Sharing rule identification for general collective consumption models[*]

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Abstract

We propose a method to set identify bounds on the sharing rule for a general collective household consumption model. Unlike the effects of distribution factors, the level of the sharing rule cannot be uniquely identified without strong assumptions on preferences across households. Our new results show that, though not point identified without these assumptions, strong bounds on the sharing rule can be obtained. We get these bounds by applying revealed preference restrictions implied by the collective model to the household's continuous aggregate demand functions. We obtain informative bounds even if nothing is known about whether each good is public, private, or assignable within the household, though

having such information tightens the bounds. We apply our method to US PSID data, obtaining narrow bounds that yield useful conclusions regarding the effects of income and wages on intrahousehold resource sharing, and on the prevalence of individual (as opposed to household level) poverty.

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1 Introduction

The collective model has become increasingly popular for analyzing household consumption behavior. Becker (1973, 1981) first considered collective household models, in which the household is characterized as a collection of individuals, each with well defined objective functions that interact to generate household level decisions. In a consumption setting, the model assumes that expenditures on each good and service the household buys are the outcome of multi-person decision making, in which each individual household member is characterized by his or her own rational preferences. Following Chiappori (1988, 1992), 'rational' group consumption is defined as any Pareto efficient outcome of a within-group bargaining process. This collective approach contrasts with the conventional unitary approach, which models households as if they were single decision makers.[1]

An intrinsic feature of the collective model is the so-called sharing rule, which governs the within-household distribution of resources. This sharing rule is often interpreted as an indicator of the relative bargaining power of individual household members. Unlike other measures of power such as Pareto weights, an attractive feature of the resource sharing rule is that it is expressed in monetary terms. The sharing rule is also useful for recovering information about the economic well being of household members. For example, Lise and Seitz (2011) use sharing rule estimates to recover the population distribution of income across individuals rather than across households; Browning, Chiappori and Lewbel (2013) combine the sharing rule with other information to recover "indifference scales" that measure the welfare implications of changes in household composition; and Dunbar, Lewbel and Pendakur (2013) use sharing rule estimates to back out rates of child poverty. See also Browning, Bourguignon, Chiappori and Lechene (1994), Chiappori, Fortin and Lacroix (2002), Blundell, Chiappori and Meghir (2005), Lewbel and Pendakur (2008), Couprie, Peluso and Trannoy (2010), Bargain and Donni (2012) and Cherchye, De Rock and Vermeulen (2012a) for various applications of the collective consumption model that make use of the sharing rule concept.

In empirical analyses, the sharing rule is generally not observed. Typically, the

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[1]Chiappori (1988) only assumes Pareto efficiency of the intra-household allocation of resources. In this sense, he generalized the work of Manser and Brown (1980) and McElroy and Horney (1981) who focused on the empirical implications of specific bargaining rules such as Nash bargaining.

only information available is total household expenditures on each good or service the household buys, along with general household characteristics, like demographic composition, information on wages, income, holdings of durables, and wealth measures. Even when detailed intrahousehold consumption data are available, the sharing rule can still be hard to measure because of shared consumption, public goods, and externalities within the household.

Information regarding the sharing rule is often obtained by making use of "distribution factors", in the terminology of Browning, Bourguignon, Chiappori, and Lechene (1994) or Bourguignon, Browning and Chiappori (2009) (these also roughly correspond to what McElroy (1990) calls extra-environmental parameters). Distribution factors are observed household or neighborhood characteristics that affect Pareto weights within a household's optimization model but not the preferences of individual household members or the household budget set. A well known result in the literature is that changes in the sharing rule, resulting from changes in distribution factors, can be identified given household level demand functions. However, the level of the sharing rule is not itself identified. See, for example, Chiappori and Ekeland (2009) for a general statement and proof of this result.[2]

This nonidentification is unfortunate, because many of the uses of sharing rule estimates, such as calculation of poverty lines, indifference scales, and distributions of income and welfare, depend on the level of the sharing rule. A few different responses to this nonidentification result have been proposed. The commonest response is to ignore the problem, and only report estimates of the impact of distribution factors on the sharing rule (see, e.g., Browning, Bourguignon, Chiappori and Lechene, 1994, Chiappori, Fortin and Lacroix, 2002, or Blundell, Chiappori and Meghir, 2005). A second approach is to try to collect more information on the consumption of individual household members (see, e.g., Cherchye, De Rock and Vermeulen, 2012a, and Menon, Pendakur, and Perali, 2012). This method is inherently limited by the difficulty of measuring the fraction of shared goods that are consumed by each individual, and cannot deal with potential externalities within a household. A third response is to make additional identifying assumptions. These assumptions hypothesize that some features of individuals' preferences remain the same across households of different compositions (first proposed in Browning, Chiappori and Lewbel, 2013; see also Lewbel and Pendakur, 2008, and Bargain and Donni, 2012).

In this paper, we follow a completely different approach. We return to the standard Chiappori framework, where all that can be observed is household level demand

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[2]An additional obstacle to the use of distribution factors for identification is that distribution factors are required to only affect allocations but not tastes. This feature of not affecting tastes can be difficult to verify. For example, relative ages of the members could affect bargaining power, but age is also likely to affect members' tastes for goods, and would therefore not be valid as a distribution factor. Another example is that some studies use relative wages as distribution factors. In our analysis wages (while appearing in our model) are not distribution factors, because in addition to possibly affecting allocations, they also affect the consumption of leisure. Wages can only be distribution factors in models that assume separability of leisure from consumption.

functions, and no additional assumptions are made. We combine this standard framework with a revealed preference approach in the tradition of Samuelson (1938) and Houthakker (1950). We show that, although the sharing rule cannot be point identified, the household's demand functions do provide information regarding sharing, and that information can be used to calculate highly informative bounds on the sharing rule. In short, we show that the sharing rule can be usefully set identified. Moreover, in contrast to the rest of the literature, this set identification does not require distribution factors, the availability of detailed intrahousehold allocation data, or additional identifying assumptions on behavior across households of varying composition.

We provide a practical method for calculating upper and lower bounds on the resource shares of each individual in a household, consistent with the collective consumption model. The method allows for the presence of both public and private goods within the household, and does not require the public or private nature of any good to be specified a priori. However, if a subset of goods is known to be private, then we can use this information to tighten the bounds. Similarly, the method allows goods to have externalities within the household, so one member's consumption of a good can affect the utility of other members.

Our method begins by estimating the observable demand functions of households. These estimates can embody restrictions that are implied by rational collective household behavior, such as the absence of money illusion and the SR1 condition discussed below. Sharing rule bounds are then obtained by combining the information given by a household's demand function with inequality restrictions implied by revealed preference theory as applied to individual household members. Typically revealed preference restrictions are applied to a finite number of observed demand bundles, but we apply them to the entire demand function. Much of what makes our bounds usefully narrow is both that they make use of entire demand functions, and those demand functions themselves incorporate useful information on household characteristics and theoretical restrictions like the SR1 condition for collective household behavior.

Other papers that combine estimated demand responses with revealed preference restrictions are Blundell, Browning and Crawford (2003, 2008), though both the goals and methodology of their papers differ from ours. They assume a unitary rather than collective model of consumption behavior, so their model does not contain a sharing rule and cannot be used to analyze intrahousehold allocation issues. Their goal is to obtain bounds on demand functions that are consistent with the unitary model, when faced with a limited number of price regimes. They apply revealed preference restrictions to estimated Engel curves, rather than to demand curves. They estimate demands as functions of total expenditures separately in each of a limited number of (observed) price regimes, and they impose revealed preference restrictions (assuming the household behaves as a single utility maximizing consumer) to bound demand functions. In contrast, we estimate household demands as functions of both total expenditures and prices, and then impose revealed preference restrictions at the level of individual household members to obtain bounds on the sharing rule in a non-unitary setting. What the two approaches have in common is that they both use estimated

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demand functions instead of observed data points to tighten bounds associated with revealed preference restrictions.

While not actually identifying the sharing rule, many papers propose tests (or checks) of whether household demands are consistent with the Chiappori model of rational, Pareto efficient group consumption. Browning and Chiappori (1998) provide a differential characterization of the general collective consumption model.[3] They find that household behavior is consistent with Pareto efficient group consumption only if there exists a household pseudo-Slutsky matrix that can be decomposed as the sum of a symmetric negative semi-definite matrix and a matrix of rank 1 (in the case of two household members, i.e. the so-called SR1 condition). Chiappori and Ekeland (2006) show that this condition, together with homogeneity and adding up, is also (locally) sufficient for the existence of individual utility functions and Pareto weights that reproduce the observed household behavior. Following in the Afriat (1967), Diewert (1973) and Varian (1982) tradition, Cherchye, De Rock and Vermeulen (2007, 2010, 2011) work with discrete sets of price and quantity bundles reflecting households' expenditure choices. They derive revealed preference characterizations of several versions of the collective model, including the general version that we consider here. In particular, Cherchye, De Rock and Vermeulen (2011) show how some information regarding the sharing rule can be recovered using this pure revealed preference characterization, but only with discrete sets of price and quantity bundles, and under the assumption that the public or private nature of the goods is known. In contrast, the present paper exploits the greater information that is available in continuous demand functions for a general collective model.

To illustrate the practical usefulness of our method, we apply it to a sample of American households drawn from the 1999-2009 Panel Study of Income Dynamics (PSID). We find that our methodology yields quite narrow bounds. For example, we obtain less than a 5 percentage point difference between the upper and lower bounds on individual resource shares for more than 75% of the childless couples in our sample. We apply these results to draw a variety of meaningful and robust conclusions regarding the effects of household income and relative wages on intrahousehold resource shares, and on the prevalence of individual poverty. For example, 11% of our childless couples have incomes below a two-person poverty line, but taking the individual allocations of resources within households into account, our bounds show that 15% to 18% of individuals are below the corresponding poverty line for individuals. Moreover, it turns out that poverty is more prevalent among women in childless couples than among men in childless couples. We obtain qualitatively similar results for couples with children. This shows that our bounds are tight enough to reveal a substantial impact of intrahousehold resource share allocations on poverty measures.

The rest of this paper is organized as follows. Section 2 introduces the general col-

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[3] The term 'differential' refers to the fact that the characterization is obtained by differentiating the functional specifications of the fundamentals of the model (e.g. the utility functions or demands of the household members) as in the calculation of Slutsky matrices.

lective consumption model and the corresponding sharing rule representation. Section 3 discusses revealed preference restrictions. Section 4 considers sharing rule identification. We also introduce extensions yielding tighter bounds in settings where the private consumption of some goods is assignable to individual household members. Section 5 presents our empirical results, and Section 6 concludes.

2 The collective model and the sharing rule

This section formally presents the collective household model, and introduces the sharing rule representation that will be used in the following sections.

2.1 A general collective model

We consider a household with two individuals (1 and 2) who consume a set of goods $N = \{1, ..., |N|\}$. We focus on two member households only for notational convenience. All results can be generalized towards households with any finite number of members. Let $\mathbf{q} = \mathbf{g}(\mathbf{p}, y)$ denote a household demand function that defines a quantity bundle $\mathbf{q} \in \mathbb{R}_+^{|N|}$ as a function of prices $\mathbf{p} \in \mathbb{R}_{++}^{|N|}$ and income $y \in \mathbb{R}_{++}$.[4]

We focus on the most general version of the collective model discussed by Browning and Chiappori (1998) and Chiappori and Ekeland (2006, 2009). This model allows every good to have both private and publicly consumed components, so

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H \text{ with } \mathbf{q}^c \in \mathbb{R}_+^{|N|} \ (c = 1, 2, H),$$

with $\mathbf{q}^1$ and $\mathbf{q}^2$ the privately consumed quantities of individuals 1 and 2, and $\mathbf{q}^H$ the publicly consumed quantities. For example, if the first good in bundle $\mathbf{q}$ is gasoline, the first elements of $\mathbf{q}^1$ and $\mathbf{q}^2$ would be the quantities used by household members 1 and 2, respectively, while driving alone, and the first element of $\mathbf{q}^H$ would be the quantity gasoline they used while riding in their car together.

Next, define general (strongly concave and differentiable) utility functions U^1 and U^2 for the members, so

$$U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) \text{ and } U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right).$$

These utility functions allow for (positive) externalities associated with the privately consumed goods, e.g., $\mathbf{q}^2$ appears in U^1 because member 1 can derive utility from having member 2 consume $\mathbf{q}^2$, and vice versa.[5] Finally, the model assumes a Pareto

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[4]We refer to y as income for short, but more precisely y is the total amount of household resources that are devoted to purchasing goods and services in a given time period. This may differ from actual income due to saving or borrowing, or from implicitly buying flows of consumption from durables, and may include foregone labor income if leisure is included among consumed goods and services.

