

Identifying Sharing Rules in Collective Household Models

an overview

Arthur Lewbel
Boston College

revised March 2016

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In addition to current research not yet in working paper form, papers discussed here include:

Laurens, C., B. De Rock, A. Lewbel, and F. Vermeulen, (2015) "Sharing Rule Identification for General Collective Consumption Models," *Econometrica*, 83, 2001-2041..

Dunbar, G., A. Lewbel and K. Pendakur (2013), "Children's Resources in Collective Households: Identification, Estimation and an Application to Child Poverty in Malawi," *American Economic Review*, 103, 438-471.

Browning, M., P.-A. Chiappori and A. Lewbel (2013), "Estimating consumption economies of scale, adult equivalence scales, and household bargaining power" *Review of Economic Studies* 80, 1267-1303.

Lewbel, A. and K. Pendakur (2008), "Estimation of collective household models with Engel curves", *Journal of Econometrics*, 147, 350-358.

Identifying Sharing Rules in Collective Household Models

What percentage of a married couple's expenditures are controlled by the husband?

How much money does a couple save on consumption goods by living together versus living apart?

What share of household resources go to children?

How much income would a woman living alone require to attain the same standard of living that she'd have if she were married?

Goals: 1. Empirically tractable. 2. Identify resource shares (bargaining), joint consumption, household member's indifference curves and indifference scales. 3. Avoid untestable cardinalization assumptions.

Overview - What We Assume Households Do

Couples for now: f female and m male (will add children later).

1. Household buys a bundle of goods z .
2. Converts z to private good equivalents $x = F^{-1}(z)$.
3. Divides bundle x into $x = x^f + x^m$ (Pareto efficient).
4. Each member gets utility $U^f(x^f)$, $U^m(x^m)$.

F is the "consumption technology function"

If good j is purely private, then $z_j = x_j = x_j^f + x_j^m$

If good j is purely public, then $z_j = x_j^f = x_j^m$

More generally, goods are partly shared. Example: A couple rides together in their car 30% of the time. Then for consumption of gasoline j ,

$z_j = x_j / 1.3$, so $x_j^f + x_j^m = 1.3z_j$.

If externalities then x^f also in U^m and x^m also in U^f .

Models

Start from Becker (1965, 1981). Assume Pareto efficient households (rules out many noncooperative models).

Define:

d = distribution factors = observables that only affect allocations between members, not utility of either.

μ = Pareto weight function.

U^f , U^m utility functions of women and men, respectively.

Couples will maximize $\mu U^f + U^m$.

Pareto weight μ interpreted as relative bargaining power, but depends on how utility is cardinalized.

Will later define resource share η that doesn't depend on unknowable cardinalizations.

Models

Model C (Chiappori 1988, Bourguignon and Chiappori 1994, Browning, Bourguignon, Chiappori, and Lechene 1994, Browning and Chiappori 1998). Every good is either purely private (private bundles z^f, z^m) or purely public (public good bundle X) $z = (z^f + z^m, X)$, $p = (p_z, p_x)$.

$$\max_{z^f, z^m, X} \mu(p, y, d) U^f(z^f, X) + U^m(z^m, X) \text{ such that } p'_z(z^f + z^m) + p'_x X = y$$

Solve for observable household demands: $X = X(p, y, d)$, $z = z(p, y, d)$.
Cherchye, De Rock and Vermeulen (2011) add externalities.

Model BCL (Browning Chiappori Lewbel 2002,2014). Goods are private or partly shared (general consumption technology F), no money illusion,

$$\max_{x^f, x^m, z} \mu(p/y, d) U^f(x^f) + U^m(x^m) \text{ such that } z = F(x^f + x^m), p'z = y$$

Solve for observable household demand functions: $z = z(p, y, d)$.

Sharing Rule Definitions

Model C: Define sharing rule = wife's (conditional) share:

$\eta(p, y, d) = p'_z z^f / p'_z (z^f + z^m)$. With no public goods, η is monotonic and one to one with μ .

A private good is *assignable* if we observe which household member consumed it. If all of z^f, z^m were assignable, then Model C sharing rule would be directly observable.

Model BCL: Define sharing rule = wife's (private equivalents) share:

$\eta(p, y, d) = p'_z x^f / p'_z (x^f + x^m)$. In BCL, for any regular consumption technology function F , η is monotonic and one to one with μ .

