Delivery options and convexity in Treasury bond and note futures

Robin Grieves, Alan J. Marcus, Adrian Woodhams

A R T I C L E   I N F O
Article history:
Received 30 March 2009
Received in revised form 26 May 2009
Accepted 23 June 2009
Available online 30 July 2009

JEL classification:
G13

Keywords:
Hedging
Futures
Convexity

A B S T R A C T
Using Treasury bond and note futures to hedge fixed-income portfolios is complicated by the large number of bonds that are eligible to deliver against the contract. Grieves and Marcus [Grieves, R. and A. Marcus. (2005). Delivery options and Treasury bond futures hedge ratios. Journal of Derivatives, 13, 70–76.] show that, in some circumstances, only two bonds—those with the highest and the lowest duration—are relevant for the hedging problem, which makes computation of analytic hedge ratios tractable. We evaluate the empirical efficacy of their two-relevant-bonds model. We compare the maturities of actual cheapest-to-deliver bonds to the prediction of the two-deliverables model and calculate empirical price values of a basis point for Treasury futures contracts to determine whether contract prices display the negative convexity predicted by the model. The model has worked very well for the note contract and very poorly for the bond contract. We show that the difference in model performance is related to the shape of the yield curve.

1. Introduction

Hedging interest-rate risk using Treasury bond (T-bond) futures is complicated by the delivery options built into the contract. Any Treasury bond with maturity or time to first call of at least 15 years is eligible for delivery against the T-bond contract. Similarly, the 10-year Treasury note (T-note) contract may be settled with any note originally issued with 10 years to maturity with remaining maturity of at least six and a half years. The cheapest-to-deliver (CTD) bond or note, in conjunction with the delivery options, determines the price value of a basis point (PVBP) for the contract. The cheapest-to-deliver bond can and does change as the yield curve evolves. That prospect, known at the outset, renders the interest-rate sensitivity of the contract uncertain.

Gay and Manaster (1984) showed that delivery options in futures contracts can be modeled as exchange options. They derived a formula to account for the impact of the option on the futures price (and therefore the hedge ratio) in the special case where there are only two eligible delivery vehicles. Grieves and Marcus (2005) argued that despite the fact that many bonds are typically eligible to settle T-bond and T-note futures, it may be acceptable to act as if only two bonds may in fact be delivered, in particular, the eligible bonds with either the shortest or longest duration. Their conclusion that only two bonds are relevant follows from certain restrictions on the shape and evolution of the yield curve. Using this simplification, they derived a futures hedge ratio as a blend of the hedge ratios that would result from delivering with either of the two relevant bonds.

In this paper, we revisit the hedging problem from an empirical point of view. We calculate the PVBP of Treasury contracts in interest-rate regions where the identity of the ultimate CTD bond is most difficult to predict and compare these empirical values to theoretical ones derived from the two-relevant-bonds-only framework. The simplified model works surprisingly well for 10-year T-note futures, and the negative convexity predicted by that model is detected in empirical results. In contrast, for T-bond contracts, the shape of the yield curve is commonly far enough from Grieves and Marcus’s assumption that the two-relevant-bonds model must be replaced by numerical-method models that allow for the full set of eligible bonds. Our results shed light on the conditions in which the two-deliverables model is adequate to calculate hedge ratios.

The next section briefly reviews the hedging problem and presents the key results from the Grieves and Marcus two-deliverables model. Section 3 presents empirical estimates of contract PVBP and compares them to the simplified theoretical model. Section 4 concludes.

2. The hedging problem

A fixed-income manager seeking to hedge interest-rate exposure can sell T-bond futures contracts. The risk-minimizing number of contracts is obtained by dividing the PVBP of the underlying cash position (i.e., the change in dollar value resulting from a 1 basis point change in yield) by the PVBP of a futures contract.
If the identity of the CTD bond could be determined in advance, then the PVBP of the contract would be easy to compute as the converted, forward PVBP of $100,000 par of that bond. The conversion factor used to adjust for the relative value of each eligible delivery bond is the price (as a multiple of par value) the bond would have if the yield curve were flat at 6%. However, because actual yield curves depart from this simple case, the ratio of market values to conversion factors varies across bonds. The cheapest-to-deliver issue at contract maturity has the lowest ratio of price-to-conversion factor. Before maturity, the most likely CTD is considered to be the bond with the highest implied repo rate. However, as the yield curve evolves, the cheapest-to-deliver bond may change, so its ultimate identity is currently unknown.