[5]Beckerian caring preferences are a special case of the general preferences we consider here. Specifically, caring preferences correspond to the model $U^m\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) = F^m(V^1(\mathbf{q}^1, \mathbf{q}^H), V^2(\mathbf{q}^2, \mathbf{q}^H))$,

efficient intrahousehold allocation, i.e. there exists a Pareto weight $\mu\left(\mathbf{p}, y\right) \in \mathbb{R}_{++}$ such that $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H = \mathbf{g}\left(\mathbf{p}, y\right)$ for

$$\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) = \arg \max_{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H} [U^1\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) + \mu\left(\mathbf{p}, y\right) U^2\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) \text{ s.t.} \qquad (1)$$

$$\mathbf{p}'\left(\mathbf{x}^1 + \mathbf{x}^2 + \mathbf{x}^H\right) \le y, \ \mathbf{x}^c \in \mathbb{R}_+^{|N|} \ (c = 1, 2, H)].$$

The Pareto weight function μ represents the 'bargaining power' of member 2 relative to member 1, in the sense that the larger the weight, the greater is the extent to which household resources are allocated based on member 2's preferences. The Pareto weight can vary with prices, household income and (like member utility functions) could also depend on other exogenous variables such as demographic characteristics of the household members. Although not necessary for our identification theorems, in our later empirical application we assume that the household does not suffer from money illusion, which implies that $\mu\left(\mathbf{p}, y\right)$ is homogeneous of degree zero in $\mathbf{p}, y$.

As noted in the introduction, some identification results are achieved in the collective household model literature by assuming the existence of distribution factors, that is, additional exogenous variables that affect Pareto weights but do not affect the members' utility functions or the budget set. Our model permits but does not require or assume the existence of distribution factors.

2.2 Sharing rule representation

We now describe a sharing rule representation of household demand behavior that is consistent with the model introduced above.[6] Intuitively, this representation provides a decentralized interpretation of the household's optimization problem as defined in (1). Specifically, it represents Pareto efficient household behavior as equivalent to the outcome of a two-step allocation procedure. In the first step, the so-called *sharing rule* distributes the aggregate household income y across the group members, defining individual income shares y^1 and y^2 such that $y = y^1 + y^2$. In the second step, each individual maximizes her/his own utility function subject to her/his own income share and evaluated at her/his own Lindahl prices associated with privately and publicly consumed quantities. The Lindahl prices represent each individual's marginal willingness to pay for the different quantities. We are not assuming that households literally allocate resources using this procedure. Rather, the sharing rule representation simply states that the outcome of the household's allocation process defined in (1) is mathematically equivalent to this two step allocation procedure.

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where F^m is an increasing function and V^i are standard utility functions. See also Chiappori (1988, 1992) for a more detailed discussion.

[6]See Chiappori (1988, 1992) for a detailed discussion on the equivalence between the characterizations of Pareto efficient consumption behavior in (1) and (2). Chiappori concentrated on a simplified setting with privately consumed quantities without externalities. However, extending his argument to our setting is relatively straightforward.

To formalize the sharing rule representation, for each individual m ($m = 1, 2$) consider an individual demand function $\mathbf{g}^m$ that defines a quantity bundle $\widetilde{\mathbf{q}}^m \in \mathbb{R}_+^{|N|}$ as a function of individual prices $\mathbf{p}^{m,1} \in \mathbb{R}_+^{|N|}$, $\mathbf{p}^{m,2} \in \mathbb{R}_+^{|N|}$, $\mathbf{p}^{m,H} \in \mathbb{R}_+^{|N|}$ and individual income $y^m \in \mathbb{R}_{++}$. Specifically let $\widetilde{\mathbf{q}}^m = \mathbf{g}^m(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ where $\widetilde{\mathbf{q}}^m = \widetilde{\mathbf{q}}^{m,1} + \widetilde{\mathbf{q}}^{m,2} + \widetilde{\mathbf{q}}^{m,H}$ with

$$(\widetilde{\mathbf{q}}^{m,1}, \widetilde{\mathbf{q}}^{m,2}, \widetilde{\mathbf{q}}^{m,H}) = \arg \max_{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H} [U^m(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H) \text{ s.t.} \tag{2}$$

$$(\mathbf{p}^{m,1})' \mathbf{x}^1 + (\mathbf{p}^{m,2})' \mathbf{x}^2 + (\mathbf{p}^{m,H})' \mathbf{x}^H \leq y^m, \mathbf{x}^c \in \mathbb{R}_+^{|N|} \ (c = 1, 2, H)].$$

Consider $\mathbf{q}^1$, $\mathbf{q}^2$ and $\mathbf{q}^H$ (with $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$) that solve (1). The same quantities solve (2) (and thus $\mathbf{q} = \widetilde{\mathbf{q}}^m$) if

$$\mathbf{p}^{m,c} = U_{\mathbf{q}^c}^m / \lambda^m, \tag{3}$$

where $U_{\mathbf{q}^c}^m$ is the gradient of the function U^m defined at $\mathbf{q}^c$ ($c = 1, 2, H$), λ^1 is the optimizing value of the Lagrange multiplier for the budget constraint in (1), and $\lambda^2 = \lambda^1 / \mu$. By this construction, each vector $\mathbf{p}^{m,c} \in \mathbb{R}_+^{|N|}$ represents individual m's marginal willingness to pay for $\mathbf{q}^c$.

Pareto efficiency implies $\mathbf{p}^{1,c} + \mathbf{p}^{2,c} = \mathbf{p}$ ($c = 1, 2, H$) by construction, so $\mathbf{p}^{m,c}$ equals the Lindahl price vector for individual m associated with $\mathbf{q}^c$. Thus, under these prices the maximization program (2) corresponds to the second step of the two-step procedure described above (given y^1 and y^2 defined in the first step). Slightly abusing our notation, the sharing rule can be defined as the construction of y^1 and y^2 that solves

$$(y^1, y^2) = \arg \max_{\widetilde{y}^1, \widetilde{y}^2} \left[\begin{array}{c} U^1(\mathbf{g}^1(\mathbf{p}^{1,1}, \mathbf{p}^{1,2}, \mathbf{p}^{1,H}, \widetilde{y}^1)) + \mu(\mathbf{p}, y) U^2(\mathbf{g}^2(\mathbf{p}^{2,1}, \mathbf{p}^{2,2}, \mathbf{p}^{2,H}, \widetilde{y}^2)) \\ \text{s.t. } \widetilde{y}^1 + \widetilde{y}^2 \leq y \end{array} \right]. \tag{4}$$

As noted in the introduction, the sharing rule is often used in place of the Pareto weight as a bargaining power measure, where the higher the relative income share y^m / y of member m is, the greater is his/her bargaining power in the household.[7] The sharing rule concept is often a more useful measure than the Pareto weight μ in empirical applications, because it is independent of cardinal representations of preferences. In addition, since y^1 and y^2 are measured in the same units as income y, they can be interpreted as measures of the wealth or poverty of individual household members.

3 Revealed preferences

Typical household data sets provide enough information to empirically estimate the household demand function $\mathbf{g}(\mathbf{p}, y)$, but not the individual demand functions $\mathbf{g}^m$ or income shares y^m ($m = 1, 2$). Our primary goal is then to recover information regarding

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[7]See, for example, Browning, Chiappori and Lewbel (2013) for a detailed discussion on the relation between relative income shares y^m / y in (2) and the bargaining Pareto weight μ in (1).

the sharing rule that defines the individual incomes y^1 and y^2 given $\mathbf{g}\left(\mathbf{p}, y\right)$. In this section we provide a revealed preference characterization of the collective consumption model. The inequalities obtained by this characterization will then be used to identify our bounds on the sharing rule.

3.1 Basic concepts

In this subsection we define some standard concepts in the revealed preference literature, as applied to an individual's demand function $\mathbf{g}^m$. For given $\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}$ and y^m, define the budget set

$$B\left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m\right) = \{\mathbf{x} \in \mathbb{R}_+^{|N|} | \mathbf{x} = \mathbf{x}^1 + \mathbf{x}^2 + \mathbf{x}^H,$$
$$\left(\mathbf{p}^{m,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}^{m,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}^{m,H}\right)' \mathbf{x}^H \leq y^m\}.$$

We now define the concept of the revealed preference relation associated with the demand function $\mathbf{g}^m$, denoted by $R_o^{\mathbf{g}^m}$.

Definition 1 (direct revealed preference) *Let $\mathbf{g}^m$ be an individual demand function. The direct revealed preference relation associated with $\mathbf{g}^m$ is defined by: for all $\mathbf{x}$, $\mathbf{z} \in \mathbb{R}_+^{|N|}$: $\mathbf{x} R_o^{\mathbf{g}^m} \mathbf{z}$ if there exist $\mathbf{p}^{m,c} \in \mathbb{R}_+^{|N|}$ ($c = 1, 2, H$) and $y^m \in \mathbb{R}_{++}$ such that $\mathbf{x} = \mathbf{g}^m\left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m\right)$ and $\mathbf{z} \in B(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ with $\mathbf{x} \neq \mathbf{z}$.*

In words, quantity bundle $\mathbf{x}$ is revealed preferred to another bundle $\mathbf{z}$ if $\mathbf{z}$ belonged to the budget set $B(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ under which $\mathbf{x}$ was chosen by equaling $\mathbf{g}^m(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$. Here $\mathbf{x} R_o^{\mathbf{g}^m} \mathbf{z}$ because the individual could have afforded to choose $\mathbf{z}$ but did not do so, choosing $\mathbf{x}$ instead. The revealed preference concept in Definition 1 differs from the standard one only in that, for our collective setting, revealed preferences are defined at the level of an individual household member m, while in a standard unitary context they are defined at the level of the aggregate household. Correspondingly, we consider preferences that pertain to decomposed quantity bundles, which are evaluated at individual prices $\mathbf{p}^{m,1}$, $\mathbf{p}^{m,2}$ and $\mathbf{p}^{m,H}$ for individual m. The associated quantity decomposition appears from the definition of the budget set $B\left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m\right)$.

We can now define the Weak Axiom of Revealed Preference (WARP; after Samuelson, 1938) for an individual m.

Definition 2 (WARP) *Let $\mathbf{g}^m$ be an individual demand function. This function $\mathbf{g}^m$ satisfies WARP if the relation $R_o^{\mathbf{g}^m}$ is asymmetric.*

Our sharing rule identification method will exploit the empirical implications of WARP in the context of the collective household consumption model.

As shown by Houthakker (1950), utility maximization generally implies that the Strong Axiom of Revealed Preference (SARP) holds. SARP extends WARP by also exploiting transitivity of preferences. We could readily reformulate Proposition 2 below, showing how to include SARP restrictions in our identification bounds in theory.

However, we only exploit WARP and not SARP in our application because it would be much more difficult if not completely intractable to fully operationalize transitivity when empirically implementing our identification procedure. It might be possible in practice to incorporate some but not all of the additional restrictions implied by SARP, by only considering the transitivity conditions implied by revealed preference sequences of a given fixed maximum length. See, for example, Echenique, Lee and Shum (2011), who apply a fixed length SARP procedure, albeit in a simpler context than ours. In their application, they found that considering SARP with sequences of fixed maximum length instead of WARP did not substantively affect their empirical results. Also, the loss of information in only considering WARP rather than SARP will be zero under conditions described by Uzawa (1960) and Bossert (1993).

3.2 Characterizing the collective model

We have described revealed preference relations in terms of individual demand functions $\mathbf{g}^m$, which are not observed. We now define the concept of 'admissible' individual demand functions. Roughly, this concept captures all possible specifications of the (unknown) individual demand functions that are consistent with the (known) household demand function. We will then apply revealed preference relations to admissible demand functions to obtain sharing rule bounds.

