.
- So partition $\mathcal{D}$ as $\mathcal{D}_1, \mathcal{D}_2, \ldots$ and N^O as $N^O_1, N^O_2, \ldots$ as follows:

  Step 1.

  $$\mathcal{D}_1 = \arg\min_{\mathcal{J}\subseteq\mathcal{D}} f\left(\mathcal{J}, N^O\right) \quad \text{and} \quad N^O_1 = C\left(\mathcal{D}_1, N^O\right)$$

  Step k.

  $$\mathcal{D}_k = \arg\min_{\mathcal{J}\subseteq\mathcal{D}\setminus\bigcup_{\ell=1}^{k-1}\mathcal{D}_\ell} f\left(\mathcal{J}, N^O \setminus \bigcup_{\ell=1}^{k-1} N^O_\ell\right)$$

  $$N^O_k = C\left(\mathcal{D}_k, N^O \setminus \bigcup_{\ell=1}^{k-1} N^O_\ell\right)$$

The Egalitarian Utility

- Construct the vector $u^E = (u_i^E)_{i \in N}$ as follows:
  1. For any overdemanded patient and perfectly-matched patient $i \in N \setminus N^U$,
  $$u_i^E = 1.$$
  2. For any underdemanded patient i whose odd component left the above procedure at Step $k(i)$,
  $$u_i^E = f(\mathcal{D}_{k(i)}, N^O_{k(i)}).$$
- *Theorem 4.* The vector u^E is feasible.

  Two major challanges in the proof:
  1. Construction of an allocation matrix that yields the egalitarian utilities.
  2. Construction of a lottery that yields this allocation matrix.

Lorenz Domination

- **Notation:** For any utility profile u, re-order individual utilities in an increasing order $\left(u^{(t)}\right)_{t\in\{1,\ldots,n\}}$ such that

$$u^{(1)} \leq u^{(2)} \leq \cdots \leq u^{(n)}$$

- u *Lorenz dominates* v iff
  1. $\sum_{s=1}^{t} u^{(s)} \geq \sum_{s=1}^{t} v^{(s)}$ for all t, and
  2. $\sum_{s=1}^{t} u^{(s)} > \sum_{s=1}^{t} v^{(s)}$ for some t.

- *Theorem 5.* The utility profile u^E Lorenz dominates any other feasible utility profile (efficient or not).

Efficiency and Incentives

- *Theorem 6.* The egalitarian mechanism is not only Pareto efficient but also it makes it a dominant strategy for a patient to reveal both

  a. her full set of compatible kidneys, and

  b. her full set of available donors.

Simulations

Assuming blood-unrelated pairs:

# Pairs	2-way	2&3-way	2&3&4-way	Unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

Simulations with New England Patient Data

- Data provided by NEOB for recent incompatible pairs: Out of 45 pairs
  * 8 patients matched if only two pair exchanges are allowed,
  * 11 patients matched if two or three pair exchanges are allowed, and
  * 12 patients matched if any size exchange is allowed.

Proposal:

New England Program for Kidney Exchange

Presented to the RTOC

September 20, 2004

(Approved, Started to Collect Patients, First Exchange Pending)

Frank Delmonico (NEOB and MGH)
Al Roth
Susan Saidman (MGH)
Tayfun Sönmez
Utku Ünver