Grieves and Marcus obtain the analytic simplification necessary to value the delivery option by assuming that the yield curve is always flat (although its level can evolve stochastically). In this case, only two eligible bonds are relevant: the ones with the longest and shortest duration. For yields above 6%, all issues cheapen compared to the benchmark 6% yield, but the longest-duration deliverable cheapens most, making it CTD. For yields below 6%, all issues richen, but the shortest-duration deliverable richens least, making it CTD.

The gently curved dark line in Fig. 1 depicts the PVBP of T-note contracts calculated by Grieves and Marcus. The exchange option is evaluated using the shortest (six-and-a-half-year) and longest (10-year) issues as the two eligible deliverables. The PVBP derived from their model, which accounts for the delivery option, is a continuous blend of the PVBP of the two relevant deliverables. As the level of the yield curve evolves, the value of the delivery option responds continuously, implying that the futures hedge ratio is also a continuous function of yields.

In contrast, methodologies limited to analysis of the currently cheapest-to-deliver bond imply discrete changes in hedge ratios as yields cross the 6% threshold and the identity of the CTD bond changes. Fig. 1 also presents (in lighter curves) contract PVBPs derived from this sort of single-deliverable model. The discontinuity in the PVBP at a yield of 6% reflects the shift of the CTD bond between the longest- and shortest-duration bonds.

If the ultimate delivery bond were known in advance, the futures price would be a uniformly convex function of interest rates, reflecting the convexity of that bond. The contract PVBP would therefore decline monotonically as yields increased. The increasing slope of portions of the PVBP curve in Fig. 1 therefore indicates negative convexity in the futures price. This is the signature effect of a model in which more than one eligible bond may be optimal to deliver. As in Burghardt, Belton, Lane and Papa (1994), the PVBP of the futures contract exhibits negative convexity for a wide range of yields. The upward-sloping portion of the PVBP curve essentially straddles the 6% threshold corresponding to the coupon of the notional delivery bond. This also is the region in which the ultimate delivery bond is most uncertain, or equivalently, where the delivery option is most valuable and has the greatest impact on futures prices.

The uncertainty in the ultimate delivery vehicle and the associated negative convexity are important features of these contracts, particularly with respect to hedging applications. If there were only one deliverable, then treating futures as levered long positions in the underlying would work fine. One could hedge or build synthetic positions that would behave just like the cash bond. Negative convexity requires monitoring and updating of hedges or synthetics with much more diligence and frequency.

When yield curves are not flat, however, it is conceivable that eligible bonds other than the extreme-duration pair may be CTD, and the relationship between hedge ratios and yields may be far more complex than the one depicted in Fig. 1. In particular, consider the potential effect of a non-flat yield curve. Suppose that in a high-yield environment (specifically, a yield curve uniformly above 6%), the yield curve is downward sloping. The high level of the yield curve would tend to favor a long-maturity delivery bond. As noted, longer-duration bonds will cheapen the most as yields rise. But the downward slope of the yield curve implies that yields on shorter-maturity bonds are higher than those on longer-maturity bonds. As prices and yields are inversely related, this implies lower ratios of price-to-conversion factor for the shorter-term bonds, which makes them more attractive as delivery vehicles.

Conversely, suppose yields are uniformly below 6%, but the yield curve is upward sloping. The low level of yields favors short-duration bonds for delivery (as their prices will rise less compared to a flat 6% yield curve scenario), but the positive slope means that longer-duration bonds will have higher yields and thus lower prices, which offsets the level effect. This analysis implies that the efficacy of the two-relevant-bonds model depends crucially on the extent to which high yields are associated with downward-sloping curves and low yields with upward-sloping curves. In the end, the relative impact of slope versus level is an empirical question.

### 3. Empirical results

The flat yield curve model makes two predictions. First, it suggests that only the two extreme-duration bonds will be CTD. Second, it implies that the bond or note futures price will, unlike any of the particular bonds in the deliverable basket, exhibit negative convexity. This section examines the empirical validity of these conclusions.