Both definitions are generally monotonic in Pareto weights, and so may be interpreted as measures of bargaining power in models where a bargaining game is assumed (assuming the game has efficient outcomes).

Earlier Sharing Rule Identification Results

Main identification result (early forms Chiappori 1992, Chiappori, Blundell, Meghir 2002, general form: Chiappori and Ekeland 2009): In model C with or without public goods, given just the household demand function $z = z(p, y, d)$, resource share levels $\eta(p, y, d)$ are not identified, but derivatives of $\eta(p, y, d)$ are (generically) identified

Application/variant of this result: Lechene and Attanasio (2010) see a cash transfer to households that leaves food shares unchanged. Transfer changes y but could also be a d . Infer $\partial\eta/\partial d$ must be nonzero to offset transfer's effect through y on shares.

Without further assumptions, is also true for BCL that $\eta(p, y, d)$ is not identified, since not identified in the purely private goods model, which is a special case of both BCL and C.

Problem: many welfare/policy calculations depend on identifying η , not on just $\partial\eta/\partial d$, e.g., poverty rates, inequality measures, indifference scales.

Goal - Identify Sharing Rule Levels η , not just $\partial\eta/\partial d$

Alternative methods for identifying η :

1. Collect extensive intrahousehold (assignable) consumption data: Cherchye, De Rock and Vermeulen (2010), Menon, Pendakur, Perali (2012). Can't deal with sharing and externalities.
2. Obtain bounds on shares: Cherchye, De Rock and Vermeulen (2011), Cherchye, De Rock, Lewbel and Vermeulen (2013).
3. Restrict individual preferences across households of different compositions: Browning Chiappori Lewbel (BCL 2002, 2013), Couprie (2007), Lewbel and Pendakur (LP 2008), Bargain and Donni (2009, 2012), Couprie, Peluso and Trannoy (2010), Lise and Seitz (2011).
4. Restrict sharing rules, assume preference similarities: Dunbar, Lewbel, and Pendakur (DLP 2013).
5. Restrict sharing rules and use distribution factors: Dunbar, Lewbel, and Pendakur (DLP 2016).

Bounds

Naive bounds: a lower bound on each individual j 's resource share is cost of j 's private, assignable consumption divided by total household consumption. We can do better.

Drop distribution factors d for now. Suppose we could see purchased bundles z_1^j, \dots, z_n^j of an individual j living alone in price/income regimes $p_1/y_1, \dots, p_n/y_n$.

If j maximizes utility, then these bundles would satisfy revealed preference inequalities derived from Samuelson (1938), Houthakker (1950), Afriat (1967), Diewert (1973), and Varian (1982).

For a known vector valued function M^j , write all these inequalities as

$$0 \leq M^j \left(\{z_i^j, p_i/y_i\}_{i=1, \dots, n} \right)$$

Bounds continued

A single consumer j satisfies $0 \leq M^j \left(\{z_i^j, p_i / y_i\}_{i=1, \dots, n} \right)$.

Assume model C (leave out public goods for now), and we observe a couple purchase bundles z_1, \dots, z_n in regimes $p_1 / y_1, \dots, p_n / y_n$. Then there must exist z_1^f, \dots, z_n^f such that resource shares η_1, \dots, η_n satisfy inequalities

$$0 \leq M^f \left(\{z_i^f, p_i / \eta_i y_i\}_{i=1, \dots, n} \right), \quad 0 \leq z_i^f \leq z_i,$$
$$0 \leq M^m \left(\{z_i - z_i^f, p_i / (1 - \eta_i) y_i\}_{i=1, \dots, n} \right).$$

Min and max η_1, \dots, η_n over all possible z_1^f, \dots, z_n^f that satisfy these inequalities to get bounds on resource shares η_1, \dots, η_n .

Bounds continued

Extension 1: Include public goods and externalities. Can get informative bounds even if we do not know which goods are private and which are public, and which have externalities.

Extension 2: Observe/estimate couple's demand functions $z = z(p, y)$. Then can obtain inequalities for every possible p/y point to get tighter bounds.

Extension 3: Browning and Chiappori (1998) show SR1 (symmetry plus rank one) is the restriction on a couple's demand system implied by Pareto efficiency. Can estimate $z = z(p, y)$ imposing SR1 to further tighten bounds.