Definition 3 (admissible individual demands) *For a given household demand function $\mathbf{g}$, the pair of individual demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ are admissible if, for all $\mathbf{p}$ and y,*

$$\mathbf{g}\left(\mathbf{p},y\right) = \mathbf{g}^1\left(\mathbf{p}^{1,1},\mathbf{p}^{1,2},\mathbf{p}^{1,H},y^1\right) = \mathbf{g}^2\left(\mathbf{p}^{2,1},\mathbf{p}^{2,2},\mathbf{p}^{2,H},y^2\right),$$

for some $\mathbf{p}^{m,c}\left(m=1,2;\ c=1,2,H\right)$ and y^m such that

$$y^1 + y^2 = y \ \text{and} \ \mathbf{p}^{1,c} + \mathbf{p}^{2,c} = \mathbf{p}, \ \text{with} \ y^m \in \mathbb{R}_{++} \ \text{and} \ \mathbf{p}^{m,c} \in \mathbb{R}_+^{|N|}.$$

Let $Q\left(\mathbf{g}\right)$ represent the collection of all admissible individual demands $\mathbf{g}^1$ and $\mathbf{g}^2$, i.e.,

$$Q\left(\mathbf{g}\right) = \{\left(\mathbf{g}^1,\mathbf{g}^2\right) \mid \mathbf{g}^1 \ \text{and} \ \mathbf{g}^2 \ \text{are admissible for the household demand function} \ \mathbf{g}\}.$$

We can now define the condition for a collective rationalization that we will use for identification. Basically, this condition states that for the given household demand function $\mathbf{g}$, there must exist at least one specification of admissible individual demand functions that solves (2).

Definition 4 (collective rationalization) *Let $\mathbf{g}$ be a household demand function. A pair of utility functions U^1 and U^2 provides a collective rationalization of $\mathbf{g}$ if there exist admissible individual demand functions $\left(\mathbf{g}^1,\mathbf{g}^2\right) \in Q\left(\mathbf{g}\right)$ such that, for each m,*

$$\mathbf{g}^m\left(\mathbf{p}^{m,1},\mathbf{p}^{m,2},\mathbf{p}^{m,H},y^m\right) = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$$

for

$$(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H) = \arg \max_{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H} [U^m (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H) \;\; s.t.$$

$$(\mathbf{p}^{m,1})' \mathbf{x}^1 + (\mathbf{p}^{m,2})' \mathbf{x}^2 + (\mathbf{p}^{m,H})' \mathbf{x}^H \leq y^m].$$

We have the following result, which establishes a revealed preference characterization of the collective consumption model under consideration. (The appendix contains the proofs of our main results.)

Proposition 1 *Consider a household demand function* $\mathbf{g}$*. There exists a pair of utility functions* U^1 *and* U^2 *that provides a collective rationalization of* $\mathbf{g}$ *only if there exist admissible individual demand functions* $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ *that both satisfy WARP.*

In theory, we can construct bounds by enumerating every element of $Q(\mathbf{g})$ and evaluating every possible sharing rule consistent with each of these elements. However, such an enumeration is not tractable, so the aim now is to construct a computationally feasible procedure for obtaining these bounds.

4 Sharing rule identification

Consider a household demand function $\mathbf{g}$. Suppose that we evaluate a situation E that is characterized by prices $\mathbf{p}_E$ and household income y_E. The household's demand in this particular situation is observed and equals $\mathbf{g}(\mathbf{p}_E, y_E)$. The (set) identification question asks for bounds on the individual incomes (y_E^1 and y_E^2) that are consistent with a collective rationalization of the observed household demand $\mathbf{g}$. Our procedure will start from the characterization given in Proposition 1. Essentially, we define lower bounds y_E^{l1} and y_E^{l2} and upper bounds y_E^{u1} and y_E^{u2} so that

$$y_E^{l1} < y_E^1 < y_E^{u1} \text{ and } y_E^{l2} < y_E^2 < y_E^{u2}. \tag{5}$$

By construction, these bounds will simultaneously apply to all possible specifications of admissible individual demand functions.

4.1 Identification in theory

To sketch the basic idea of our approach, suppose for now that the individual demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ were known (in addition to knowing the household demand function $\mathbf{g}$). Then we would also know the individual prices $\mathbf{p}^{m,c}$ ($m = 1, 2$; $c = 1, 2, H$) and income y^m that imply $\mathbf{g}(\mathbf{p}, y) = \mathbf{g}^m(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ for each $\mathbf{p}$ and y. See Definition 3. So if the demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ were known, then the income shares would be known.

Although in practice we do not know the true $\mathbf{g}^1$ and $\mathbf{g}^2$, what we do know is that they must be admissible. Let $\mathbf{q}_E = \mathbf{g}(\mathbf{p}_E, y_E) = \mathbf{g}^m(\mathbf{p}_E^{m,1}, \mathbf{p}_E^{m,2}, \mathbf{p}_E^{m,H}, y_E^m)$. A necessary

condition for a given pair of functions $\mathbf{g}^1$ and $\mathbf{g}^2$ to be admissible, and hence consistent with a collective rationalization of $\mathbf{g}$ is

$$
y_E^1 \leq \inf_{\mathbf{x}_1^1, \mathbf{x}_1^2, \mathbf{x}_1^H} [\left(\mathbf{p}_E^{1,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_E^{1,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_E^{1,H}\right)' \mathbf{x}_1^H | \mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E] \text{ and}
$$

$$
y_E^2 \leq \inf_{\mathbf{x}_2^1, \mathbf{x}_2^2, \mathbf{x}_2^H} [\left(\mathbf{p}_E^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_E^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_E^{2,H}\right)' \mathbf{x}_2^H | \mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E]. \tag{6}
$$

This necessary condition for a collective rationalization of $\mathbf{g}$ follows immediately from the WARP conditions in Proposition 1. The right hand sides of the inequalities in (6) define upper bounds y_E^{u1} and y_E^{u2} for the income shares y_E^1 and y_E^2, i.e. $y_E^1 < y_E^{u1}$ and $y_E^2 < y_E^{u2}$. Lower bounds then follow from $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$.

Since we do not know the functions $\mathbf{g}^1$ and $\mathbf{g}^2$, we obtain bounds by considering all possible admissible individual demands $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$. The tightest bounds will be obtained if the (positive) differences $y_E^{u1} - y_E^{l1}$ and $y_E^{u2} - y_E^{l2}$ are as small as possible. Substituting $y_E - y_E^{u2}$ for y_E^{l1} in $y_E^{u1} - y_E^{l1}$ (or substituting $y_E - y_E^{u1}$ for y_E^{l2} in $y_E^{u2} - y_E^{l2}$) obtains that these tightest bounds correspond to the smallest value of $y_E^{u1} + y_E^{u2} - y_E$. Since y_E is a constant, we will aim at minimizing the sum $y_E^{u1} + y_E^{u2}$ in what follows. (In this respect, also observe that minimizing $(y_E^{u1} + y_E^{u2})$ is equivalent to maximizing $(y_E^{l1} + y_E^{l2})$.)

Summarizing, the upper bounds y_E^{u1} and y_E^{u2} are required to solve

$$
\sup_{(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})} \inf_{\substack{\mathbf{x}_1^1, \mathbf{x}_1^2, \mathbf{x}_1^H \\ \mathbf{x}_2^1, \mathbf{x}_2^2, \mathbf{x}_2^H}} (y_E^{u1} + y_E^{u2}) \tag{P.0}
$$

s.t.

$$
y_E^{u1} = \left(\mathbf{p}_E^{1,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_E^{1,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_E^{1,H}\right)' \mathbf{x}_1^H,
$$

$$
y_E^{u2} = \left(\mathbf{p}_E^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_E^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_E^{2,H}\right)' \mathbf{x}_2^H,
$$

$$
\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E,
$$

$$
\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E.
$$

The supremum operator in this objective makes that the upper bounds y_E^{u1} and y_E^{u2} apply to any possible specification of $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$. Corresponding lower bounds are defined as $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$. It directly follows that a collective rationalization of the data is possible only for individual income shares y_E^1 and y_E^2 that meet (5).

It follows from our discussion of the preceding program P.0 that, if the solution value of program P.0 does not exceed y_E, then it is impossible to specify income shares y_E^1 and y_E^2 that meet (5). If this happens then the demand function $\mathbf{g}$ cannot be collectively rationalized, meaning that the collective model is rejected for the given household demand function $\mathbf{g}$. Analogous results apply to the programs that we present below.

4.2 Identification in practice

The program P.0 described above is not yet empirically tractable, because it requires direct consideration of all the (infinitely many) elements of $Q(\mathbf{g})$. In this section, based on program P.0 we provide a program describing bounds that can be applied in empirical practice. Our procedure begins by characterizing the bundles $\mathbf{x}_1$ and $\mathbf{x}_2$ that satisfy $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ in program P.0, in terms of the observed household demand function $\mathbf{g}$.

Proposition 2 *Let $\mathbf{g}$ be a household demand function. Then, we have $\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E$ ($m, l \in \{1, 2\}$, $m \neq l$) for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2)$ $\in Q(\mathbf{g})$ that satisfy WARP if $\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1)$ and $\mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2)$ such that*

$$y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2) \ \text{ and } \ y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1). \tag{C}$$

This proposition shows that as long as condition C holds, we can conclude that $\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E$. This result makes it possible to compute the upper bounds y_E^{u1} and y_E^{u2} (and corresponding lower bounds $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$) through the following programming problem:

$$\min_{\mathbf{p}_1, \mathbf{p}_2, y_1, y_2} (y_E^{u1} + y_E^{u2}) \tag{P.1}$$

$$\text{s.t.}$$

$$y_E^{u1} = \mathbf{p}_E' \mathbf{x}_1, \ y_E^{u2} = \mathbf{p}_E' \mathbf{x}_2, \tag{P.1-1}$$

$$y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2), \ y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1), \tag{P.1-2}$$

$$\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1), \ \mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2). \tag{P.1-3}$$

To see how this program works, observe first that, as in program P.0, the objective minimizes the sum $(y_E^{u1} + y_E^{u2})$ by suitably selecting an $\mathbf{x}_1$ and $\mathbf{x}_2$, as defined by $\mathbf{p}_1$, $\mathbf{p}_2$, y_1 and y_2 using (P.1-3). The lower the value of the objective function, the tighter are the sharing rule bounds. Next, because of Proposition 2, the constraint (P.1-2) implies $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ or $\mathbf{x}_1 R_o^{\mathbf{g}^2} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^1} \mathbf{q}_E$. Without loss of generality, we assume $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$. Because $\mathbf{x}_m R_o^{\mathbf{g}^m} \mathbf{q}_E$, we need

$$y_E^m < \left(\mathbf{p}_E^{m,1}\right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2}\right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H}\right)' \mathbf{x}_m^H \tag{7}$$

as can be seen by comparing this with (6) above. By construction, we have $\left(\mathbf{p}_E^{m,1}\right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2}\right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H}\right)' \mathbf{x}_m^H \leq \mathbf{p}_E' \mathbf{x}_m$ for any specification of $\mathbf{p}_E^{m,c}$ and $\mathbf{x}_m^c$ ($c = 1, 2, H$).

A feature of this program is that we avoid having to specify $\mathbf{p}_E^{m,c}$ and $\mathbf{x}_m^c$, by instead using just the upper bound $y_E^{um} = \mathbf{p}_E' \mathbf{x}_m$ in (P.1-1). This parallels the fact that we also do not need to consider particular specifications of the individual functions $\mathbf{g}^m$ in the program P.1. This is what makes the program P.1 empirically tractable. We compute the bounds y_E^{u1}, y_E^{u2}, y_E^{l1}, and y_E^{l2} by applying nonlinear programming techniques to

this program. The application of these numerical methods are simplified by the fact that both the objective and most of the constraints in program P.1 are linear. The only nonlinear constraint is (P.1-3), since household demand functions **g** are generally nonlinear.

4.3 Extensions: using more information to tighten bounds

In empirical applications, it is often reasonable to assume that a subset of goods is privately consumed without externalities, while the nature of the other goods is unknown. This type of information is often exploited to aid in the identification of collective household models. Particular examples of privately consumed goods include assignable goods and exclusive goods. An assignable good is one where information about who consumes various quantities of the good is known to the researcher. An example may be clothing. If information is collected about whether purchased clothing is male or female, then that information may be used to assign these clothing expenditures to the husband or wife. Closely related are exclusive goods, which are goods that are only consumed by one household member. For example, it is often assumed that an individual's leisure in a labor supply setting is exclusively consumed by that individual. If assignable or exclusive goods do not entail externalities, then they appear only in the utility function of one household member, which aids in identification (see Bourguignon, Browning and Chiappori, 2009, for more discussion). This section considers including such information on privately consumed goods to tighten sharing rule bounds.

