Tricks With Hicks: The EASI Demand System

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Consumer demand systems

log price vector $\mathbf{p}$, log quantity vector $\mathbf{q}$, Log total expenditure x
Budget shares $\mathbf{w} = \exp(\mathbf{q} + \mathbf{p} - x)$

Max utility $u = U(\mathbf{q})$ such that $\exp(\mathbf{q})' \exp(\mathbf{p}) \leq \exp(x)$
Solution is quantity Marshallian demand functions $\mathbf{q} = \mathbf{q}(\mathbf{p}, x)$,
or in budget share form $\mathbf{w} = \exp[\mathbf{q}(\mathbf{p}, x) + \mathbf{p} - x] = \mathbf{w}(\mathbf{p}, x)$

Log cost function $x = C(\mathbf{p}, u)$
Hicksian demand functions $\mathbf{w} = \omega(\mathbf{p}, u) = \mathbf{w}[\mathbf{p}, C(\mathbf{p}, u)]$
Shephard's lemma $\omega(\mathbf{p}, u) = \nabla_{\mathbf{p}} C(\mathbf{p}, u)$

Add in preference heterogeneity: observable taste shifters $\mathbf{z}$ and unobservable (random utility) parameters ε
$u = U(\mathbf{q}, \mathbf{z}, \varepsilon), \quad \mathbf{w} = \mathbf{w}(\mathbf{p}, x, \mathbf{z}, \varepsilon)$
In practice usually estimate $\mathbf{w} = \mathbf{w}(\mathbf{p}, x, \mathbf{z}) + \mathbf{e}$,
Coherency, invertibility, support of $\mathbf{e} = \mathbf{e}(\mathbf{p}, x, \mathbf{z}, \varepsilon)$?

Demand systems $\mathbf{w} = \mathbf{w}(\mathbf{p}, x, \mathbf{z}, \varepsilon)$ are numerically difficult:
Many, often collinear, prices
Many nonlinear constraints (integrability, Slutsky symmetry)
Data require flexible price responses,
complicated income response (Engel curve) shapes
Reconciling model errors with unobserved preference heterogeneity

The goal here is to construct demand systems that, like the Almost Ideal:
have Diewert flexible price responses and
are nearly linear in coefficients (numerically tractible)

But also have:
Unrestricted Engel curves (can be nonparametric, any rank)
and model errors $\mathbf{e}$ = random utility parameters ε

Demand System Functional Forms

Cobb Douglas (1928), Geary (1951 LES) Stone (1954 LES), Arrow, Chenery, Minhas, and Solow (1961 CES), Theil (1965 Rotterdam), Armington (1969 CES), Diewert (1971 generalized leontief), Christensen, Jorgenson, and Lau (1975 translog), Howe, Pollak and Wales (1979 QES), Deaton and Muellbauer (1980 AID), Elbadawi, Gallant, and Souza (1983 fourier), Banks, Blundell, Lewbel (1997 QUAIDS), Pendakur and Sperlich (2005, closest to our EASI), many others.

Engel Curves

Engel (1857, 1895), Allen and Bowley (1935), Working (1943), Leser (1963), Houthakker and Taylor (1966), Gorman (1981), Bierens and Pott-Buter (1990 first nonparametric regression in economics?), Lewbel (1991), Blundell, Duncan and Pendakur (1998), Pendakur (1999), Koenker and Hallock (2001 quantile), Blundell, Chen and Kristensen (2007).

Random Utility - Unobserved Taste Heterogeneity - Coherence

McFadden and Richter (1971, 1990), Brown and Walker (1989), van Soest, Kapteyn, and Kooreman (1993), Brown and Matzkin (1998), Lewbel (2001), Matzkin (2005), Beckert and Blundell (2008).

Deaton and Muellbauer's "Almost Ideal" demand system

Budget shares $\mathbf{w}$, log prices $\mathbf{p}$,
Log total expenditure x,
Log cost function $x = C(\mathbf{p}, u)$.
$\mathbf{w} = \omega(\mathbf{p}, u) = \nabla_{\mathbf{p}} C(\mathbf{p}, u)$.

$$C(\mathbf{p}, u) = \exp(\mathbf{c}'\mathbf{p})\, u + \mathbf{p}'\left(\mathbf{a} + \tfrac{1}{2}\mathbf{B}\mathbf{p}\right)$$
$$\omega(\mathbf{p}, u) = \exp(\mathbf{c}'\mathbf{p})\, u\mathbf{c} + \mathbf{a} + \mathbf{B}\mathbf{p}$$

Marshallian demands:
$$\mathbf{w} = \mathbf{a} + \mathbf{B}\mathbf{p} + \mathbf{c}y$$
$$y = x - \mathbf{p}'\left(\mathbf{a} + \tfrac{1}{2}\mathbf{B}\mathbf{p}\right)$$

Approximate AI, $\widetilde{y} = x - \mathbf{w}'\mathbf{p}$

Tricks With Hicks

Budget shares $\mathbf{w}$, log prices $\mathbf{p}$, Random utility parameters $\boldsymbol{\varepsilon}$, Log total expenditure x, Log cost function $x = C(\mathbf{p}, u, \boldsymbol{\varepsilon})$.

Shephard's lemma: $\mathbf{w} = \omega(\mathbf{p}, u) = \nabla_{\mathbf{p}} C(\mathbf{p}, u, \boldsymbol{\varepsilon})$.

In Hicks demands, easy to have flexible $\mathbf{p}$ and u effects, linear in parameters, and additive errors. e.g., C a poly in $\mathbf{p}, u$ plus $\mathbf{p}'\boldsymbol{\varepsilon}$:

$$C = \mathbf{p}' \left(\mathbf{a} + \frac{1}{2}\mathbf{B}\mathbf{p} + \mathbf{b}u + \frac{1}{2}\mathbf{C}\mathbf{p}u + \mathbf{c}u^2 + ... + \boldsymbol{\varepsilon} \right)$$

$$\mathbf{w} = \omega(\mathbf{p}, u, \boldsymbol{\varepsilon}) = \mathbf{a} + \mathbf{B}\mathbf{p} + \mathbf{b}u + \mathbf{C}\mathbf{p}u + \mathbf{c}u^2 + ... + \boldsymbol{\varepsilon}$$

Implicit Marshallian Demands

Problem with Hicks: u not observed.
Solution: Implicit Marshallian Demands

The idea: construct $C(\mathbf{p}, u, \varepsilon)$ so that
$u = g\left[\omega(\mathbf{p}, u, \varepsilon), \mathbf{p}, x\right]$ for a simple g.

Then let $y = g(\mathbf{w}, \mathbf{p}, x)$ and estimate
Implicit Marshallian demand functions:
$\mathbf{w} = \omega(\mathbf{p}, y, \varepsilon)$

In our applications y is linear in x, and
$y \approx$ a log money metric utility measure
so will call y log real expenditures.

Trivial Implicit-Marshallian Example:
$x = C(\mathbf{p}, u, \varepsilon) = u + \mathbf{p}' [\mathbf{m}(u) + \varepsilon]$

By Shephard's lemma,
$\mathbf{w} = \omega(\mathbf{p}, u, \varepsilon) = \mathbf{m}(u) + \varepsilon$, so
$u = x - \mathbf{p}' [\mathbf{m}(u) - \varepsilon] = x - \mathbf{p}'\omega(\mathbf{p}, u, \varepsilon)$,
So let $y = g(\mathbf{w}, \mathbf{p}, x) = x - \mathbf{p}'\mathbf{w}$.
y is Stone index deflated x

Implicit-Marshallian budget shares are

$$\begin{aligned} \mathbf{w} &= \mathbf{m}(x - \mathbf{p}'\mathbf{w}) + \varepsilon \\ \mathbf{w} &= \mathbf{m}(y) + \varepsilon \end{aligned}$$

Arbitrary Engel curves, additive ε, but
has no price effects (except through y).