Note: We base bounds on WARP (weak axiom of revealed preference) inequalities applied to each household member. Imposing SARP may be numerically infeasible.

Bounds - Empirical Results

Some empirical results from Cherchye, De Rock, Lewbel and Vermeulen (2013). 865 childless working couples with full reporting, from 1999-2009 US PSID. Goods are each spouse's leisure (assumed private and assignable), food, housing, and other (allowed to have both public and private components and externalities).

Naive: Lower bound is own leisure only, upper is all expenditure except spouse's leisure.

RP1: Reveled preference bounds from nonparametrically estimated couple's demand system.

RP2: Bounds from parametric (QUAIDS) demand system with no utility restrictions imposed.

RP3: Bounds applied to parametric (QUAIDS) system with SR1 restrictions imposed.

Bounds - Empirical Results

Naive: lower is leisure only, upper is all expenditure except spouse's leisure.

RP1: nonparametrically estimated couple's demand system.

RP2: parametric demands with no utility restrictions.

RP3: parametric demands with SR1 restrictions imposed.

	Naive bounds	RP1 bounds	RP2 bounds	RP3 bounds
Mean	17.52	11.33	9.72	3.44
First quartile	12.12	6.54	3.93	1.49
Median	15.78	11.67	8.91	2.78
Third quartile	21.68	15.13	14.53	4.74

Table: Percentage point differences between upper and lower bounds on the female relative income share

BCL Model

Drop distribution factors d , not needed.

BCL demands $z = h(p/y)$ obtained from

$$\max_{x^f, x^m, z} \mu(p/y) U^f(x^f) + U^m(x^m) \text{ such that } z = F(x^f + x^m), p'z = y$$

For $j = f, m$, define $x^j = h^j(p/y)$ as demands from $\max_{x^j} \{U^j(x^j) \mid px^j = y\}$.

Define indirect utility functions $V^j(p/y) = U^j[h^j(p/y)]$.

Working Towards Identification - Duality

BCL show duality: A shadow (Lindahl) price vector $\pi(p/y)$ and a sharing rule $0 < \eta(p/y) < 1$ exist such that

$$x^f\left(\frac{p}{y}\right) = h^f [\pi / (\eta y)], \quad x^m\left(\frac{p}{y}\right) = h^m [\pi / ((1 - \eta) y)], \quad h\left(\frac{p}{y}\right) = F\left(h^f + h^m\right)$$

f maximizes her utility by buying $x_f = h^f [\pi / (\eta y)]$ at shadow prices π with share η of y . m does same with share $1 - \eta$. Adds up to household buying bundle $z = F(x^f + x^m)$.

Pareto weight μ connected to resource share η by

$$\mu = - \left[\partial V^f(\pi / (\eta y)) / \partial \eta \right] / \partial V^m[\pi / ((1 - \eta) y)] / \partial \eta$$

So: given demands h , h^f , h^m , if π and η are identified, everything (ordinal) is identified.

Questions:

1. How to identify demand functions h , h^f , h^m ?
2. Given h , h^f , h^m , are π , η identified? Address this first.

BCL - Generic Point Identification

Claim: If functions h , h^f , h^m known, then functions x^m , x^f , F , and η are "generically" identified (generic as in Chiappori and Ekeland 2009).

Proof sketch: let $\rho = p/y$. h , h^f , h^m known. Given any $\bar{F} \in \Omega_F$, let

$$\bar{\pi}(\rho) = \frac{D\bar{F}(x)' \cdot \rho}{x' D\bar{F}(x)' \cdot \rho}, \text{ evaluated at } x = \bar{F}^{-1}[h(\rho)],$$

$$\bar{x}(\rho, \eta) = h^m[\bar{\pi}(\rho)/(1 - \eta)] + h^f[\bar{\pi}(\rho)/\eta],$$

$$\bar{\eta}(\rho) = \arg \min_{\eta^* \in [0,1]} \max |\bar{x}(\rho, \eta^*) - \bar{F}^{-1}[h(\rho)]|.$$

and define \tilde{F} by $\tilde{F}[\bar{x}(\rho, \bar{\eta}(\rho))] = h(\rho)$. This defines a mapping $\tilde{F} = \mathcal{T}(\bar{F})$. True F is fixed point, and true η is $\bar{\eta}$ with $\bar{F} = F$.