4.3.1 Private goods without externalities

Let N_A be a subset of private goods without externalities and let N_B be the subset of other goods, so $N = N_A \cup N_B$. For any good $n \in N_A$, we can add the following condition to the set of collective rationalization conditions in Definition 4. Letting $(\mathbf{a})_n$ denote the nth entry of vector $\mathbf{a}$, for every $n \in N_A$,

$$\left(\mathbf{p}^{1,2}\right)_n = 0 \text{ (or } \left(\mathbf{p}^{1,1}\right)_n = (\mathbf{p})_n) \text{ and } \left(\mathbf{p}^{2,1}\right)_n = 0 \text{ (or } \left(\mathbf{p}^{2,2}\right)_n = (\mathbf{p})_n). \qquad (8)$$

Intuitively, this condition says that, because there are no consumption externalities for good n, the willingness to pay of household member m for member l's consumption of good n is zero. Formally, condition (8) can be obtained directly from the definition of $\left(\mathbf{p}^{m,c}\right)_n$ in (3).

Using (8), we can immediately reformulate program P.1 as follows:

$$\min_{\mathbf{p}_1, \mathbf{p}_2, y_1, y_2} \left(y_E^{u1} + y_E^{u2} \right) \tag{P.2}$$

s.t.

$$y_E^{u1} = \sum_{n \in N_A} (\mathbf{p}_E)_n \left(\mathbf{x}_1^1 \right)_n + \sum_{n \in N_B} (\mathbf{p}_E)_n \left(\mathbf{x}_1 \right)_n, \tag{P.2-1}$$

$$y_E^{u2} = \sum_{n \in N_A} (\mathbf{p}_E)_n \left(\mathbf{x}_2^2 \right)_n + \sum_{n \in N_B} (\mathbf{p}_E)_n \left(\mathbf{x}_2 \right)_n, \tag{P.2-2}$$

$$y_1 \geq \mathbf{p}_1' \left(\mathbf{q}_E + \mathbf{x}_2 \right), \; y_2 \geq \mathbf{p}_2' \left(\mathbf{q}_E + \mathbf{x}_1 \right), \tag{P.2-3}$$

$$\left(\mathbf{x}_k \right)_n = \left(\mathbf{x}_k^1 \right)_n + \left(\mathbf{x}_k^2 \right)_n \quad (k = 1, 2; \; n \in N_A), \tag{P.2-4}$$

$$\mathbf{x}_1 = \mathbf{g} \left(\mathbf{p}_1, y_1 \right), \; \mathbf{x}_2 = \mathbf{g} \left(\mathbf{p}_2, y_2 \right).$$

Similar to before, the constraint (P.2-3) implies $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$. Thus, we again get the condition (7). In this case, we have

$$\left(\mathbf{p}_E^{m,m} \right)_n \left(\mathbf{x}_m^m \right)_n = (\mathbf{p}_E)_n \left(\mathbf{x}_m^m \right)_n \text{ and } \left(\mathbf{p}_E^{m,l} \right)_n \left(\mathbf{x}_m^m \right)_n = 0 \; (m \neq l) \text{ if } n \in N_A,$$

while

$$\left(\mathbf{p}_E^{m,1} \right)_n \left(\mathbf{x}_m^1 \right)_n + \left(\mathbf{p}_E^{m,2} \right)_n \left(\mathbf{x}_m^2 \right)_n + \left(\mathbf{p}_E^{m,H} \right)_n \left(\mathbf{x}_m^H \right)_n \leq (\mathbf{p}_E)_n \left(\mathbf{x}_m \right)_n \text{ if } n \in N_B.$$

Therefore, we can use

$$\left(\mathbf{p}_E^{m,1} \right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2} \right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H} \right)' \mathbf{x}_m^H \leq \sum_{n \in N_A} (\mathbf{p}_E)_n \left(\mathbf{x}_m^m \right)_n + \sum_{n \in N_B} (\mathbf{p}_E)_n \left(\mathbf{x}_m \right)_n,$$

which yields (P.2-1) and (P.2-2) instead of (P.1-1) as before. Note that for $n \in N_A$ the privately consumed quantities $(\mathbf{x}_k^1)_n$ and $(\mathbf{x}_k^2)_n$ are not given a priori and are therefore defined within program P.2 (subject to the constraint (P.2-4)). Like the previous program, we can solve program P.2 by nonlinear programming techniques.

4.3.2 Assignable and exclusive goods

So far, we have used the fact that goods $n \in N_A$ only affect the utility of one household member (i.e., exclusivity), but we have not yet exploited assignability of these goods to individual household members. To make use of assignability, let $N_{Am} \subseteq N_A$ represent the set of goods that are assignable (or exclusive) to member m. Then, we get

$$\left(\mathbf{x}_k^m \right)_n = \left(\mathbf{x}_k \right)_n \text{ if } n \in N_{Am}.$$

Using this, we obtain the following extension of Proposition 2.

Proposition 3 *Let $\mathbf{g}$ be a household demand function. Then, we have $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that satisfy*

WARP if $\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1)$ *and* $\mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2)$ *such that one of the following conditions holds:*

$$y_1 \geq \mathbf{p}_1'(\mathbf{q}_E + \mathbf{x}_2) \quad and \tag{C.1}$$
$$\sum_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n \geq \mathbf{p}_2'\mathbf{x} - \sum_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n \quad for \ \mathbf{x} = \mathbf{x}_1, \mathbf{q}_E,$$

$$y_2 \geq \mathbf{p}_2'(\mathbf{q}_E + \mathbf{x}_1) \quad and \tag{C.2}$$
$$\sum_{n \in N_{A_1}} (\mathbf{p}_1)_n (\mathbf{x}_1)_n \geq \mathbf{p}_1'\mathbf{x} - \sum_{n \in N_{A_2}} (\mathbf{p}_1)_n (\mathbf{x})_n \quad for \ \mathbf{x} = \mathbf{x}_2, \mathbf{q}_E,$$

or

$$\sum_{n \in N_{A_1}} (\mathbf{p}_1)_n (\mathbf{x}_1)_n \geq \mathbf{p}_1'\mathbf{q}_E - \sum_{n \in N_{A_2}} (\mathbf{p}_1)_n (\mathbf{q}_E)_n \quad and \tag{C.3}$$
$$\sum_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n \geq \mathbf{p}_2'\mathbf{q}_E - \sum_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{q}_E)_n.$$

The essential difference between this result and Proposition 2 is that, in contrast to condition C, the new conditions C.1, C.2 and C.3 'assign' preference relations to individual household members (i.e. we get $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$), based on the assignable goods.

Using Proposition 3, we must consider three nonlinear programs (in addition to program P.2) to define y_E^{u1}, y_E^{u2}, y_E^{l1}, y_E^{l2}. Each program has the same structure as P.2, except that the condition C in the constraint (P.2-3) is replaced by one of the conditions C.1-C.3. The best upper and lower bounds then correspond to minimum and maximum values defined over the three programs.

4.4   A stylized example

We end this theoretical section with a stylized example that demonstrates the mechanics of program P.1. This example shows that it is possible for our method to yield arbitrarily narrow bounds on the income shares, even when no good is a priori specified as public or private.

For this particular example assume we have three goods so $|N| = 3$, and, letting $(\mathbf{q})_n$ denote the quantity of the nth good in $\mathbf{q}$, assume member utility functions (unknown to the researcher) are given by

$$U^1 = (\delta A) \cdot (\mathbf{q})_1 + (B - (\delta - 1)A) \cdot (\mathbf{q})_2 + C \cdot (\mathbf{q})_3 \text{ and} \tag{9}$$
$$U^2 = (B - (\delta - 1)A) \cdot (\mathbf{q})_1 + \delta A \cdot (\mathbf{q})_2 + C \cdot (\mathbf{q})_3, \tag{10}$$

where δ, A, B and C are positive real numbers specified below. Non-negative consumption externalities require $(B - (\delta - 1)A)$ to be positive. Intuitively, one may think of goods 1 and 2 as leisure of the first and the second household member, while good 3 represents other household consumption.

Next, assume the (unknown to the researcher) Pareto weight specification is

$$\mu = 0 \text{ if } (\mathbf{p})_1 > (\mathbf{p})_2, \ \mu = \infty \text{ if } (\mathbf{p})_2 > (\mathbf{p})_1 \text{ and } \mu = 1 \text{ if } (\mathbf{p})_1 = (\mathbf{p})_2. \qquad (11)$$

In the example where goods 1 and 2 are leisure, this corresponds to a model where the household member with the higher wage exercises complete control over the household's consumption allocation, while both members have equal bargaining weight if their wages are equal.[8]

For given ϵ, assuming $0 < \epsilon < 1$, specify δ, A, B and C such that

$$C < \delta A < A + B < C\frac{1 + 2\epsilon}{1 + \epsilon} \text{ and } (A + B)\frac{2 + 2\epsilon}{2 + 3\epsilon} < \delta A.$$

Then, for $(B - (\delta - 1)A)$ positive but sufficiently small, it is easily verified that the model specification in (9) and (11) generates the following results.

For

$$\mathbf{p}_E = (0.5 + \epsilon, 0.5 + \epsilon, 1 + \epsilon) \text{ and } y_E = 1$$

the household demand function $\mathbf{g}$ generated by this model is

$$\mathbf{q}_E = \mathbf{g}\left(\mathbf{p}_E, y_E\right) = \left(0, 0, \frac{1}{1 + \epsilon}\right),$$

while for

$$\mathbf{p}_1 = \left(1 + \epsilon, \frac{\epsilon}{2}, 1 + \epsilon\right) \text{ and } y_1 = 1 + \epsilon,$$

$$\mathbf{p}_2 = \left(\frac{\epsilon}{2}, 1 + \epsilon, 1 + \epsilon\right) \text{ and } y_2 = 1 + \epsilon,$$

the household demand function is

$$\mathbf{x}_1 = \mathbf{g}\left(\mathbf{p}_1, y_1\right) = (1, 0, 0) \text{ and } \mathbf{x}_2 = \mathbf{g}\left(\mathbf{p}_2, y_2\right) = (0, 1, 0).$$

Note that

$$y_1 = 1 + \epsilon, \ \mathbf{p}_1'\mathbf{x}_2 = \frac{\epsilon}{2}, \ \mathbf{p}_1'\mathbf{q}_E = 1,$$

$$y_2 = 1 + \epsilon, \ \mathbf{p}_2'\mathbf{x}_1 = \frac{\epsilon}{2}, \ \mathbf{p}_2'\mathbf{q}_E = 1,$$

so that constraint (P.1-2) is indeed satisfied.

---

[8]The assumption that $\mu = 0$ if a household member does not have any control over the household's allocation is made for convenience. It can be easily shown that the argument holds for a Pareto weight that is arbitrarily close to 0. This is of course an unrealistically extreme example, but we are only using it to make a point about the potential informativeness of our bounds procedure.

Applying our program P.1 to this household demand function yields bounds

$$y_E^{u1} = \mathbf{p}'_E \mathbf{x}_1 = 0.5 + \epsilon, \ y_E^{u2} = \mathbf{p}'_E \mathbf{x}_2 = 0.5 + \epsilon,$$
$$y_E^{l1} = y_E - y_E^{u2} = 0.5 - \epsilon, \ y_E^{l2} = y_E - y_E^{u1} = 0.5 - \epsilon.$$

These upper and lower bounds become arbitrarily tight as ϵ gets arbitrarily small.

This example generated household demand functions in which, for some values of prices, only one of the three goods was demanded in nonzero quantities. This was done only for mathematical simplicity. It would be possible, with more complicated models, to generate examples where quantities are always nonzero while bounds are still arbitrarily narrow.

While this example shows that it is possible in theory to obtain arbitrarily narrow bounds without any information on the privateness or assignability of any good, in practice, such information will typically be helpful in tightening bounds. This will be the case for the empirical application that we present in the next section.

5 Application

5.1 Set-up

We apply our method to a model of joint labor supply and consumption decisions of couples with and without children. Specifically, at observed individual wage rates and market prices, we consider the allocation of a household's full income to both spouses' leisure and to a set of three types of nondurable consumption goods. Here full income is defined as the sum of both spouses' maximum possible labor income and nonlabor income (excluding savings and expenditures on durables). While we rely on the standard assumption of separability between the modeled commodities and durable goods, we explicitly allow for non-separability between the spouses' leisure and nondurables consumption.

This joint consumption-labor supply setting contains substantial price and wage variation, which is useful for obtaining informative sharing rule bounds. Also, this application allows us to consider various assumptions regarding the nature of the different goods. For example, we examine how much the bounds tighten when we treat each individual's leisure as an exclusive good without externalities. Lise and Seitz (2011) similarly use labor supply to identify resource shares, but for identification their results depend on strong functional form assumptions, as well as restrictions across households like those in Browning, Chiappori, and Lewbel (2013).