![Fig. 1. PVBP of 10-year note futures. The price value of a basis point for a T-note futures contract, computed incorporating the value of the delivery option, is given by the curved line. The PVBP is continuous and relatively flat with respect to changes in market yields. The downward-sloping lines are PVBPs derived from the single-deliverable model and are discontinuous at a yield of 6%.](image-url)
We begin by computing a times series of empirical PVBP\(s\) for Treasury note and bond contracts. They are calculated as the dollar change in futures contract value (price change times 100)\(^2\) divided by the basis point change in reference yield. For example, a PVBP of $70 for 10-year note futures implies that futures prices must change by 7 cents ($0.07) for each basis point change in reference yields. For a note selling near par value, this is equivalent to a modified duration of 7.

To assess futures contract hedge ratios at each yield, we need a plot of contract PVBP\(s\) against yield levels. If the two-deliverable-bonds model is adequate, we should observe an upward-sloping segment when yields are near the benchmark 6% coupon rate.

We begin with the March 2000 contracts, which were the first to have a 6\% notional coupon. Prior to 2000, the notional coupon had been 8\%. The benchmark coupon was reset to 6\% in recognition that “normal” market yields had shifted downward in the prior decade. The March 2000 contracts became “front” contracts on December 1, 1999, the first delivery day for the December 1999 contracts. Therefore, our sample starts at the end of November, which coincides with hedgers rolling their hedges into the December contract.

We wish to limit our sample to periods in which the ultimate delivery bond is difficult to predict in advance. We also want our observations to come from a continuous time series with no gaps between observations. We restrict the sample for each contract to periods when bond and note yields were in the vicinity of 6\%. This is the best region to test the two-relevant-bonds simplification because, in that model, if yields depart dramatically from 6\%, uncertainty concerning the delivery bond falls substantially, and the delivery option becomes moot. Because note and bond yields have evolved differently, however, our empirical analysis for each will span different time periods and produce different numbers of observations.

We focus our attention on yields between 5.5\% and 6.5\%. Because yields for both notes and bonds declined to levels far below 6\% beginning in December 2000 for 10-year notes and in August 2002 for bonds, our samples end in those months. Ten-year on-the-run yields fell below 5.5\% in December 2000 and have since remained below that level, except for a few days in late May 2001. Over the 1999–December 2000 period, 10-year on-the-runs traded at yields between 5.33\% and 6.79\%. The highest 10-year yield in this period was 6.79\%, still not far from the notional 6\% threshold.\(^3\)

Our bond series also begins in December 1999. On-the-run 30-year bond yields dropped below 5.5\% in July 2002 and remained there, other than for a few days in May 2004. Therefore, our bond sample period is December 1999 through July 2002. Over this period, 30-year on-the-run yields ranged between 4.79\% and 6.75\%. During the same time span, the CTD yield ranged from 5.18\% to 6.85\%. The numbers typically the last delivery date with upward-sloping yield curves.

We calculate PVBP\(s\) by dividing futures price changes by changes in yields. While one could calculate PVBP\(s\) daily, matching daily price changes to daily yield changes, this procedure would result in highly noisy estimates of PVBP, as any extraneous source of change in reported price (for example, rounding error, non-synchronous prices, or bid–ask spread) would result in large impacts on the ratio \(\Delta P / \Delta Y\). For example, on a day when reference yields changed by 1 basis point (i.e., 0.0001), any reported price change would be multiplied by 1000 to find the PVBP. Any pricing error would thus be magnified 1000 times.

To limit such measurement error, we look at futures contract price changes across more substantial 10 basis point yield changes for the

\(^2\) Ten-year note futures and 30-year bond futures have a notional par value of $100,000, and their prices are quoted in dollars per $100 par value. A 1\% change in prices means $1000 change in contract value.

\(^3\) In some figures below, our plots use the beginning yield of the CTD note rather than the on-the-run yield. The CTD often trades at higher yields than the on-the-runs, so there are a few observations with yields above 6.75\%.

10-year on-the-runs and the 30-year on-the-runs. Over the full sample period for the note contracts, we identify 38 non-overlapping episodes of 10 basis point movements in 10-year on-the-run yields. Similarly, for the bond contract, we identify 85 non-overlapping episodes of 10 basis point movements in 30-year on-the-run yields. Each episode allows us to compute a PVBP.

For each date that begins at the start of a time span with a 10 basis point move in on-the-run yields, we gather pricing data for all issues deliverable into the front contract. We identify the cheapest-to-deliver using standard methodology (see Burghardt et al., 1994). We then use the CTD bond to calculate the PVBP (i.e., \(\Delta P / \Delta Y\)) of the futures price (see Table 1 for the step-by-step procedure). Because 10 basis point yield movements may play out over several days or even weeks, however, futures price movements for each observation have both predictable and unpredictable components.\(^4\) For hedging purposes, we need to strip away the predictable component of price changes and focus on the stochastic portion.