Efficient Kidney Exchange: Coincidence of Wants in a Structured Market

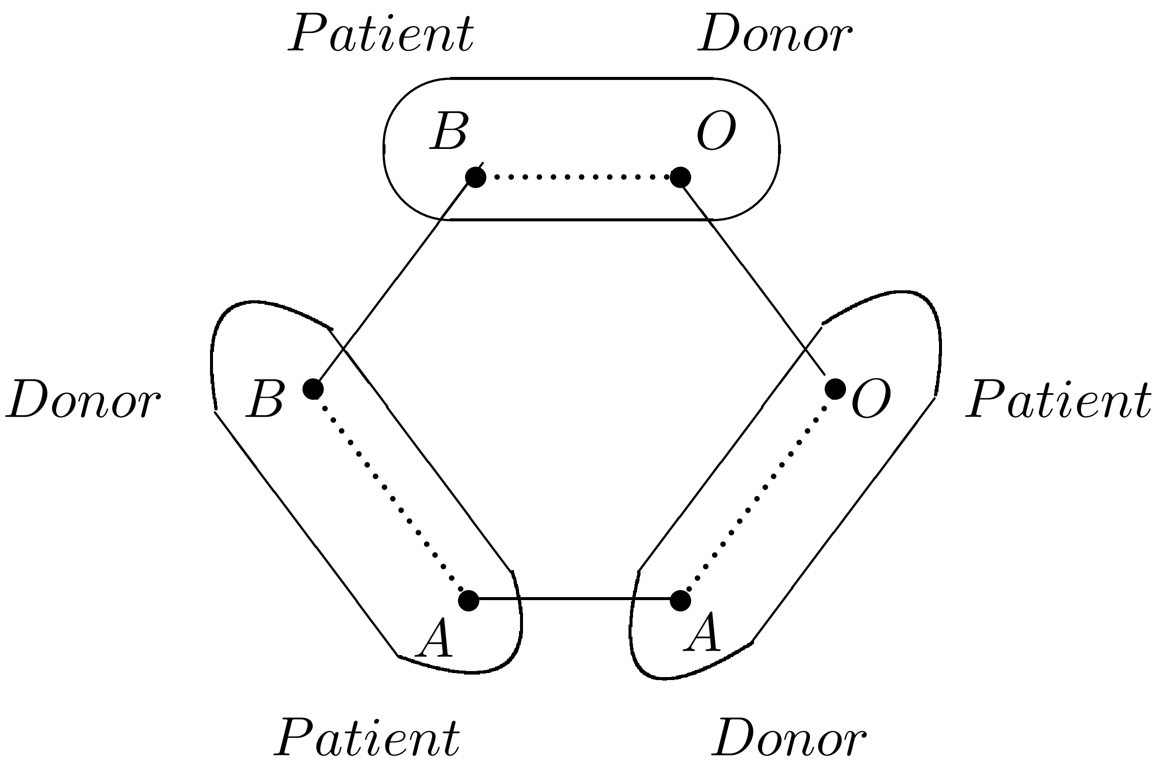
Alvin Roth, Tayfun Sönmez, Utku Ünver

- **Example:** Consider a population of 7 incompatible patient-donor pairs consisting of:
  - O-A, O-B (difficult to match O patients)
  - A-B, A-B, B-A (more A-B than B-A pairs)
  - A-A, (an odd number of A-A pairs)
  - B-O (scarce O donor)
- If only two-way exchanges are possible: 4 transplants