Some implementations of collective consumption models treat wages as distribution factors, thereby assuming that wages only affect Pareto weights. These models implicitly assume that consumption goods are separable from leisure in utility functions, and only deal with the consumption component of utility. In our application, we use wages as prices (of leisure), which may therefore affect both individual demand functions via

the budget constraint, as well as appearing directly as arguments in the household's Pareto weights. As pointed out above, our methodology does not require existence of any distribution factors, though if any are available they could be readily incorporated into the analysis.

5.2 Data

We apply our sharing rule identification method to a sample of households drawn from the 1999-2009 Panel Study of Income Dynamics (PSID). This widely used data set began in 1968 with a representative sample of over 18,000 individuals living in 5,000 households in the United States. The data set contains information about employment, income, wealth and socio-demographic variables of these individuals and their descendants. Moreover, since 1999 the panel also contains expenditures on a detailed set of consumption categories (see Blundell, Pistaferri and Saporta-Eksten, 2014, for more details on this consumption data).

In each year of data, we select households that are couples where both adult members (aged between 25 and 65) are participating in the labor market, so both are on an intensive margin regarding their supply of labor. The self-employed are excluded to avoid issues regarding the imputation of wages and the separation of consumption from work-related expenditures. In what follows, we will conduct separate analyses for a sample of childless households and a sample of households with children. For this last sample, we restricted the age of children to under 18 years.

Finally, we deleted households with important missing information (mostly, incomplete information on expenditures) and trimmed out a few households with extremely high or low expenditures or wages. We do not exploit the panel aspect of this dataset, because the number of periods for which we have detailed consumption data is limited and yields a very unbalanced panel. Our resulting data set consists of a first sample of 865 observations on childless couples and a second sample of 1390 observations of households with children.

We analyze the allocation of the households' full income to both spouses' leisure and to three categories of consumption: food (which includes food at home and food outside the home), housing (rent or rent equivalent, home insurance and utilities) and other goods (health, transportation, education and child care). These consumption categories together comprise all the consumption available in the PSID. Our five categories of commodities provide sufficient relative price and wage variation to obtain significantly informative bounds, while avoiding modeling issues associated with zero consumption of some goods, which can arise at more disaggregate levels of consumption.[9]

Tables 1 and 2 provide summary statistics on the relevant data for the two samples

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[9]Zeros arising from corner solutions (such as nonparticipation on the labor market or non-consumption of some goods) do not interfere with the revealed preference characterization of the collective model that underlies our bounds identification method. However, such zeros complicate estimation of household demand functions of prices and income. To deal with these issues, one might proceed along the lines of Donni (2003) or Blundell, Chiappori, Magnac and Meghir (2007).

at hand. Wages are net hourly wages. Leisure and annual hours worked are measured in hours per year. To compute leisure we assume that an individual needs 8 hours per day for non market labor requirements like sleeping (and so could work at a job for at most 16 hours per day), and that (s)he can potentially work for 52 weeks a year.[10] Leisure per year, defined as time that could potentially have been spent on market labor but was not, therefore equals 112 available hours per week minus average hours worked per week, all multiplied by 52 weeks. Full income and consumption expenditures are measured in nominal dollars per year. The prices of our three nondurable goods are region-specific consumer price indices that have been constructed by the Bureau of Labor Statistics.

	Mean	Std.dev.	Min.	Max.
Expenditures on food	6,096	4,550	120	60,000
Expenditures on housing	18,296	13,110	2,620	99,540
Expenditures on other goods	15,048	15,389	240	140,816
Female leisure	4,109	503	2,078	5,771
Male leisure	3,611	503	327	5,537
Female annual hours worked	1,714	503	53	3,746
Male annual hours worked	2,213	503	287	5,497
Full income	235,855	117,378	75,620	716,814
Female wage	22.61	14.26	3.13	113.90
Male wage	28.43	18.82	3.43	140.77
Price food	190.38	18.79	159.87	222.27
Price housing	197.42	26.63	153.08	253.06
Price other goods	204.96	32.31	148.92	243.45
Age female	48.7	8.1	25	64
Age male	50.2	8.0	25	65
Grade education female	13.9	2.0	6	17
Grade education male	13.7	2.2	6	17
Home owner	0.94	0.25	0	1

Table 1: Descriptive statistics - childless couples

5.3   Demand estimation

A crucial ingredient to our identification method is $\mathbf{g}\left(\mathbf{p}, y\right)$, the household's vector-valued demand function with respect to both spouses' leisure and the three non-durables. We consider four different estimators for this household level demand system:

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[10]Subtracting time needed to sleep from total time available is common in this type of studies without an explicit model of home production. See, for example, Hoynes (1996), Keane and Moffitt (1998) and Lise and Seitz (2011).

	Mean	Std.dev.	Min.	Max.
Expenditures on food	7,398	4,374	120	55,800
Expenditures on housing	18,776	13,552	1,800	114,692
Expenditures on other goods	15,243	13,522	550	142,540
Female leisure	4,260	577	1,765	5,804
Male leisure	3,623	457	0	5,726
Female annual hours worked	1,563	577	20	4,059
Male annual hours worked	2,201	457	98	5,824
Full income	235,367	121,303	70,731	840,296
Female wage	20.82	13,97	3,05	134,53
Male wage	29.16	19,65	3,08	193,56
Price food	187.41	18.42	159.87	222.27
Price housing	193.74	26.20	153.08	253.06
Price other goods	199.66	32.46	148.92	243.45
Age female	41.2	6.3	25	61
Age male	42.9	6.7	25	65
Grade education female	14.3	2.0	3	17
Grade education male	14.0	2.2	3	17
Homeowner	0.92	0.27	0	1
Number of children	1.8	0.8	1	8

Table 2: Descriptive statistics - couples with children

(i) a fully nonparametric demand system, (ii) a flexible parametric demand system, (iii) a parametric system imposing (differential based) parameter restrictions that are implied by the collective consumption model and (iv) the same parametric system in which we add taste shifters to capture heterogeneity across our households.

The function $\mathbf{g}$ is a five element vector, but only four elements need to be estimated because the fifth element is determined given the others by the budget constraint $\mathbf{p'g} = y$, where $\mathbf{p}$ is the vector of individual wages and prices and y is full income. This is known in the consumer demand literature as the "adding up" constraint. For our nonparametric demand system, we estimate the first four elements of $\mathbf{g}$ (i.e. spouses' leisure and consumption of food and housing) as the fitted values of nonparametrically regressing the household's quantities of these goods on both members' deflated wages, on the deflated prices of food and housing, and on the deflated household's full income. Each of these regressors was obtained by dividing nominal wages, prices and full income by the price of the third nondurable (other goods). This deflation by the price of the remaining good in the system imposes the homogeneity of demand implied by the absence of money illusion. The nonparametric regressions we use are Nadaraya-Watson kernel estimators with a Gaussian product kernel.

The three parametric demand systems we estimate are versions of Banks, Blundell and Lewbel's (1997) Quadratic Almost Ideal Demand System (QUAIDS). Thanks to its flexibility, QUAIDS is one of the most widely used demand systems in empirical analyses that are based on the unitary model. However, as demonstrated by Browning and Chiappori (1998), it can also be used to analyze collective household behavior. In particular, these authors derived restrictions that the QUAIDS parameters must satisfy if household demand is to be consistent with the collective model. In our application, we estimated QUAIDS both with and without these restrictions. This will also allow us to assess whether the parameter estimates of the QUAIDS demand system are consistent with the theoretical implications of the collective model.

Specifically, the second demand system in our analysis is QUAIDS without any restrictions imposed other than the same adding-up and homogeneity constraints that are also imposed in the nonparametric demand system. Under QUAIDS, the budget share of commodity i $(i = 1, ..., 5)$ takes the parametric form

$$w_i = \alpha_i + \beta_i \ln\left[\frac{y}{a(\mathbf{p})}\right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{\ln\left[\frac{y}{a(\mathbf{p})}\right]\right\}^2 + \sum_{j=1}^{5} \gamma_{ij} \ln p_j, \qquad (12)$$

where p_j is the j'th element of $\mathbf{p}$ and where

$$\ln a\left(\mathbf{p}\right) = \alpha_0 + \sum_{i=1}^{5}\alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5}\gamma_{ij}\ln p_i \ln p_j,$$

$$b\left(\mathbf{p}\right) = \prod_{i=1}^{5}p_i^{\beta_i},$$

$$\lambda\left(\mathbf{p}\right) = \sum_{i=1}^{5}\lambda_i \ln p_i.$$

The parameters α_i, β_i, λ_i and γ_{ij} $(\forall i, j)$ are to be estimated. We follow Banks, Blundell and Lewbel (1997) by setting the parameter α_0 to a value just below the lowest value of $\ln y$ in the data. The adding-up constraint implies that $\sum_i \alpha_i = 1$, $\sum_i \beta_i = 0$, $\sum_i \lambda_i = 0$ and $\sum_i \gamma_{ij} = 0$ $(\forall j)$, while homogeneity requires $\sum_j \gamma_{ij} = 0$ $(\forall i)$. Adding-up is then satisfied by construction. We impose homogeneity by estimating the system in terms of deflated prices and deflated full income (in line with the nonparametric system discussed above). Assuming additive errors, the system is estimated by means of Nonlinear Least Squares (NLS). The fitted value of the quantity consumed of a particular good is obtained by multiplying its fitted value of the budget share by full income divided by the good's own price.

Our third demand system estimator exploits the (differential) behavioral implications of the collective model by imposing restrictions on the above QUAIDS. As discussed in the Introduction, Browning and Chiappori (1998) showed that a necessary condition for the household demand function $\mathbf{g}$ to be consistent with the collective model with two decision makers is that it satisfies SR1, i.e. that the associated pseudo-Slutsky matrix, defined as $\mathbf{S} = \frac{\partial \mathbf{g}(\mathbf{p},y)}{\partial \mathbf{p}'} + \frac{\partial \mathbf{g}(\mathbf{p},y)}{\partial y}\mathbf{g}\left(\mathbf{p},y\right)'$, can be decomposed into the sum of a symmetric negative semi-definite matrix and a matrix of rank 1. They also show that in the QUAIDS model, this SR1 condition holds if and only if the same decomposition applies to the matrix $\Gamma = (\gamma_{ij})$, which equivalently requires that the rank of the (real) antisymmetric matrix $\mathbf{M} = \Gamma - \Gamma'$ is at most 2. Let m_{ij} denote an element of $\mathbf{M}$. Lemma 3 of Browning and Chiappori shows that this rank condition holds if and only if, for all i, k such that $k > i > 2$,

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}} \tag{13}$$

where, without loss of generality, m_{12} is assumed to be different from zero. To obtain our SR1-restricted QUAIDS parameters, we estimated the above described budget share equations (12) by means of NLS while imposing the equality restrictions in (13).

Finally, for our fourth demand system, we still impose these SR1 restrictions, but we also add a couple of taste shifters to account for observable heterogeneity across households. Our taste shifters are the age of the husband and a dummy for home

ownership (denoted by t_1 and t_2, respectively).[11] We only add these two taste shifters and not more because they appear to be particularly important, and the dimensionality of our model is already relatively high given our sample sizes. The new budget share equations are obtained by replacing the above constant α_i by

$$\alpha_i = \alpha_{i,0} + \alpha_{i,1}t_1 + \alpha_{i,2}t_2.$$

This is the same way Banks, Blundell and Lewbel (1997) add taste shifters. As before, we estimate these budget share equations using NLS while imposing the SR1 equality restrictions in (13).

When constructing bounds based on all of these estimated demand systems, we imposed the restriction that male and female leisure are each private, assignable, and do not generate externalities. We did not place any restriction on the nature of our three nondurable goods, which allows the consumption of each of these goods to be public, private (with or without externalities), or both.[12] We coded both the demand estimation step and the bounds calculations in MATLAB, using TOMLAB/SNOPT to solve our nonlinear programs. In the supplemental materials to the paper, we provide more details on our estimates of the different demand systems, together with our codes.

5.4 Empirical results

We assume a collective model with two decision makers. This is not a constraint for our first sample, which consists of childless couples. But for the second sample, this means that we model the expenditures on children as consumption goods that provide utility to both parents, as in Blundell, Chiappori and Meghir (2005) and Cherchye, De Rock and Vermeulen (2012a). This is a common assumption in the empirical collective household model literature. An alternative approach would have been to treat children as additional consumers with their own utility functions and Pareto weights in the collective model, as in Bargain and Donni (2012) or Dunbar, Lewbel, and Pendakur (2013).[13] In principle, our identification method described in Section 4 can be generalized towards collective models with more than two decision makers. But, to ease both exposition and computational requirements, we do not consider this extension in our current application.