Define ESI (Exact Stone Index) demand system as having $u = y = x - \mathbf{p}'\mathbf{w}$. Previous slide was an example of an ESI system. Compare ESI to Deaton and Muellbauer's (1980) AID. Unlike AID, for ESI deflated expenditures $y = x - \mathbf{p}'\mathbf{w}$ is the exact right deflator, not an approximation.

Theorem: All ESI demands are implausible; budget shares are unchanged when all prices are squared.

To get a useful system, generalize the ESI idea: Define EASI, an Exact Affine Stone Index demand system, as one where y is an affine transform of Stone deflated x:

$$u = y = t(\mathbf{p}) + s(\mathbf{p})\left[x - \mathbf{w}'\mathbf{p}\right]$$

Appendix has some theorems describing all cost functions that are ESI or EASI.

Let $\mathbf{z}$ = characteristics (age, family type)
Nice class of EASI cost functions:

$$C(\mathbf{p}, u, \mathbf{z}, \varepsilon) = u + \mathbf{p}'\mathbf{m}(u, \mathbf{z}) + T(\mathbf{p}, \mathbf{z}) + S(\mathbf{p}, \mathbf{z})u + \mathbf{p}'\varepsilon$$

$\mathbf{z}$ is a vector of observed characteristics.
Has Implicit Marshallian demands

$$\mathbf{w} = \mathbf{m}(y, \mathbf{z}) + \nabla_{\mathbf{p}} T(\mathbf{p}, \mathbf{z}) + \nabla_{\mathbf{p}} S(\mathbf{p}, \mathbf{z})y + \varepsilon$$

$$y = \frac{x - \mathbf{p}'\mathbf{w} - T(\mathbf{p}, \mathbf{z}) + \mathbf{p}'\left[\nabla_{\mathbf{p}} T(\mathbf{p}, \mathbf{z})\right]}{1 + S(\mathbf{p}, \mathbf{z}) - \mathbf{p}'\left[\nabla_{\mathbf{p}} S(\mathbf{p}, \mathbf{z})\right]}$$

$\mathbf{m}(y, \mathbf{z}) \longrightarrow$ general Engel curves, $T(\mathbf{p}, \mathbf{z}) \longrightarrow$ general price effects
$S(\mathbf{p}, \mathbf{z})y \longrightarrow$ interactions, $\varepsilon \longrightarrow$ model error = random utility

Proposed parametric model is a semiparametric sieve approximation of above, with high order polynomial for $\mathbf{m}(y, \mathbf{z})$, and low order for $T(\mathbf{p}, \mathbf{z})$ and $S(\mathbf{p}, \mathbf{z})$.

EASI model we use empirically has Cost function

$$C(\mathbf{p}, u, \mathbf{z}, \varepsilon) = u + \mathbf{p}' \left[\sum_{r=0}^{5} \mathbf{b}_r u^r + \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{z}u \right] + \frac{1}{2} \sum_{l=0}^{L} z_l \mathbf{p}' \mathbf{A}_l \mathbf{p} + \frac{1}{2} \mathbf{p}' \mathbf{B} \mathbf{p} u + \mathbf{p}' \varepsilon.$$

Has Implicit Marshallian demands:

$$\mathbf{w} = \sum_{r=0}^{5} \mathbf{b}_r y^r + \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{z}y + \sum_{l=0}^{L} z_l \mathbf{A}_l \mathbf{p} + \mathbf{B}\mathbf{p}y + \varepsilon.$$

log real expenditures:

$$y = g(\mathbf{w}, \mathbf{p}, x, \mathbf{z}) = \frac{x - \mathbf{p}'\mathbf{w} + \sum_{l=0}^{L} z_l \mathbf{p}' \mathbf{A}_l \mathbf{p}/2}{1 - \mathbf{p}' \mathbf{B} \mathbf{p}/2}.$$

Theorems give parameter restrictions for global regularity (homogeneity, Slutsky, etc.,.) and coherency with $\varepsilon \perp x, \mathbf{p}, \mathbf{z}$ (requires $x, \mathbf{p}, \mathbf{z}, \varepsilon$ have bounded support).

Features of our EASI model

$$\mathbf{w} = \sum_{r=0}^{5} \mathbf{b}_r y^r + \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{z}y + \sum_{l=0}^{L} z_l \mathbf{A}_l \mathbf{p} + \mathbf{B}\mathbf{p}y + \varepsilon.$$

where y is affine in $x - \mathbf{p}'\mathbf{w}$:

Diewert flexible interactions of $y, \mathbf{p}, \mathbf{z}$.
Engel curves are arbitrary functions in $y, \mathbf{z}$. No Gorman type rank restrictions.

Additive errors are (coherent, invertible) random preference heterogeneity.

Like AID, is linear in parameters up to y.

Closed form expressions for consumer surplus, cost of living indices, etc.,.

Estimation

Approximate model: $\widetilde{y} = x - \mathbf{p}'\overline{\mathbf{w}}$

$$\mathbf{w} \approx \sum_{r=0}^{5} \mathbf{b}_r \widetilde{y}^r + \mathbf{Cz} + \mathbf{Dz}\widetilde{y} + \sum_{l=0}^{L} z_l \mathbf{A}_l \mathbf{p} + \mathbf{Bp}\widetilde{y} + \varepsilon$$

Can estimate by separate equation linear Ordinary Least Squares or SUR with symmetry constraint on $\mathbf{A}_l$ and $\mathbf{B}$.

Constraints on $\mathbf{A}_l$ matrices are linear. Can do constrained SUR, same asymptotically as normal ε ML.

Can use approximate model as starting values for exact model.

Exact model is

$$\mathbf{w} = \sum_{r=0}^{5} \mathbf{b}_r y^r + \mathbf{Cz} + \mathbf{Dz}y + \sum_{l=0}^{L} z_l \mathbf{A}_l \mathbf{p} + \mathbf{Bp}y + \boldsymbol{\varepsilon}$$

$$y = \frac{x - \mathbf{p}'\mathbf{w} + \sum_{l=0}^{L} z_l \mathbf{p}'\mathbf{A}_l \mathbf{p}/2}{1 - \mathbf{p}'\mathbf{Bp}/2}$$

Use GMM (or nonlinear 3SLS) for to handle nonlinearity (one dimensional, only in y), as well as endogeneity in y and possible heteroskedasticity in $\boldsymbol{\varepsilon}$.

Assume $E(\boldsymbol{\varepsilon} \mid x, \mathbf{p}, \mathbf{z}) = \mathbf{0}_J$. Let $\mathbf{r} = (r_1, \ldots, r_M)'$ be bounded functions of $x, \mathbf{p}, \mathbf{z}$. (we let $\mathbf{r}$ be all the regressors in the approximate model. Then for $\ell = 1, \ldots, M$ use $E(\boldsymbol{\varepsilon} r_\ell) = 0$ as moments for GMM, or assuming $\boldsymbol{\varepsilon} \perp x, \mathbf{p}, \mathbf{z}$, use $\mathbf{r}$ as 3SLS instruments.