The predictable portion of the price change is due to the convergence between futures and cash prices as contract maturity draws nearer. Fewer days from settlement to delivery means smaller carry embedded in the futures price. Similarly, the embedded option value (basis net of carry) becomes smaller with remaining time to delivery. Of course, the value of carry changes as term repo rates change, and the embedded option values also change as pricing volatility changes.

Because we seek to hedge unexpected changes in price as yields change, we subtract decreases in carry (at constant repo rates) and decreases in embedded options (with linear amortization) from the realized price change over the time period necessary for yields to change by 10 basis points. Although the time spans for analysis are determined by 10 basis point changes in on-the-run yields, we calculate PVBP\(s\) as futures price changes divided by the change in yield of the CTD at the beginning date of each time span. The yield changes are highly correlated, but they are not identical.

Fig. 2 displays the PVBP\(s\) for 10-year note contracts versus the beginning yield for the CTD bond. The observations in Fig. 2 are divided into three groups according to the term to maturity of the CTD. Even though CTD determination is tied to duration, we will couch a substantial portion of our discussion in terms of term to maturity. We are doing this because term to maturity is more intuitive when discussing the deliverable basket and because the mapping from term to maturity to duration for the deliverable basket is highly, if imperfectly, correlated. The number and severity of duration crossings (see Li, Grieves & Griffiths, 2007) is limited. Markers show the third of the observations from which each point comes. Consistent with the Grieves–Marcus (2005) model, the delivery notes in low-yield scenarios were the short-maturity (and short-duration) eligible

\(^4\) The time for 10 basis point changes varies considerably, from one day to more than eight weeks, with an average of 11.4 days for bonds and the same for 10s.

\[
\text{Table 1} \\
\text{Steps in calculating empirical PVBP\(s\) for Treasury futures contracts.}
\]

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Identify relevant time spans. We chose times over which yields for on-the-run 10s and bonds changed by 10 basis points or more.</td>
</tr>
<tr>
<td>2.</td>
<td>Collect futures price data for the front contract at the beginning and the same contract at the end of each time span.</td>
</tr>
<tr>
<td>3.</td>
<td>Collect prices for all issues in the deliverable basket at the beginning of the span.</td>
</tr>
<tr>
<td>4.</td>
<td>Identify the CTD on the beginning date and find its beginning yield to maturity.</td>
</tr>
<tr>
<td>5.</td>
<td>Calculate carry for the CTD from the beginning to the predicted delivery date, typically the last delivery date with upward-sloping yield curves.</td>
</tr>
<tr>
<td>6.</td>
<td>Calculate basis net of carry for the CTD, which is the value of the delivery options.</td>
</tr>
<tr>
<td>7.</td>
<td>Find the ending yield for the original CTD, which might have been replaced as yields changed.</td>
</tr>
<tr>
<td>8.</td>
<td>Calculate the total price change for the futures contract. Reduce that price change for carry over the span and linear amortization of basis net of carry over the span to calculate the “price surprise.”</td>
</tr>
<tr>
<td>9.</td>
<td>Divide the futures price surprise by the CTD yield change to calculate PVBP for that span.</td>
</tr>
</tbody>
</table>
notes, whereas in high-yield scenarios, the long-maturity (long-duration) note was delivered. At yields just above 6%, medium-maturity notes were more commonly delivered. The negative convexity predicted by the two-deliverables model in the interval surrounding 6% is clear. The PVBP generally increases with market yields.

The success of the two-deliverables model for the note contract is also evident in Fig. 3, which superimposes Fig. 2 on top of Fig. 1. The smooth curve is the plot of PVBP as a function of yield from the Grieves–Marcus model, while the points scattered around that curve are empirical observations of PVBP-yield pairs. While there is noise in the empirical estimates, it is clear that the scatter of points follows the curve from the model.

However, the model is far less successful in describing the bond contract. Fig. 4 displays the PVBP for the bond contract versus beginning yield for the CTD. In contrast to Fig. 2 for the note contract, here there is no hint of negative convexity, and the markers for CTD are a complete jumble, with no tendency for the maturity (and duration) of the CTDs to vary systematically with the level of market rates. Here, again, the maturities and durations of the CTDs are closely correlated.