$$(B - O, O - B); (B - A, A - B)$$

- If three-way exchanges are feasible as well: 6 transplants

$$(B - O, O - A, A - B); (B - A, A - A, A - B)$$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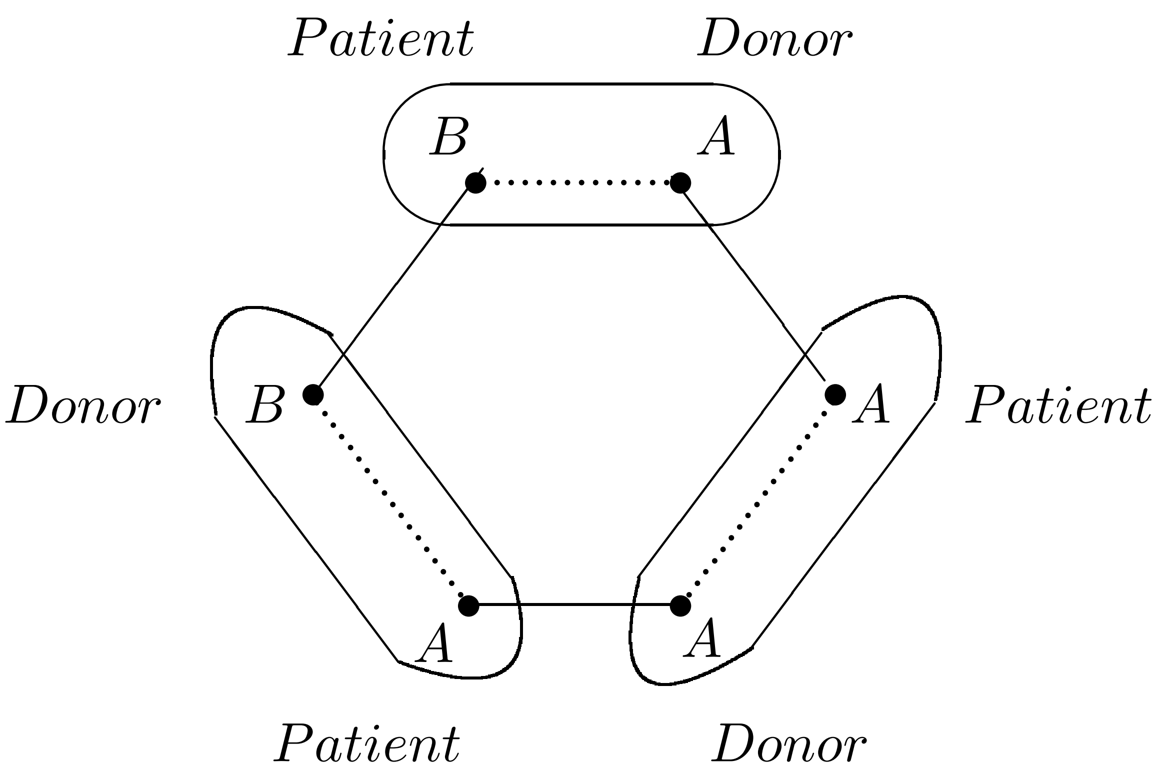
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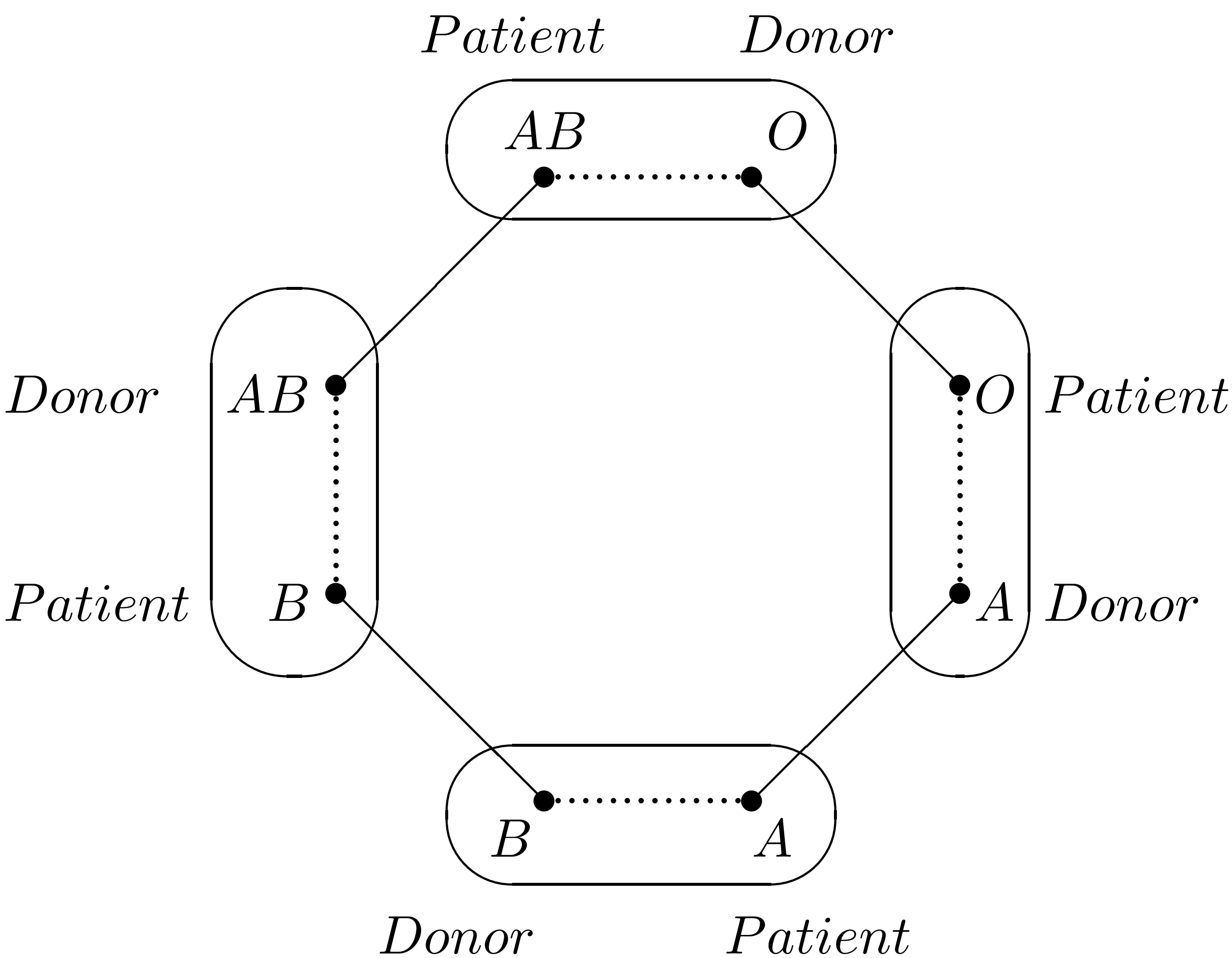
4-Way Exchanges Add Much Less



Conclusion

- Opportunity to use tools of market/mechanism design in the health sector.
- In the present context not only economic theory provides guidence to solve an important real-life problem, but also the real-life problem helps advancing economic theory.