Empirical results

Canadian Expenditure Surveys. 12 years, 4 regions, in 1969 to 1999. urban, rental-tenure, singles. 4,847 observations, 48 price regimes.

9 Categories:
food-in, food-out, rent, clothing, household operation, household furnishing & equipment, transport operation, recreation, personal care.

z is age, sex, car dummy, social assistance dummy, time.
Also, numerically scale y so y^5 is ok.

Tables and Figures at the end of these slides. Summarize some highlights for now.

8 equations $\times$ 4847 observations per equation.
Model has 576 parameters, minus 196 imposing symmetry.
Aproximate SUR was very close to exact 3SLS.
Exact model is so close to linear it converged in just three iterations.

Table 2 - homoskedastic Wald test p-values:
Symmetry and overidentification: .014, .021
(marginal rejection given sample size)
zeroing cross $y, \mathbf{p}, \mathbf{z}$ term matrices .000
(except for $y\mathbf{p}$ interaction, .060)
zeroing higher than y^2, 4 goods have $< .001$

Table 3 - approximate model and ignoring endogeneity work fine:
$x - \mathbf{p}'\mathbf{w}$, and $x - \mathbf{p}'\overline{\mathbf{w}}$ have .998 correlation with true y, even after removing any time trends.

Table 4 - price effects ok, Slutsky terms ok.
example: rent own-price compensated semi-elasticity .063. A compensated 10% rent increase raises rent share by .63%

Significant cross price effects, e.g., clothing share compensated rent cross-price semi-elasticity is -0.066

matrix B shows price effects of x.
Rent own-price B is .088, positive means increasing substitution.
Rent own price Slutsky elasticities:
At 5th percentile of x elasticity is -0.080.
At 95th percentile of x elasticity is -0.236.

Figures show a wide variety of significantly nonquadratic Engel curves.

Consumer Surplus Experiment

Have shares $\mathbf{w}$ at $\mathbf{p}_0$, change to $\mathbf{p}_1$.
EASI Log true cost of living change is simple:

$$C(\mathbf{p}_1, u, \mathbf{z}, \varepsilon) - C(\mathbf{p}_0, u, \mathbf{z}, \varepsilon) = (\mathbf{p}_1 - \mathbf{p}_0)'\mathbf{w} \\ + (\mathbf{p}_1 - \mathbf{p}_0)' \left(\sum z_l \mathbf{A}_l + \mathbf{B}y\right) (\mathbf{p_1} - \mathbf{p_0})/2$$

First term $(\mathbf{p}_1 - \mathbf{p}_0)'\mathbf{w}$ is just the Stone index based cost of living index: a share weighted geometric mean of price changes.

The $z_l \mathbf{A}_l + \mathbf{B}y$ term is price substitution effects

Experiment (see last figure): extending 15% sales tax to rent.
Results: Substitution effects are small but systematic (black squares). The effects of unobserved preference heterogeneity ε are large (empty circles vs. filled circles).

Additional Results in the paper

Closed form Marshallian and Hicksian elasticity expressions.

Consumer Surplus theory

Existence of general $y = g(x, \mathbf{w}, \mathbf{p})$.

Global regularity conditions.

Coherence and invertibility of ε.

Closure Under Unit Scaling.

Conditions for Shape Invariance.

Properties of associated indirect utility and Marshallian demands.

Table 1: Data Descriptives

Variable		Mean	Std Dev	Minimum	Maximum
budget shares	Food-at-Home	0.14	0.08	0.00	0.61
	Food-Out	0.08	0.07	0.00	0.63
	Rent	0.37	0.12	0.01	0.89
	Household Oper	0.07	0.04	0.00	0.61
	Household Furneq	0.04	0.05	0.00	0.52
	Clothing	0.08	0.06	0.00	0.48
	Transport Oper	0.11	0.08	0.00	0.60
	Recreation	0.08	0.07	0.00	0.59
	Personal Care	0.03	0.02	0.00	0.21
log-prices	Food-at-Home	-0.05	0.43	-1.41	0.34
	Food-Out	0.04	0.49	-1.46	0.53
	Rent	-0.08	0.40	-1.27	0.37
	Household Oper	-0.06	0.45	-1.40	0.32
	Household Furneq	-0.05	0.32	-0.94	0.20
	Clothing	0.04	0.35	-0.94	0.34
	Transport Oper	-0.07	0.57	-1.53	0.57
	Recreation	0.01	0.40	-1.04	0.42
	Personal Care	-0.03	0.38	-1.11	0.29
demographics	age-40	0.71	11.89	-15.00	24.00
	male	0.51	0.50	0.00	1.00
	car-owner	0.42	0.49	0.00	1.00
	social asst	0.27	0.44	0.00	1.00
	time	88.99	8.73	69.00	99.00
log-expenditure	x	-0.11	0.59	-2.75	1.66
(median-norm'd)	$x-\mathbf{p'w}$	-0.07	0.44	-1.70	1.44

Table 2: Model Tests (Wald- or J-tests)					
model	test of	parameters	df	Test Stat	p-value
asymmetric with with y^{-1}, y^{6}	symmetry	$\mathbf{A}_l=\mathbf{A}_l'$ for all l; $\mathbf{B}=\mathbf{B}'$	196	241.8	0.014
	symmetry	$\mathbf{A}_l=\mathbf{A}_l'$ for all l	168	194.1	0.082
	symmetry	$\mathbf{B}=\mathbf{B}'$	28	24.4	0.661
	exclusion	$\mathbf{B=0}$	64	82.5	0.060
	exclusion	$\mathbf{A}_l=\mathbf{0}$ for all l	384	636.0	0.000
	exclusion	$\mathbf{B=0}$, $\mathbf{A}_l=\mathbf{0}$ for all l	448	999.7	0.000
	exclusion	y^{-1}	8	15.6	0.049
	exclusion	y^{6}	8	20.6	0.008
	exclusion	y^{5}	8	21.0	0.007
	exclusion	y^{-1}, y^{6}	16	28.3	0.029
	exclusion	y^{-1}, y^{6}, y^{5}	24	75.5	0.000
symmetric with without y^{-1}, y^{6}	exclusion	$\mathbf{B=0}$	36	51.7	0.043
	exclusion	$\mathbf{A}_l=\mathbf{0}$ for all l	216	646.2	0.000
	exclusion	$\mathbf{B=0}$, $\mathbf{A}_l=\mathbf{0}$ for all l	252	710.6	0.000
	exclusion	y^{5}	8	45.8	0.000
	non-quadratic	Food-at-Home	3	16.9	0.001
	non-quadratic	Food-Out	3	10.3	0.016
	non-quadratic	Rent	3	94.4	0.000
	non-quadratic	Household Oper	3	22.7	0.000
	non-quadratic	Household Furneq	3	3.4	0.340
	non-quadratic	Clothing	3	9.4	0.025
	non-quadratic	Transport Oper	3	6.8	0.079
	non-quadratic	Recreation	3	28.9	0.000
	overidentification	*J-* test (with sym **pz)**	196	238.4	0.021
	overidentification	*J-* test (with sym. **pz,** y^{-1}, y^{6})	212	273.4	0.003