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[11]Including the ages of both spouses resulted in a multicollinearity problem, because of the strong correlation between these two variables.

[12]Without an assignable good, one cannot infer the identity (i.e., male or female) corresponding to a particular income share. By using leisure as an assignable good for each spouse, we can separately identify (bounds on) male and female income shares, which is useful for our analyses in Sections 5.4.2 and 5.4.3. Still, we also experimented with estimates that allowed all goods to be public, private, or both, and allowed all to have externalities, but we found that this most general possible specification often failed to provide useful bounds, likely due to the limited size of the available data set.

[13]Our empirical application included leisure, with prices given by wages, but our general method does not require this. Since the price of leisure (wages) is not observed for children, utility for children would need to be based only on their consumption of goods, and so we would not be able to measure the impact of children's leisure on their utility or on resource shares.

5.4.1 Comparisons of sharing rule bounds

We assumed leisure is private and assignable, so the value of a household member's leisure is a lower bound on that member's share of full income. This 'naive' lower bound assigns all of the household's nondurable consumption to the other household member. Similarly, a naive upper bound gives a household member his/her leisure and all of the household's nondurable consumption. These naive bounds do not make use of any revealed preference restrictions associated with the collective household model.

We now have five different estimates of sharing rule bounds: naive bounds, and bounds based on each of the four demand system estimators described in the previous section. We can evaluate the usefulness of our method, and the informativeness of the assumptions underlying each of the demand system estimates, by comparing the bounds obtained using each estimator to the naive bounds. In this section only, we carry out this comparison using just our sample of childless couples. This provides a cleaner assessment of the empirical usefulness of our method than the analysis of couples with children, because, as described earlier, the latter impose additional modeling assumptions regarding the number of decision makers in the household. However, the next section will then show that our method yields usefully informative bounds for couples with children as well.

Since resource shares sum to one, we only need to report bounds for one of the two decision makers. We therefore report just the female resource share bounds. The results for childless couples are summarized in Table 3, where the columns labeled Naive, RP1, RP2, RP3 and RP4 refer respectively to naive bounds based only on the privateness of leisure, and bounds obtained through the nonparametric demand system, QUAIDS without SR1 imposed, QUAIDS with SR1 imposed, and QUAIDS with taste shifters and SR1 imposed. Each entry of this table gives the difference between the upper and lower bound on $y^1/(y^1 + y^2)$ where member 1 is the female, i.e., bounds on the female relative income share. The bounds are reported as percentages of total income, so the largest possible difference between the lower and the upper bound that could be reported in the table is 100, and at the other extreme point identification of an income share would imply an entry of zero in the table. We calculate these estimated differences between upper and lower bounds separately for every couple in the sample, and Table 3 then gives summary statistics of the distribution (across all the couples in the sample) of these differences.

Comparing the naive and RP1 columns of Table 3 shows that our RP based method provides a substantial improvement over the naive bounds, even with fully nonparametric demand function estimates. The average difference between the upper and lower naive bounds is about 17.5 percentage points, which narrows to 9.5 percentage points using the nonparametric RP1 estimates.[14]

The RP1 and RP2 bounds are fairly similar, which indicates that our parametric QUAIDS model yields results similar to the nonparametric demand system. This shows

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[14]Also, the smallest bounds for all four of our demand systems are zero to two decimal places, showing that for some households we come very close to point identification.

that the parametric QUAIDS functional form does not impose undue restrictions on the data, since it yields roughly the same results as the nonparametric alternative. This is a useful observation since, as discussed above, this parametric specification easily allows for imposing Browning and Chiappori's (1998) SR1 condition, and for adding taste shifters.

Next, our results for the RP3 bounds are also similar to those for the RP1 and RP2 bounds. This shows that there is little gain in going from either RP1 or RP2 to RP3. The difference between RP2 and RP3 is only the imposition of the SR1 condition. Since this condition is supposed to hold anyway given the assumption of a Pareto efficient collective household, it is reassuring that our estimated demand equations and associated bounds estimates are little changed when SR1 is imposed. This is further confirmed by the nonlinear analog to the F-test of the parameter restrictions in going from RP2 to RP3, which does not reject the null hypothesis of the SR1 restrictions.[15] Note that the classic Chiappori non-identification theorem is that the SR1 restrictions (i.e., the differential restrictions implied by the collective household model) do not suffice to point identify the sharing rule.

The bounds in the last column, where taste shifters are included, are quite a bit tighter than those in the other columns. Exploiting observable heterogeneity across households in the demand estimation step appears to yield considerably more informative bounds. The average difference between lower and upper bounds in this column is below 3 percentage points. Moreover, the third quantile row of Table 3 shows that, with RP4, for more than 75% of the couples the difference between the upper and lower bounds is less than 4.5 percentage points. This shows that we can make quite precise predictions about each spouse's resource share for a substantial majority of households.

	Naive	RP1	RP2	RP3	RP4
Mean	17.52	9.52	8.93	9.04	2.91
Minimum	3.28	0.00	0.00	0.00	0.00
First quartile	12.12	5.80	3.18	3.27	0.72
Median	15.78	9.71	8.11	8.57	2.21
Third quartile	21.68	12.78	13.40	13.51	4.48
Maximum	64.03	52.54	42.41	39.95	41.66

Table 3: Percentage point differences between upper and lower bounds on the female income share - childless couples

As a further exercise, we consider the statistical precision of our sharing rule upper and lower bound estimates, by means of a bootstrap procedure.[16] For compactness, we restrict attention to the apparently most useful bounds, which are those based on RP4.

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[15]Similarly, we can conclude that the SR1 restriction cannot be rejected for the QUAIDS model with taste shifters, and for QUAIDS with and without taste shifters for the sample of couples with children. In all cases, p-values are well above any reasonable significance level.

[16]We do not attempt to derive formal limiting distribution theory for our estimates of bounds. While some asymptotic results exist for inference on set identified objects, our estimators are considerably

We construct 100 data sets by drawing randomly (with replacement) 865 households from our original data set of 865 childless couples. For each of these new datasets, we estimated QUAIDS with two taste shifters included and SR1 imposed. We then used the resulting parameter estimates to obtain RP4 sharing rule bounds. To summarize the results, we estimate bounds for nine combinations of the spouses' wages: female wage equal to the first, second or third quartile of the female wage distribution combined with male wage equal to the first, second or third quartile of the male wage distribution (with full income each time adjusted accordingly).

Table 4 shows the means and standard deviations across the bootstrap replications of the lower bounds (LB) and the upper bounds (UB) for each of these nine scenarios. All the bounds appear to be rather precisely estimated, with bootstrap standard deviations between 3.7 and 5.1 percent. If we assume normality, we can reject the assumption that spouses with very unequal wages share income equally, but we cannot reject equal sharing for spouses with similar wages. Finally, Table 4 provides evidence that spouses' shares of income increase when their relative wages increases. We will return to this relationship between relative wages and individual income shares in the next section.

Female wage/Male wage	Mean LB	Std.dev. LB	Mean UB	Std.dev. UB
Quartile 1/Quartile 1	47.39%	4.16%	50.42%	4.10%
Quartile 2/Quartile 1	51.23%	3.81%	54.58%	4.11%
Quartile 3/Quartile 1	57.84%	3.73%	62.39%	4.44%
Quartile 1/Quartile 2	41.92%	4.06%	45.43%	4.20%
Quartile 2/Quartile 2	46.22%	4.10%	50.53%	4.51%
Quartile 3/Quartile 2	51.76%	3.81%	55.86%	4.33%
Quartile 1/Quartile 3	32.99%	4.51%	39.14%	4.45%
Quartile 2/Quartile 3	39.49%	5.07%	44.72%	4.63%
Quartile 3/Quartile 3	44.63%	4.61%	48.62%	4.43%

Table 4: Bootstrap summary statistics on the female income share - childless couples

Overall, the results in Tables 3 and 4 reveal that we obtain quite narrow and hence informative sharing rule bounds. In what follows, we investigate the implications of these bounds in more detail. To focus our discussion, from here on out we restrict attention to results based on RP4 bounds. However, we found that the other three demand systems yielded qualitatively similar results. Our remaining analyses will consider both couples with and without children, obtaining informative results for both types of households.

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more elaborate than those to which existing theory can be readily applied. These bootstrap results should therefore only be interpreted informally.

5.4.2 The relation between sharing rule bounds, total income, and relative wages

We next focus on the relationships between our estimated bounds and various individual and household characteristics, for both childless couples and couples with children. Figures 1 and 2 show the relationship between the RP based bounds on the female income share in dollars, y^1, and the logarithm of the household's full income, $\ln(y)$. Each $\bullet$ and $+$ sign on the figures represents the upper and lower bound for a given household in our sample. To help visualize the results, we include trendlines showing local sample averages (i.e., nonparametric regressions) of the estimated upper and lower bounds.

Figure 1 shows that the bounds are mostly very narrow for childless couples, reflecting the results in Table 3. The trendlines are upward sloping as one would expect, showing that total resources consumed by the female increase as total household income increases, making her consumption a normal good (as opposed to an inferior good) in the household.

Figure 2 reveals roughly the same pattern for couples with children. For these households, we obtain bounds that, while still informatively narrow, are generally wider than for childless couples. One possible explanation is that even young children may have some bargaining power. If children are to some extent separate decision makers in the household, with their own bargaining weights and resource shares (as in Bargain and Donni, 2012, and Dunbar, Lewbel and Pendakur, 2013), then this makes part of total household income not directly allocatable to either the mother or the father, which could lead to wider bound estimates for their individual income shares.

Figures 3 and 4 plot the relative female income share, y^1/y, against $\ln(y)$. In both cases, the trendlines are slightly decreasing, but quite close to horizontal. This finding lends empirical support to the assumption that relative income shares do not vary with total income, which Lewbel and Pendakur (2008) and Bargain and Donni (2012) used to help point identify resource shares. These results lend even stronger support for the weaker assumption used by Dunbar, Lewbel and Pendakur (2013) to help obtain point identification, which is that relative income shares do not vary with total income at low income levels.[17]

The trendlines in Figures 3 and 4 give average upper bounds around 50% and average lower bounds around 40% to 45%. These results suggest that the income shares of females and males are not far from equal on average, though with men having slightly larger shares on average. This is confirmed by the average lower and upper bound for all the households in our sample. For childless couples, these averages are 44.04% and 46.87%, whereas for couples with children they equal 42.44% and 51.14%. However, the figures also show that some households divide their full income very unequally,