To understand why the results for the note versus bond contracts differ so dramatically, we plot some sample yield curves using the deliverable basket. These samples illustrate the problem. To see the degree to which they are representative, we then display a histogram of the CTD maturities and a scatterplot of CTD term versus beginning yield. We start with the note contract.

Fig. 5 plots yield as a function of maturity for each note in the 10-year deliverable basket in February 2000, a high-yield period with yields all above 6%. The CTD is nearly the longest maturity in the basket. The only two longer issues, the on-the-run 10s and the old on-the-run 10s typically have a repo advantage and thus trade rich, making them less attractive as delivery notes. Fig. 6 plots the 10-year deliverable basket yield curve for December 2000, a low-yield period when yields were all below 6%. As predicted, the shortest-maturity deliverable is CTD. Here, too, on-the-run 10s trade rich in the sector.

The CTD notes in Figs. 5 and 6 generally correspond to the predictions of the two-relevant-bonds model, with the short note being most attractive to deliver in a low-rate environment and the long note in a high-rate environment. Fig. 7 is a histogram of maturities of the CTD note for the 10-year note contract. Consistent with the two-relevant-bonds model, the maturity of the CTD note has a roughly “barbell-shaped” frequency distribution. Thirty-five of the 38 observations are either seven years and under or nearly nine years; very few contracts are settled with intermediate-maturity bonds.

Fig. 8 plots the term to maturity of the CTD note versus beginning yield level. The important information in this scatter diagram is the “gap,” i.e., the absence of CTD notes with intermediate maturities, consistent with the barbell in Fig. 7. Notice that the long-maturity

---

5 The outlier with PVBP of $70 per contract comes from a span when a new on-the-run 10-year note was introduced. The new on-the-run had a lower yield because of its term repo advantage. In turn, that lower yield triggered a 10 basis point movement in on-the-run yields, while the yield change for the CTD over that span was only 3.5 basis points. This observation supports our choice to use larger yield movements rather than smaller ones.
Fig. 6. Yield curve for the deliverable basket for the 10-year note contract in a low-yield month (December 2000).

Fig. 7. Histogram of cheapest-to-deliver maturities for the T-note futures contract.

Fig. 8. Scatter diagram of maturity versus starting yield for CTD note.

Fig. 9. Yield curve for the deliverable basket for the 30-year bond contract in a high-yield month (December 1999).

notes are CTD when yields are high and the short notes are CTD when yields are low.\footnote{As noted earlier, we talk in terms of maturity for convenience. The results are nearly identical when we substitute duration for maturity.} Figs. 7 and 8 both indicate that the implications of the single-month graphs, Figs. 5 and 6, for the choice of the CTD note are representative of a large number of months and not just one-off outliers.

Figs. 9 and 10 repeat this analysis for the T-bond contract. Fig. 9 plots the yield curve for eligible bonds in a high-yield month (December 1999). But the yield curve in this month was also sloping downward, precisely the high-level, negative-slope configuration that is problematic for the two-deliverables model. While high yields tend to favor the long-maturity delivery bond, the fact that long yields were lower than short ones made longer bonds higher priced compared to their conversion factors, and thus less attractive delivery choices. In the end, the CTD bond in this month turns out to have a maturity almost precisely in the middle of the distribution of eligible bonds.

Fig. 10 depicts a yield curve in a low-yield month (March 2001), but one in which the yield curve is hump-shaped. While the effect of a yield curve below 6% would by itself favor the shortest-maturity bond as the delivery vehicle, the (initially) rising yield curve favors somewhat longer-maturity bonds, as their higher yields reduce their prices compared to their conversion factors. Again, the net effect is for the CTD bond to be one of intermediate maturity. The ultimate decline in yields for maturity dates beyond 2005 makes the longest-term eligible bonds doubly unattractive delivery choices. Their long duration makes their prices more responsive to drops in yields below 6%, and the negative slope in this range of the yield curve implies a greater yield difference compared to the benchmark.

Fig. 11 is a histogram of the maturities of the CTDs for the bond contract. In contrast to the note-contract histogram, this frequency distribution is essentially bell-shaped. Nearly all of the observations are intermediate maturities, between 18 years and 24 years, with very few observations from near the extremes of 15 years or 30 years. The bimodal or barbell results for notes suggest that the yield curve was flat in the 7 to 10 year range, at least approximately, whereas the unimodal results for bonds suggest otherwise for the 10 to 30 year range.