Table 3: Variation and Correlation of *y* Components										
Variable	std dev	y	y (no **e**)	*y-bar*	*x*- **p'w**	*x*- **p'w**-*bar*	**p'e**	**p**'(**w-e**)	**p'Ap**/2	**p'Bp**/2
y	0.4440	1.0000								
y (no **e**)	0.4435	0.9988	1.0000							
y-bar	0.4514	0.9983	0.9995	1.0000						
x- **p'w**	0.4434	1.0000	0.9988	0.9983	1.0000					
x- **p'w**-*bar*	0.4508	0.9984	0.9996	1.0000	0.9983	1.0000				
p'e	0.0214	-0.0447	0.0036	0.0034	-0.0446	0.0035	1.0000			
p'(**w-e**)	0.4186	-0.0635	-0.0640	-0.0701	-0.0611	-0.0678	-0.0093	1.0000		
p'Ap/2	0.0023	0.0191	0.0188	0.0294	0.0143	0.0246	-0.0070	-0.4968	1.0000	
p'Bp/2	0.0019	-0.0196	-0.0202	-0.0146	-0.0228	-0.0177	-0.0112	-0.5774	0.6726	1.0000

Table 4: Price Effects, evaluated for reference type with median expenditure at base prices										
	Own-Price Slutsky Terms	Own-Price **B** element	Compensated Semi-Elasticities							
			Food-in	Food-Out	Rent	HH Oper	HH Furneq	Clothing	Tran Oper	Recr
Food-at-Home	-0.137	0.047	-0.025							
Std Err	*0.045*	*0.063*	*0.045*							
Food-Out	-0.120	-0.009	0.051	-0.025						
Std Err	*0.037*	*0.068*	*0.033*	*0.037*						
Rent	-0.164	0.088	-0.026	0.037	0.063					
Std Err	*0.029*	*0.047*	*0.021*	*0.021*	*0.029*					
Household Oper	-0.035	0.044	0.026	-0.002	-0.002	0.012				
Std Err	*0.026*	*0.039*	*0.026*	*0.022*	*0.014*	*0.026*				
Household Furneq	-0.034	0.037	0.110	-0.043	0.028	-0.018	-0.005			
Std Err	*0.058*	*0.114*	*0.035*	*0.032*	*0.021*	*0.026*	*0.058*			
Clothing	-0.066	-0.050	-0.050	-0.001	-0.066	-0.025	-0.023	-0.008		
Std Err	*0.046*	*0.060*	*0.030*	*0.032*	*0.020*	*0.024*	*0.038*	*0.046*		
Transport Oper	-0.105	-0.006	-0.023	-0.030	-0.019	-0.013	-0.034	0.053	0.038	
Std Err	*0.020*	*0.035*	*0.021*	*0.019*	*0.016*	*0.017*	*0.021*	*0.019*	*0.020*	
Recreation	-0.182	-0.030	-0.044	0.032	-0.021	0.034	-0.010	0.087	0.033	-0.106
Std Err	*0.053*	*0.093*	*0.034*	*0.032*	*0.023*	*0.025*	*0.045*	*0.037*	*0.022*	*0.053*

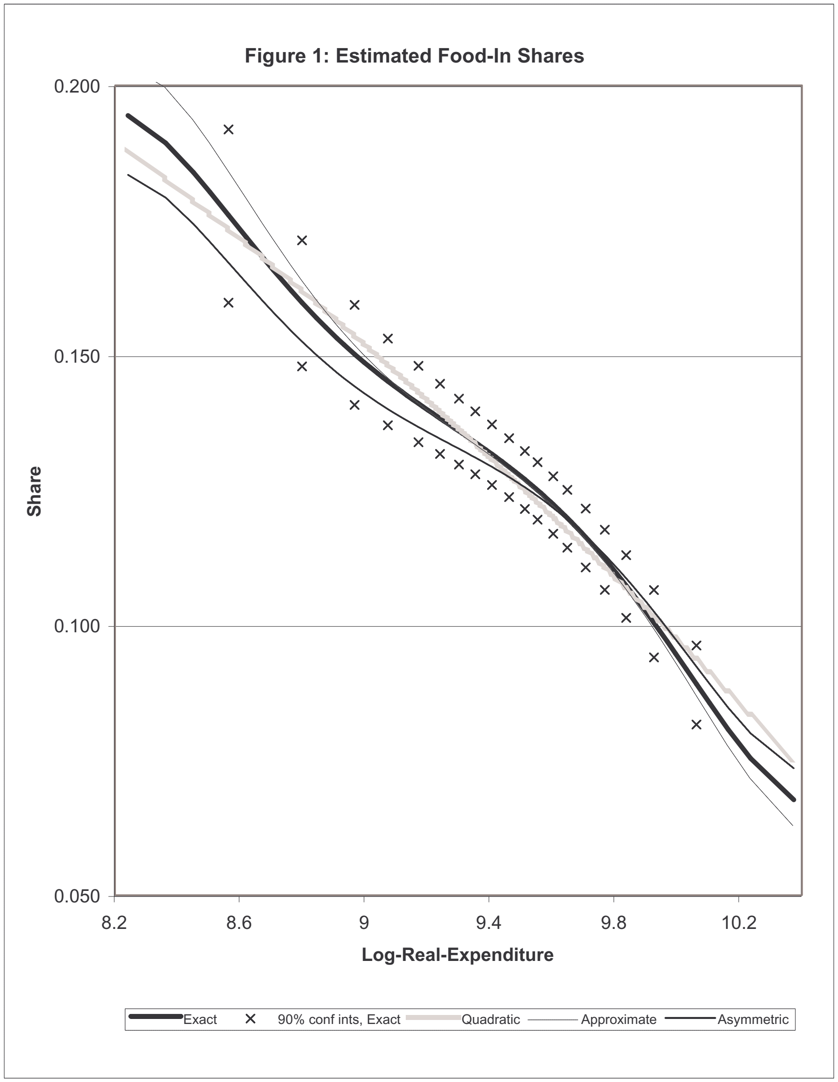
Figure 1: Estimated Food-In Shares
0.200
0.150
0.100
0.050
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
90% conf ints, Exact
Quadratic
Approximate
Asymmetric