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[17]Note, however, that these authors define publicness of goods differently than we do, and their empirical applications focus on a different definition of full income, since they do not include leisure in their consumption models. See also Menon, Pendakur, and Perali (2012), who provide additional empirical evidence supporting this assumption.
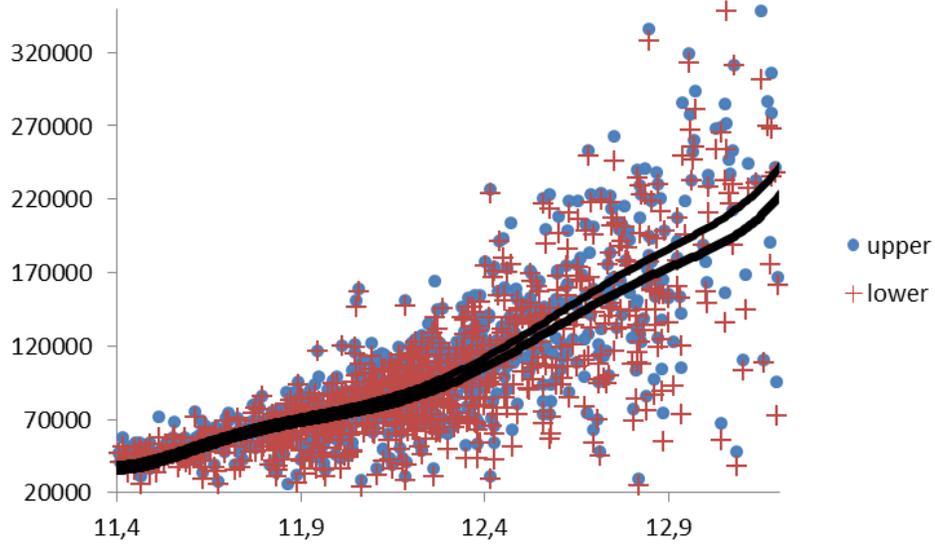
Figure 1: Absolute sharing rule bounds (Y-axis) and the logarithm of full income (X-axis) - childless couples
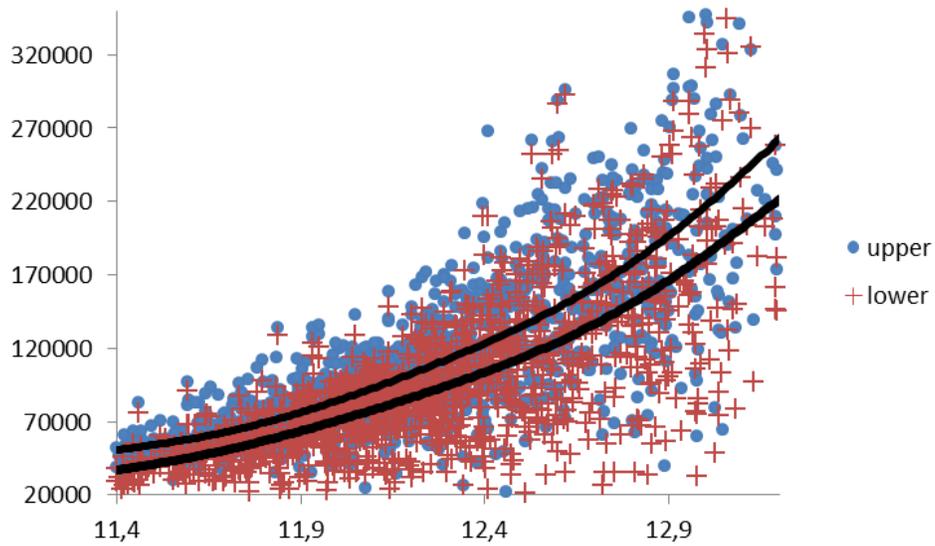

Figure 2: Absolute sharing rule bounds (Y-axis) and the logarithm of full income (X-axis) - couples with children
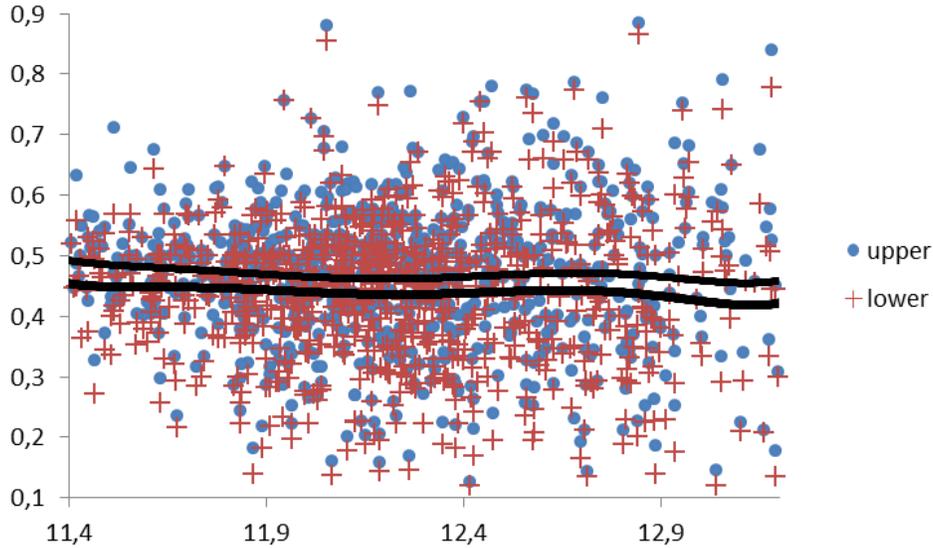
Figure 3: Relative sharing rule bounds (Y-axis) and the logarithm of full income (X-axis) - childless couples

e.g., there exist a number of households that have upper and lower bounds of the relative female income share less than 15%, and there exist other households where these bounds are both above 80%. Since leisure is assumed to be private, and is priced at an individual's own wage level, most of these extreme households are ones where the spouses have very unequal wage levels. In such households a sizeable fraction of total household nondurable consumption would need to be given to the low wage spouse to roughly equalize income shares.

We next look at the relationship between the bounds on relative female income shares and relative wages (defined as female wages divided by male wages). Figures 5 and 6 show the household specific upper and lower bounds and the corresponding trendlines. As expected, the bounds on women's resource shares tend to increase when their relative wage goes up. This result, which we obtained through our RP based approach, confirms earlier evidence found in the literature, which shows that a household member's bargaining power is generally higher when her/his relative wage is higher (see Chiappori, Fortin and Lacroix, 2002, Blundell, Chiappori, Magnac and Meghir, 2007, and Oreffice, 2011, among many others). However, as noted above, this result may also be in part an artifact of including leisure in total income and assuming leisure is private.

5.4.3 Poverty analysis

A unique advantage of models that focus on the intrahousehold allocation of resources is that they allow one to conduct welfare analyses directly at the level of individuals
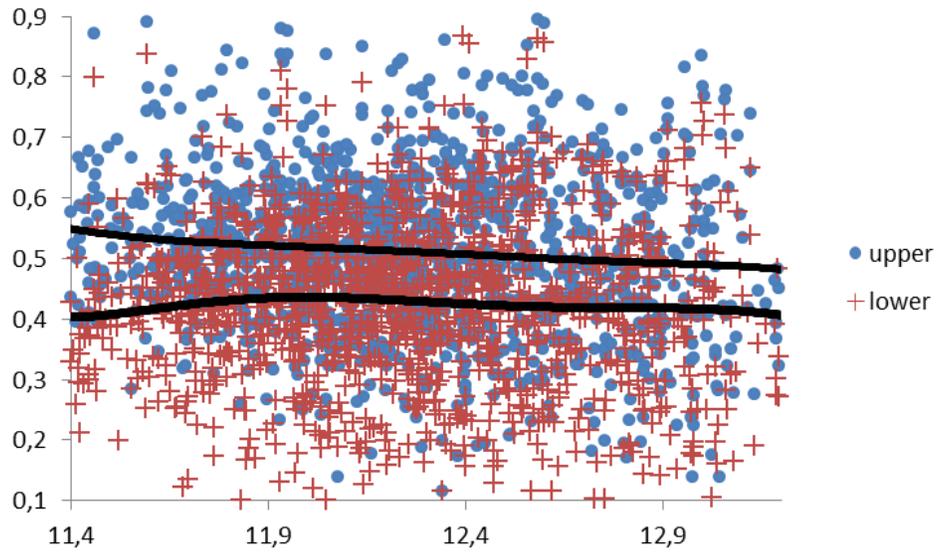
30

Figure 4: Relative sharing rule bounds (Y-axis) and the logarithm of full income (X-axis) - couples with children
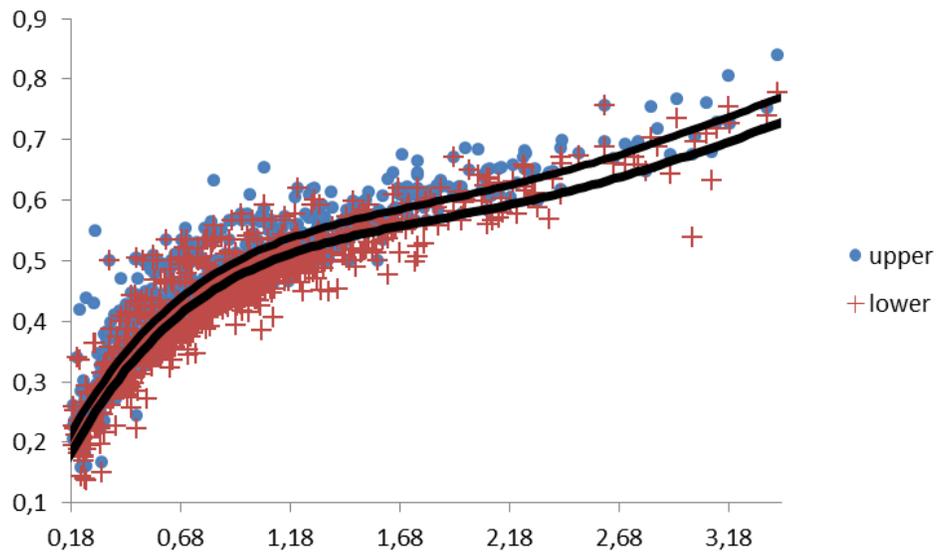

Figure 5: Relative sharing rule bounds (Y-axis) and the relative wage (wage female/wage male) (X-axis) - childless couples
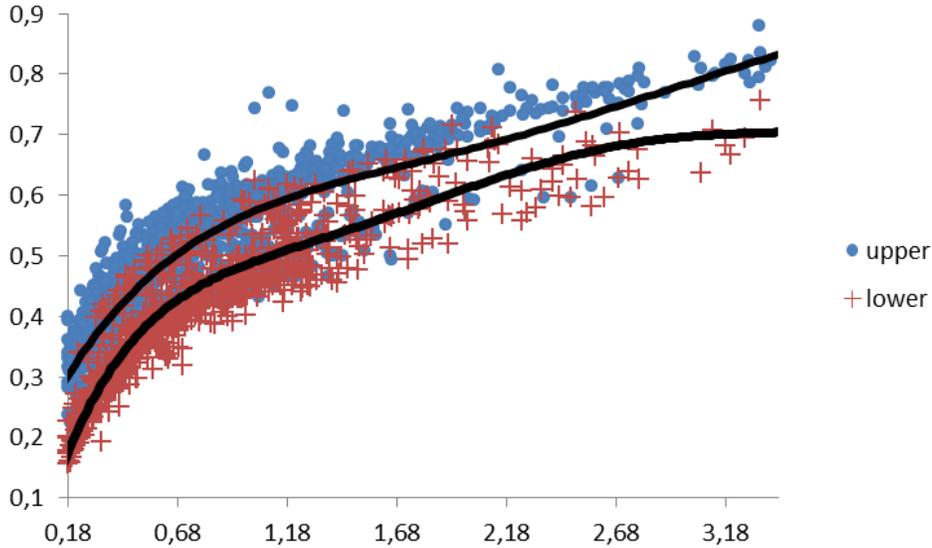
Figure 6: Relative sharing rule bounds (Y-axis) and the relative wage (wage female/wage male) (X-axis) - couples with children

rather than at the level of households. See Chiappori (1992), Blundell, Chiappori and Meghir (2005) and Browning, Chiappori and Lewbel (2013) for further discussion, and Lise and Seitz (2011), Cherchye, De Rock and Vermeulen (2012b), and Dunbar, Lewbel, and Pendakur (2013) for examples of such analyses.

Here we use our RP based bounds to conduct a poverty analysis at the level of individuals. Unlike previous studies that are based on point identified sharing rules, for our analysis we do not impose any assumptions regarding similarity of preferences across individuals, or impose functional forms at the level of individual's demand functions, or impose functional restrictions on the sharing rule.[18] We also do not require or assume the availability of distribution factors.

Tables 5 and 6 summarize the results of our poverty analysis for the two types of couples under study. The first column in both tables contains the poverty rate calculated in the usual way, i.e., the poverty rate is defined as the percentage of households (without and with children, respectively) having full income that falls below the poverty line, which we fix at 60% of the median full income in the relevant sample of households. Note that while 60% of the median income is a standard measure of relative poverty (e.g. used in the definition of OECD poverty rates), in our case the poverty rate is calculated on the basis of full income instead of (the more commonly used) earnings or total expenditures. Also, our data consists of couples where both

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[18]For example, Browning, Chiappori and Lewbel (2013) and Lise and Seitz (2011) assume similarity of preferences of (fe)male singles and preferences of (fe)male individuals in couples, while Dunbar, Lewbel and Pendakur (2013) assume restrictions upon individual preferences.

spouses participate in the labor market, and so our poverty line will be higher than a line based on data that includes households with unemployed or retired members. Our poverty line also does not make use of data on singles living alone or with children.

The second column in Tables 5 and 6 shows the incidence of poverty at the level of individuals in the sample, based on our RP income share estimates. For this and the remaining columns in the tables, an individual is labeled as poor if her/his income share falls below the individual poverty line (defined as half of the poverty line for couples used in column 1). Using our estimated income share bounds, we compute upper and lower bounds for individual poverty rates. If all couples split income perfectly equally, then these poverty rates would equal those in column 1. However, despite our earlier finding that many couples have close to equal divisions of income, we find that the individual poverty rate is above the household poverty rate. For childless couples, the lower and upper rates of individual poverty equal 15.32% and 18.55%, compared to the household rate of 11.33%. As indicated above, sharing rule bounds are somewhat wider for couples with children. Nonetheless, here too we find a similar pattern: the household poverty rate is 12.22%, whereas the lower and upper rates of individual poverty equal 12.65% and 24.84%.