Fig. 12 plots the term to maturity of the CTD bond versus its beginning yield level. The important feature of Fig. 12 is the clustering of points in the middle range of maturities. Very few observations are near the extremes, precisely the opposite pattern demonstrated in the corresponding plot for notes (Fig. 8). Here, too, the figure looks nearly identical when duration instead of maturity is on the vertical axis. As was true for notes, Figs. 11 and 12 also signify that the illustrative single-month graphs, Figs. 8 and 9, are in fact representative of many months.

The difference in results between the note and bond contracts is not intrinsic to their differing maturities. Instead, it is due to the interplay between the level and the slope of the yield curve. In our sample period, downward-sloping yield curves tended to correspond to high-yield periods, and vice versa, at least for the long-maturity segment of the yield curve. This was not true for the shorter end of the curve. Thus, the two-relevant-bonds model worked quite well for the note contract and quite poorly for the bond contract.
The empirical PVBPs reveal that the shape of the yield curve seems quite as important as its level in the determination of the CTD bond. Given the potential variety of curve shapes, the nearly universal usage of numerical methods models to determine hedge ratios is easily understood.

Is there any reason to expect different results for notes and bonds? Ilmanen (1995) explains humped yield curves with “convexity bias.” As yield volatility increases, convexity becomes more valuable because it enhances fixed-income returns. But, convexity is much more important in the bond sector, and in the longest-maturity bonds, than in the 10-year sector. We expect to find humped yield curves—highest yields at intermediate bond maturities, say 22 to 24 years—in the bond sector, but not within six and a half to 10 years.

Another possibility for the divergent results would follow from the notion of a “natural” or long-run equilibrium interest rate (for example, 6%, consistent with the Chicago Board of Trade’s contract benchmark). Given a mean-reverting rate, low-yield environments will tend to exhibit upward slopes, while high-yield environments will tend to exhibit downward slopes. This is precisely the pattern least conducive to the two-relevant-bonds simplification. Moreover, the range of eligible maturities for the note contract is fairly narrow, only six and a half to 10 years. In contrast, bonds with maturities of 15 to 30 years may be used to settle the bond contract. The impact of a non-flat yield curve can be a far more telling impact across a spread of 15 years.

Figs. 13 and 14 provide some evidence on this last point. Fig. 13 plots modified duration against maturity for the CTD note for each contract month in the sample. The absence of intermediate maturities is plainly evident here, but more to the current point is the nearly precise relationship between duration and maturity. This suggests that the yields across notes must have been relatively uniform, for highly variable yields would have added noise to the duration-versus-maturity relationship. In contrast, Fig. 14, which presents the corresponding plot for the bond contract, exhibits considerable scatter in the relationship, consistent with the hypothesis that the bond contract was subject to far more variation in yields across eligible bonds.

4. Conclusion

Using Treasury bond and note futures to hedge fixed-income portfolios is complicated by the large number of bonds that are eligible to deliver against the contract. Grieves and Marcus (2005)
show that, in some circumstances, only two bonds—those with the highest and the lowest duration—are relevant for the hedging problem, which makes computation of analytic hedge ratios tractable.

We evaluate the empirical performance of the two-relevant-bonds model. We compare the maturities of actual cheapest-to-deliver bonds to the prediction of the two-deliverables model. We find that, consistent with the model, extreme-duration bonds have generally been selected to settle the note contracts. However, the bond contract has been more frequently settled with intermediate-maturity bonds. We also calculate empirical price values of a basis point for Treasury futures contracts to determine whether they display the negative convexity predicted by the two-deliverables model. Again, the note contract is consistent with the model, clearly exhibiting negative convexity, but the bond contract exhibits none. In sum, the model has worked very well for the note contract and very poorly for the bond contract.

The difference in model performance is related to the shape of the yield curve. Whereas the yield curve, when it was near 6%, has been relatively flat for the narrow range of maturities eligible to settle the note contract, the wider maturity range admissible for the bond contract apparently has allowed for more pronounced differences in yields. These yield differences can—and have—made the ultimate CTD bond far more difficult to predict than would be the case if the yield curve were flat, which goes a long way toward explaining why numerical methods dominate hedge ratio calculations for market participants.

References