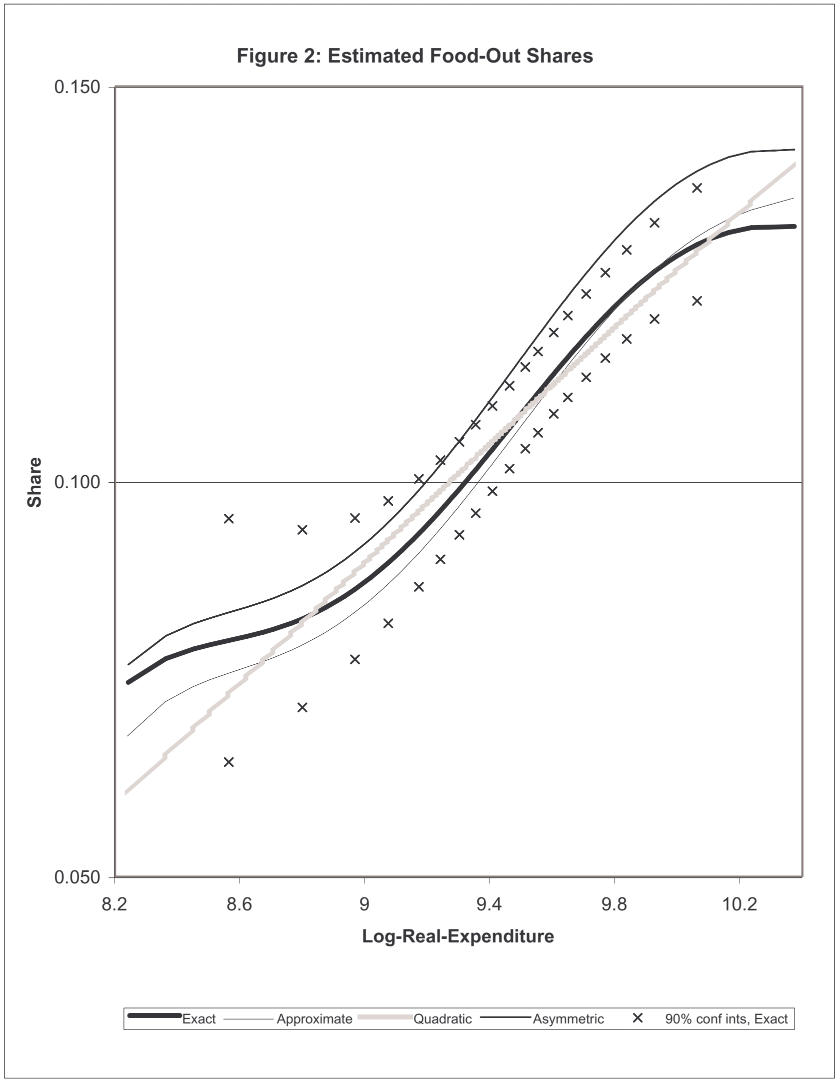
Figure 2: Estimated Food-Out Shares
0.150
0.100
0.050
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Approximate
Quadratic
Asymmetric
90% conf ints, Exact

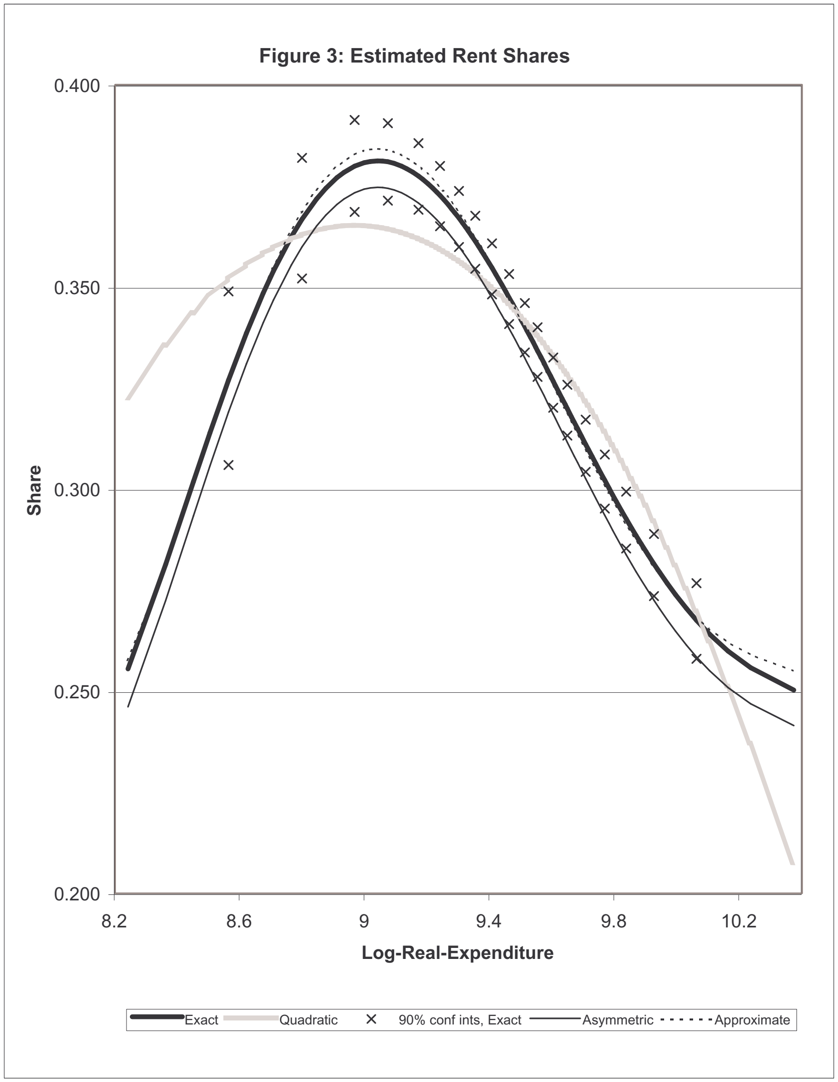
Figure 3: Estimated Rent Shares
0.400
0.350
0.300
0.250
0.200
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
90% conf ints, Exact
Asymmetric
Approximate

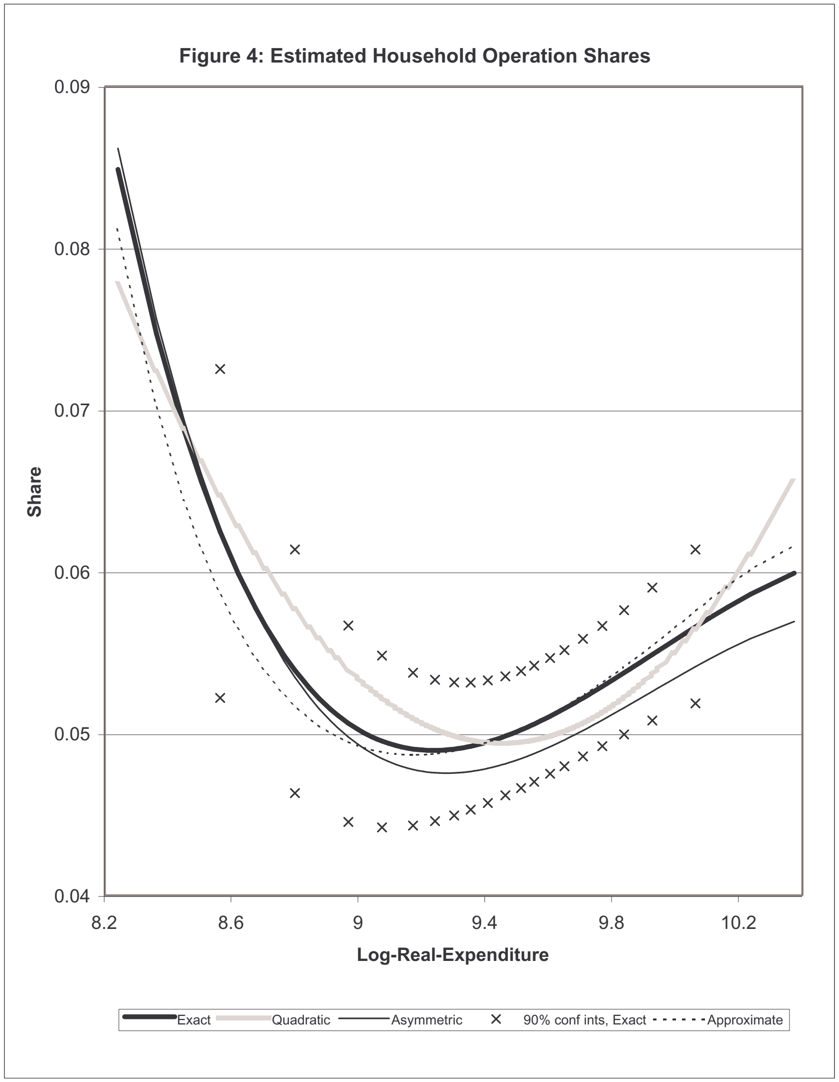
Figure 4: Estimated Household Operation Shares
0.09
0.08
0.07
0.06
0.05
0.04
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
Asymmetric
90% conf ints, Exact
Approximate