Our results imply that, due to unequal sharing of resources within households, the fraction of individuals living below the poverty line may be considerably greater than the fraction obtained by standard measures that ignore intrahousehold allocations. Thus, the incidence of poverty at the individual level may be substantially higher than is indicated by standard measures based on household level income.

The remaining columns of Tables 5 and 6 provide separate bounds on poverty rates for females and males. For childless couples, we find that the upper bound on the poverty rate for males is much lower than the lower bound for females. Next, for couples with children, we find that the respective lower and upper bounds on poverty rates for females are substantially above those for males. Overall, these results suggest that, among couples, poverty is primarily a female phenomenon in our data.

	Households	All individuals	Females	Males
Household poverty rate	11.33%	-	-	-
Lower bound	-	15.32%	20.69%	12.32%
Upper bound	-	18.55%	24.74%	12.37%

Table 5: Poverty rates - childless couples

	Households	All individuals	Females	Males
Household poverty rate	12.22%	-	-	-
Lower bound	-	12.65%	15.03%	10.28%
Upper bound	-	24.84%	29.48%	20.20%

Table 6: Poverty rates - couples with children

6 Conclusions

It has long been known that, under the general collective household model, the income sharing rule is not point identified. Past responses to this result have been to focus on features of the model that are identified (like the impacts of distribution factors), or to add strong additional assumptions on preferences or behavior to obtain point identification. In contrast, we show that, given just household level demand functions, bounds on the sharing rule can be obtained by imposing inequality restrictions that are implied by revealed preference theory as applied to the (unobserved) demand functions of individual household members. We show that informative bounds are possible even when nothing is known about the privateness or assignability of any of the goods being consumed by household members, and when no distribution factors are observed. We show how these bounds can be implemented using standard programming methods, employing household level demand functions that are estimated by standard parametric or nonparametric regression methods. We also show how bounds can be tightened by using additional information that may be available on the privateness and assignability of some goods, by the use of household level demand functions that satisfy the SR1 differential restriction implied by the collective household model, and by adding information on taste shifters.

At the practical level, we demonstrate that our identification methods are empirically tractable and yield usefully narrow bounds when applied to American households drawn from the PSID. We show the usefulness of these bounds by providing empirical analyses of the effects of household characteristics like income and relative wages on income shares, and we provide a distributional analysis of the incidence of poverty at the level of individuals rather than at the level of observed households.

In our analysis, the household demand functions are estimated, and we have only to a limited extent taken into account estimation errors. Some results exist on inference for set identified objects, including Manski (2003), Chernozhukov, Hong and Tamer (2007), Beresteanu and Molinari (2008), and Galichon and Henry (2009). In particular, the studies of Hoderlein and Stoye (2013), Kitamura and Stoye (2013) and Henry and Mourifié (2013), which focus on set identification in a revealed preference context, provide machinery that might be usable for developing inference for our analyses, which are quite a bit more complicated than existing applications to which such asymptotic theory has been successfully applied. Similar issues arise in other applications that combine demand estimation with revealed preference restrictions, such as Blundell, Browning and Crawford (2008) and Blundell, Kristensen and Matzkin (2014).

To avoid further complications, our analysis was based on a collective model with two decision makers. However, our methods immediately extend in theory to handle more than two consumers per household. Therefore, they could be used to estimate bounds on children's resource shares as well, treating children as additional consumers with their own utility functions and Pareto weights in the collective model, as in Bargain and Donni (2012) or Dunbar, Lewbel, and Pendakur (2013), but without the restrictions on preferences that these authors require. The main modification needed

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to deal with children is that their leisure could not be included in the model, because wages for children are not observed.

We obtain our bounds without observing any distribution factors. An advantage of not requiring distribution factors is that such variables are assumed to only affect allocations but not tastes, and this can be difficult to verify. For example, wages (while appearing in our model) are not distribution factors in our model, because in addition to possibly affecting allocations, wages also affect the consumption of leisure. Wages can only be distribution factors in models that assume separability of leisure from consumption. Still, if one observed distribution factors, then they could be incorporated into our analysis by conditioning the bounds calculations on the values of these factors.

Appendix

Proof of Proposition 1

This result follows from adapting the original reasoning of Samuelson (1938) to our collective setting. Specifically, Definition 4 states that a pair of utility functions U^1 and U^2 provides a collective rationalization of $\mathbf{g}$ if there exist admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ such that, for each m,

$$\mathbf{g}^m \left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m \right) = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$$

for

$$(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H) = \arg \max_{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H} [U^m \left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H \right) \text{ s.t.}$$
$$\left(\mathbf{p}^{m,1} \right)' \mathbf{x}^1 + \left(\mathbf{p}^{m,2} \right)' \mathbf{x}^2 + \left(\mathbf{p}^{m,H} \right)' \mathbf{x}^H \leq y^m].$$

Thus, for each individual m there must exist a utility function U^m such that the function $\mathbf{g}^m$ solves the corresponding maximization problem for any prices $\mathbf{p}^{m,1}$, $\mathbf{p}^{m,2}$, $\mathbf{p}^{m,H}$ and income y^m. Samuelson's (1938) argument obtains that this is possible only if the function $\mathbf{g}^m$ satisfies the WARP condition in Definition 2.

Proof of Proposition 2

As a first step, we prove (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$(y_1 \geq \mathbf{p}_1' \mathbf{x}_2 \text{ and } y_2 \geq \mathbf{p}_2' \mathbf{x}_1) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{x}_2 \text{ and } \mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{x}_1 \ (l \neq m) \right). \quad (14)$$

To obtain this result, we note that $y_1 \geq \mathbf{p}_1' \mathbf{x}_2$ implies by construction

$$\sum\nolimits_{m=1}^2 \left(\mathbf{p}_1^{m,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_1^{m,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_1^{m,H}\right)' \mathbf{x}_1^H \tag{15}$$
$$\geq \sum\nolimits_{m=1}^2 \left(\mathbf{p}_1^{m,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_1^{m,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_1^{m,H}\right)' \mathbf{x}_2^H$$

for all possible specifications of $\mathbf{p}_1^{m,c}$, $\mathbf{x}_1^c$ and $\mathbf{x}_2^c$ ($m = 1, 2;\ c = 1, 2, 3$). The inequality (15) necessarily obtains

$$\left(\mathbf{p}_1^{m,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_1^{m,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_1^{m,H}\right)' \mathbf{x}_1^H \geq \left(\mathbf{p}_1^{m,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_1^{m,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_1^{m,H}\right)' \mathbf{x}_2^H$$

for $m = 1$ *or* 2, which can also be expressed as $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{x}_2$ *or* $\mathbf{x}_1 R_o^{\mathbf{g}^2} \mathbf{x}_2$ for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$.

Now, without loss of generality, let us assume $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{x}_2$ (i.e. $m = 1$ in (14)). Then, because the functions $(\mathbf{g}^1, \mathbf{g}^2)$ satisfy WARP, we must have

$$\left(\mathbf{p}_2^{1,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_2^{1,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_2^{1,H}\right)' \mathbf{x}_1^H > \left(\mathbf{p}_2^{1,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{1,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{1,H}\right)' \mathbf{x}_2^H. \tag{16}$$

In turn, because $y_2 \geq \mathbf{p}_2' \mathbf{x}_1$, this implies

$$\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_1^H \leq \left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H,$$

or $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1$. This proves (14).

As a second step, we show (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\left(\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{x}_2 \text{ and } y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1)\right) \Rightarrow \left(\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E \ (l \neq m)\right). \tag{17}$$

To prove this result, we first observe that $y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1)$ implies

$$\sum\nolimits_{m=1}^2 \left(\mathbf{p}_2^{m,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{m,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{m,H}\right)' \mathbf{x}_2^H \tag{18}$$
$$\geq \sum\nolimits_{m=1}^2 \left(\mathbf{p}_2^{m,1}\right)' \left(\mathbf{q}_E^1 + \mathbf{x}_1^1\right) + \left(\mathbf{p}_2^{m,2}\right)' \left(\mathbf{q}_E^2 + \mathbf{x}_1^2\right) + \left(\mathbf{p}_2^{m,H}\right)' \left(\mathbf{q}_E^H + \mathbf{x}_1^H\right),$$

for all possible specifications of $\mathbf{p}_1^{m,c}$, $\mathbf{x}_1^c$, $\mathbf{x}_2^c$ and $\mathbf{q}_E^c$ ($m = 1, 2;\ c = 1, 2, H$).

Without loss of generality, let us now assume $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{x}_2$ (i.e. $m = 1$ in (17)). Like before, WARP consistency then requires (16), and combining this last inequality with (18) yields

$$\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H > \left(\mathbf{p}_2^{2,1}\right)' \mathbf{q}_E^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{q}_E^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{q}_E^H,$$

or $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$. This proves (17).

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Now, a directly similar reasoning as the one leading up to (17) yields

$$\left(\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{x}_1 \text{ and } y_1 \geq \mathbf{p}_1' \left(\mathbf{q}_E + \mathbf{x}_2\right)\right) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E \ (m \neq l)\right). \tag{19}$$

Combining (14), (17) and (19) gives the wanted result: we have $\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E$ for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that satisfy WARP if $y_1 \geq \mathbf{p}_1' \left(\mathbf{q}_E + \mathbf{x}_2\right)$ and $y_2 \geq \mathbf{p}_2' \left(\mathbf{q}_E + \mathbf{x}_1\right)$. $\blacksquare$

Proof of Proposition 3

In what follows, we only give the proof for condition C.1. The arguments for the remaining conditions C.2 and C.3 are analogous.

As a first step, we prove that (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$ and for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\sum\nolimits_{n \in N_{A_2}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}_2\right)_n \geq \mathbf{p}_2' \mathbf{x} - \sum\nolimits_{n \in N_{A_1}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}\right)_n \tag{20}$$
$$\Rightarrow \left(\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1 \text{ and } \mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E\right)$$

To obtain this result, we note that

$$\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H \geq \sum\nolimits_{n \in N_{A_2}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}_2\right)_n \tag{21}$$

for any possible specification of $\mathbf{p}_2^{2,c}$ and $\mathbf{x}_2^c$ $(c = 1, 2, H)$. Indeed, $\left(\mathbf{x}_k^2\right)_n = \left(\mathbf{x}_k\right)_n$ and $\left(\mathbf{p}_2^{2,2}\right)_n = \left(\mathbf{p}_2\right)_n$ for the assignable goods $n \in N_{A_2}$, which yields

$$\left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 \geq \sum\nolimits_{n \in N_{A_2}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}_2\right)_n,$$

while $\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H \geq 0$ by construction.

Similarly, we have (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$)

$$\mathbf{p}_2' \mathbf{x} - \sum\nolimits_{n \in N_{A_1}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}\right)_n \geq \left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}^H. \tag{22}$$

To see this, we can use an analogous argument as before to get

$$\left(\mathbf{p}_2^{1,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{1,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{1,H}\right)' \mathbf{x}^H \geq \sum\nolimits_{n \in N_{A1}} \left(\mathbf{p}_2\right)_n \left(\mathbf{x}_1\right)_n,$$

and thus

$$\mathbf{p}_2' \mathbf{x} - \sum_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n$$

$$\geq \ \mathbf{p}_2 \mathbf{x} - \left[\left(\mathbf{p}_2^{1,1} \right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{1,2} \right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{1,H} \right)' \mathbf{x}^H \right]$$

$$= \ \left(\mathbf{p}_2^{2,1} \right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2} \right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H} \right)' \mathbf{x}^H.$$

Combining (21) and (22) gives (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$)

$$\left(\mathbf{p}_2^{2,1} \right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2} \right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H} \right)' \mathbf{x}_2^H \geq \left(\mathbf{p}_2^{2,1} \right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2} \right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H} \right)' \mathbf{x}^H,$$

which yields $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$. This proves (20).

From this first step we conclude $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1$ under condition C.1. Next, we can use a similar argument as in Proposition 2 (for (17)) to obtain (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\left(\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1 \text{ and } y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2) \right) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E \right). \tag{23}$$

This gives the wanted result: we have $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that satisfy WARP if condition C.1 holds. ∎

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