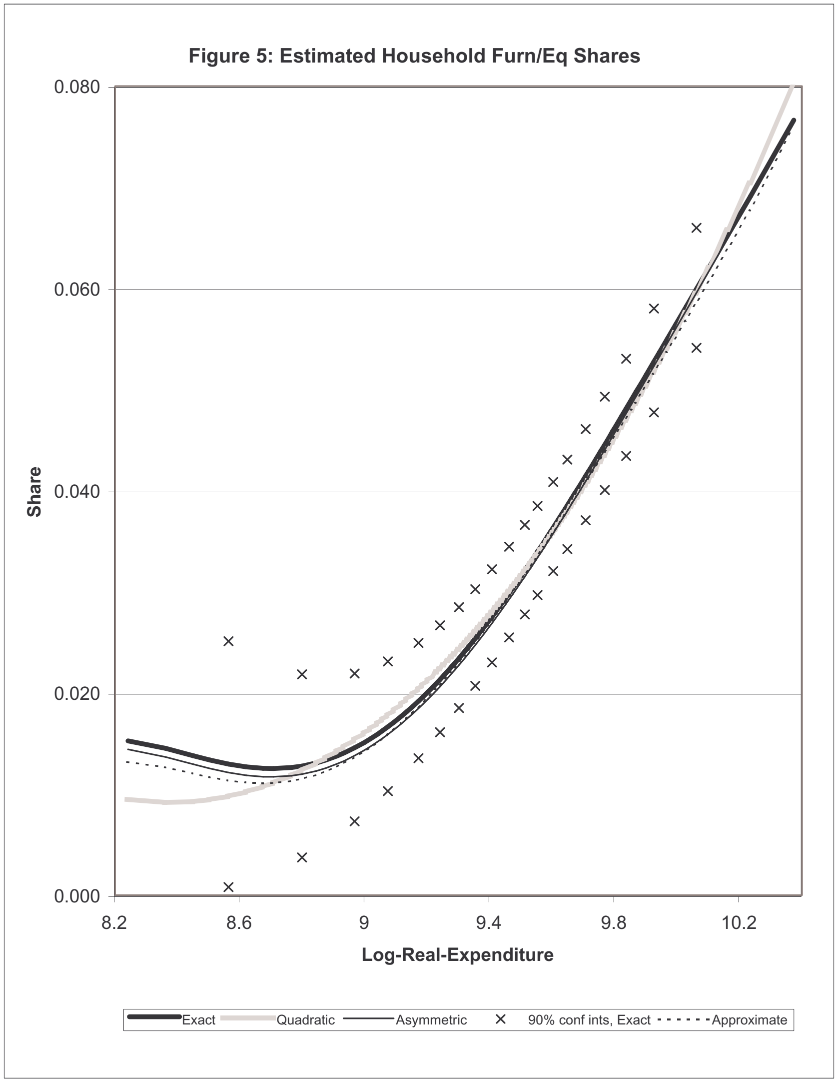

Figure 5: Estimated Household Furn/Eq Shares
0.080
0.060
0.040
0.020
0.000
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
Asymmetric
90% conf ints, Exact
Approximate

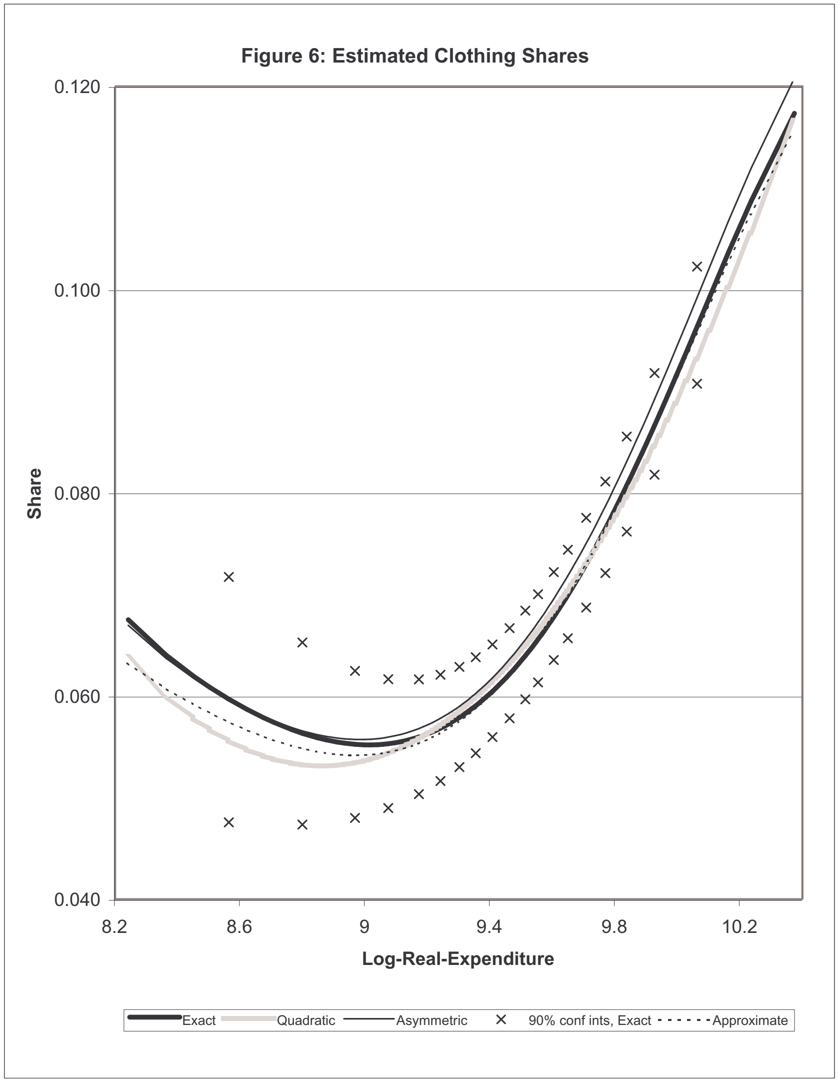

Figure 6: Estimated Clothing Shares
0.120
0.100
0.080
0.060
0.040
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
Asymmetric
90% conf ints, Exact
Approximate

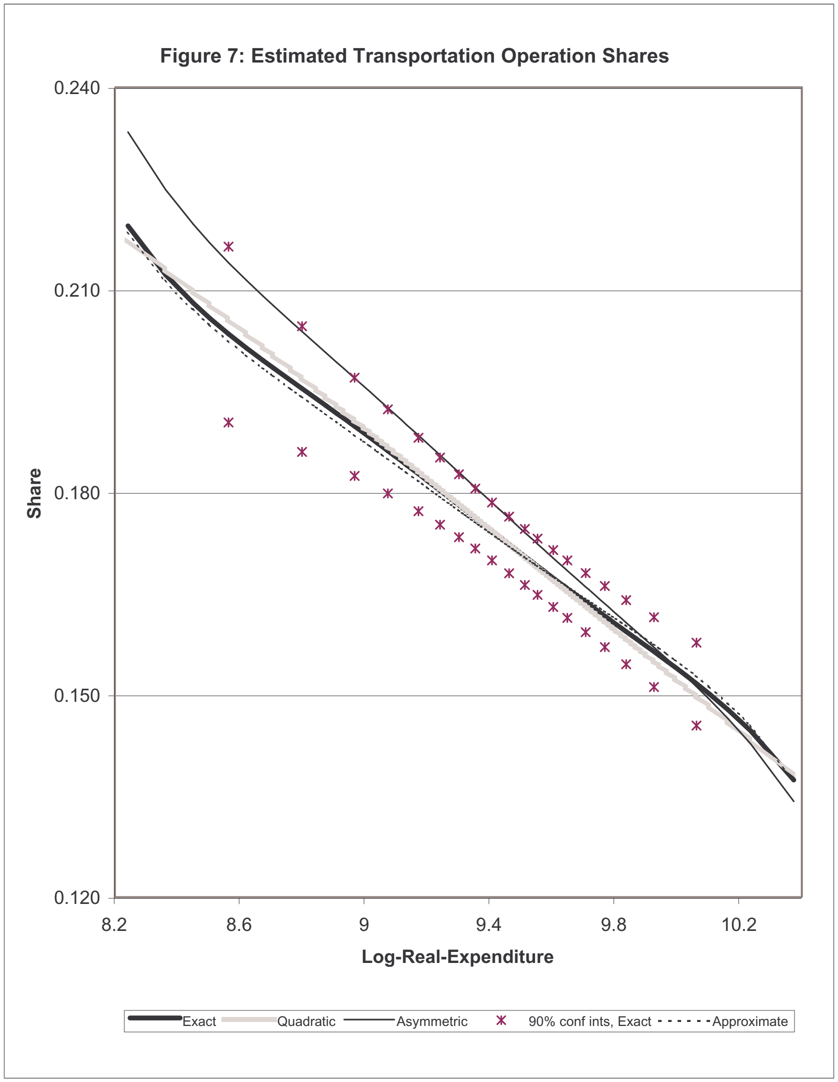
Figure 7: Estimated Transportation Operation Shares
Share
0.240
0.210
0.180
0.150
0.120
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
Asymmetric
90% conf ints, Exact
Approximate

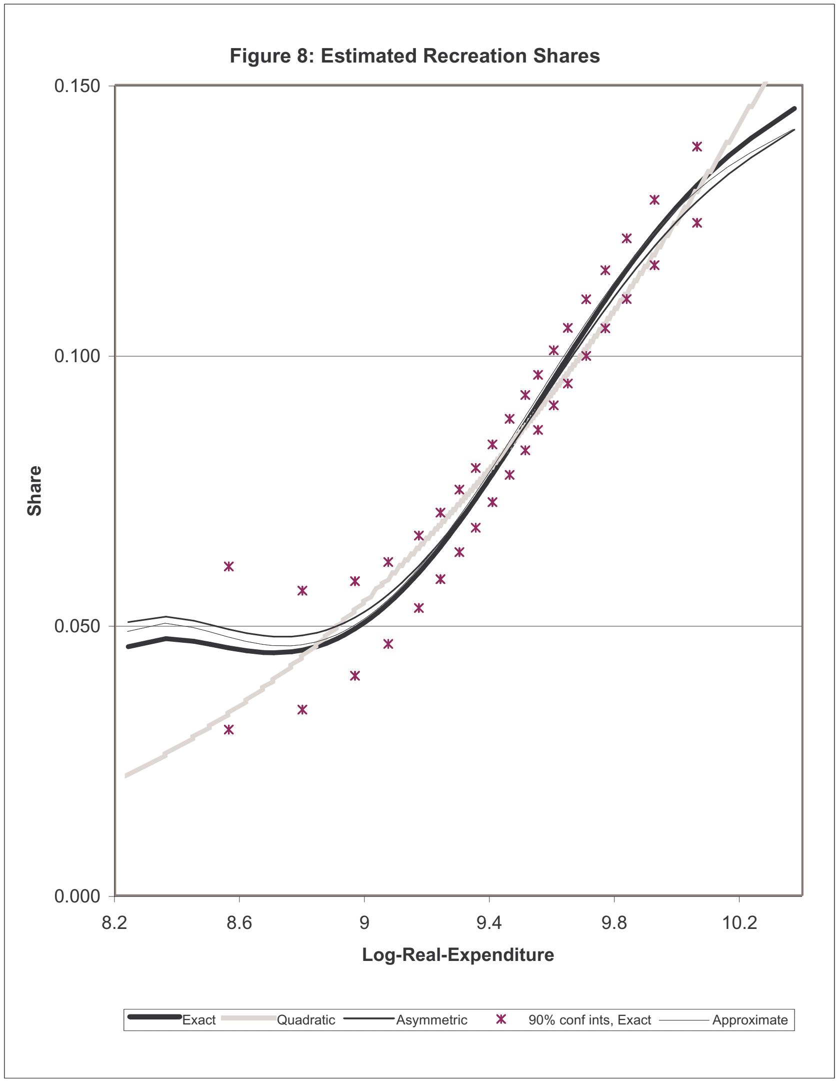
Figure 8: Estimated Recreation Shares
Share
0.150
0.100
0.050
0.000
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
Exact
Quadratic
Asymmetric
90% conf ints, Exact
Approximate

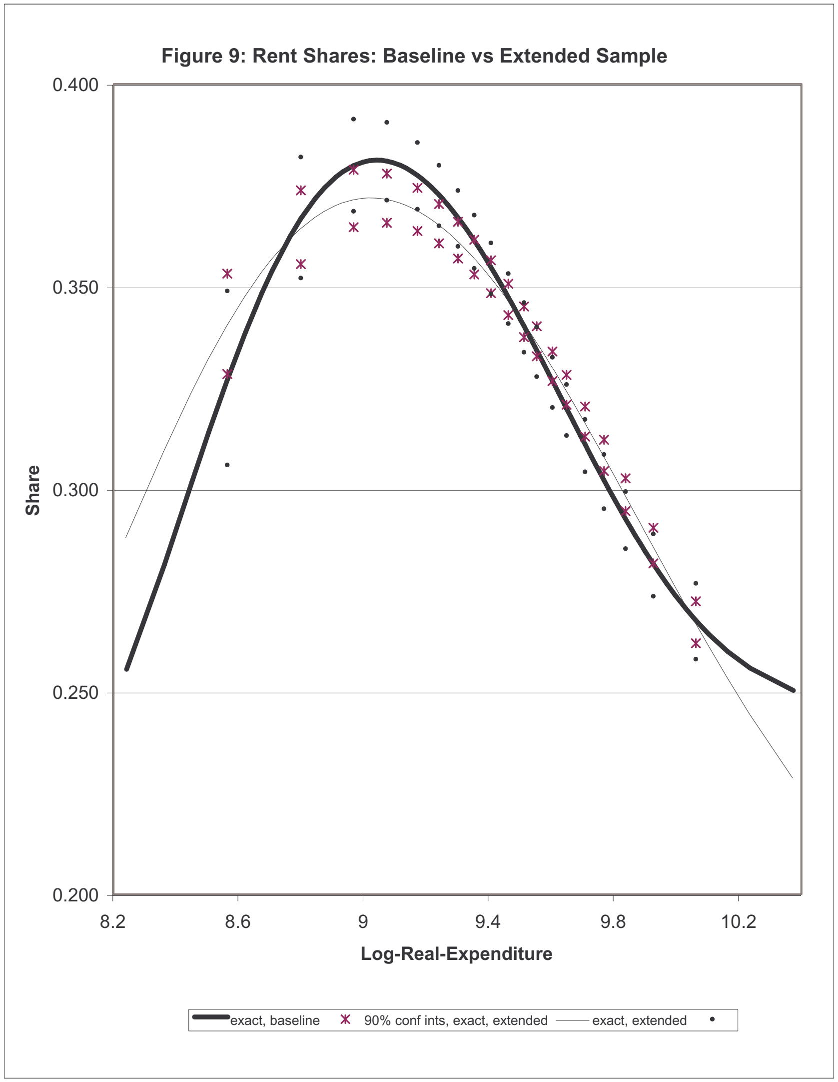
Figure 9: Rent Shares: Baseline vs Extended Sample
0.400
0.350
0.300
0.250
0.200
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
exact, baseline
90% conf ints, exact, extended
exact, extended

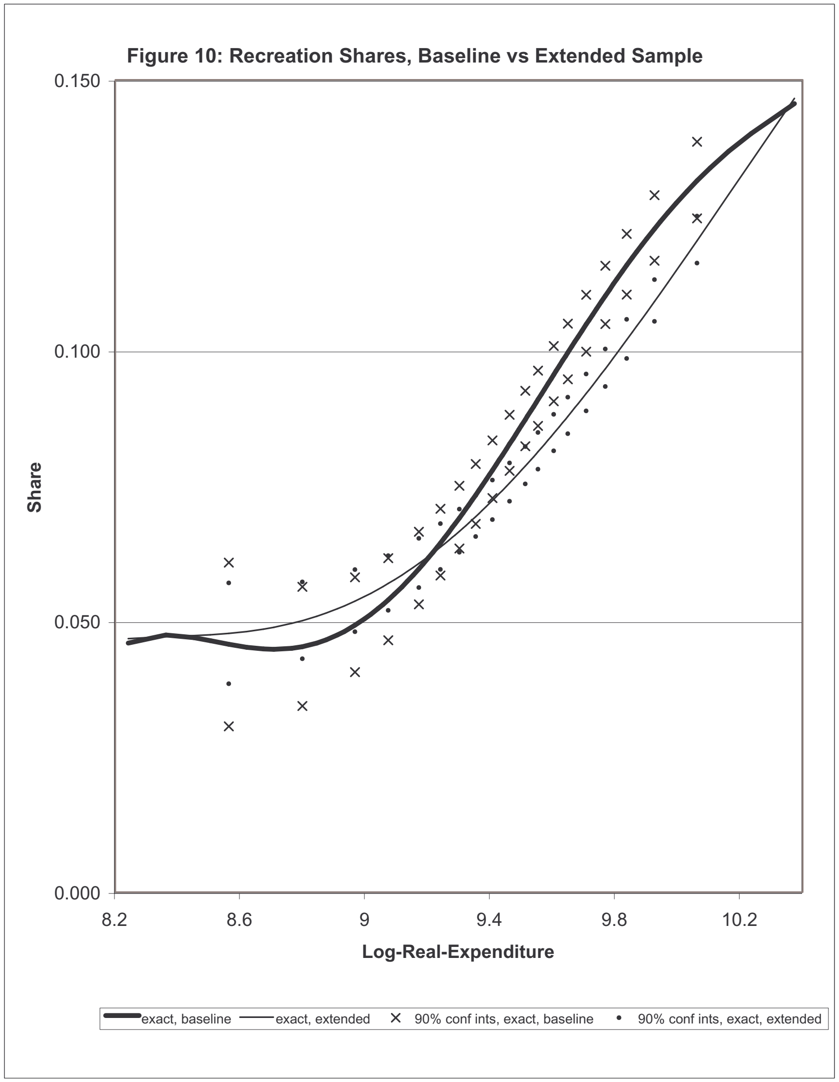
Figure 10: Recreation Shares, Baseline vs Extended Sample
0.150
0.100
0.050
0.000
Share
8.2
8.6
9
9.4
9.8
10.2
Log-Real-Expenditure
exact, baseline
exact, extended
90% conf ints, exact, baseline
90% conf ints, exact, extended

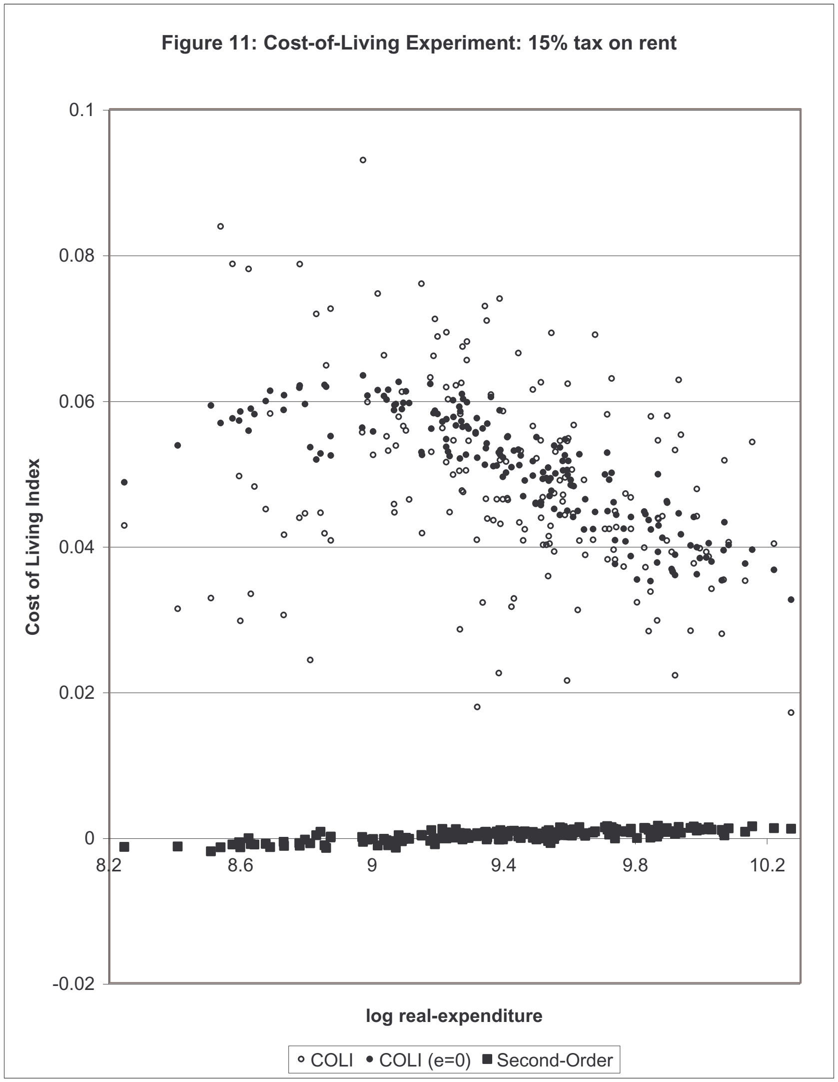
Figure 11: Cost-of-Living Experiment: 15% tax on rent
Cost of Living Index
0.1
0.08
0.06
0.04
0.02
0
-0.02
8.2
8.6
9
9.4
9.8
10.2
log real-expenditure
COLI
COLI (e=0)
Second-Order