\mathcal{T} might not be a contraction mapping. Loosely, existence of \mathcal{T} shows enough demand functions are identified to generally permit recovery of F and η ; are identified as long as the demand functions are not too simple.

BCL - Linear Consumption Technology

Assume a linear consumption technology: $z = F(x) = Ax + a$
makes shadow prices not depend on indirect utility functions:

$$\frac{\pi(p/y)}{y} = \frac{A'p}{y - a'p}$$

gives household demand functions the form:

$$\begin{aligned} z &= h(p/y) = Ah^f \left(\frac{\pi(p/y)}{\eta(p/y)y} \right) + Ah^m \left(\frac{\pi(p/y)}{(1 - \eta(p/y))y} \right) + a \\ &= Ah^f \left(\frac{A'p}{y - a'p} \frac{1}{\eta(\frac{p}{y})} \right) + Ah^m \left(\frac{A'p}{y - a'p} \frac{1}{1 - \eta(\frac{p}{y})} \right) + a \end{aligned}$$

Linear Consumption Technology - Compare BCL to Gorman

BCL with linear consumption technology F is

$$z = h(p/y) = Ah^f \left(\frac{A'p}{y - a'p} \frac{1}{\eta(\frac{p}{y})} \right) + Ah^m \left(\frac{A'p}{y - a'p} \frac{1}{1 - \eta(\frac{p}{y})} \right) + a$$

Gorman (1976) general linear technology household demand model is:

$$z = Ah^m \left(\frac{A'p}{y - a'p} \right) + a$$

Barten (1964) is Gorman's model with $a = 0$ and A a diagonal matrix.

Gorman had similar motivation for consumption technology as a model of sharing and jointness of consumption, but only in a unitary model.

Gorman makes household demands be a scaled function of one individual's demands.

Gorman differs from BCL even if members all have same preferences ($h^f = h^m$) and same equivalent incomes ($\eta = 1/2$).

Nonparametric Identification With Linear Technology

Claim: Given observable demand functions, the functions x^m , x^f , F , and η are generically identified if number of goods is $n \geq 3$.

Take $T \geq n + 10$ price vectors p^1, \dots, p^T . Then

$$z^t = Ah^f \left(\frac{A'p^t}{y - a'p^t} \frac{1}{\eta^t} \right) + Ah^m \left(\frac{A'p^t}{y - a'p^t} \frac{1}{1 - \eta^t} \right) + a$$

For each t have $(n - 1)$ independent equations, total $(n - 1) T$ equations. The unknowns are A , a , and η^t ; total $n^2 + n + T$ unknowns. With $n \geq 3$ and $T \geq n + 10$, have more equations than unknowns, so we have identification as long as the equations are linearly independent (not too simple).

Examples: Identification fails for LES h , Scaling of Barten Technology A not identified for homothetic h^i .

Example: Identification In Almost Ideal Demand System

Claim: Assume h^i is defined by budget shares $\omega^i(p/y^i) = \alpha^i + \Gamma^i \ln p + \beta^i [\ln(y^i) - c^i(p)]$ for $i = m, f$. Assume $\beta^f \neq \beta^m$ and elements of β^f , β^m , and the diagonal of A are nonzero. Then the functions x^m , x^f , F , and η are identified.

Actually substantially overidentified. Most parameters are identified from demands on just one good. Models that nest Almost Ideal like QUAIDS are also identified.

Proof sketch: Have $\pi = A'p/(1 - a'p)$ and

$$z_k = a_k + \sum_j A_{kj} \left[\frac{\eta}{\pi_j} \omega^f \left(\frac{\pi}{\eta y} \right) + \frac{1 - \eta}{\pi_j} \omega^m \left(\frac{\pi}{(1 - \eta)y} \right) \right]$$

Intercepts identify a . Coefficients of $\ln y$ identify

$\sum_j [\eta \beta_j^f + (1 - \eta) \beta_j^m] / \sum_\ell (A_{\ell j} / A_{k j}) p_\ell$. Variation in p and subscripts identifies η , β coefficients and ratios $A_{\ell j} / A_{k j}$. Levels of A_{jk} are identified from the quadratic price terms in $\sum_j A_{kj} \left[\frac{\eta}{\pi_j} c^f(\pi) + \frac{1 - \eta}{\pi_j} c^m(\pi) \right]$.

Identifying Demand Functions

$z = h(p/y)$ are household demand functions obtained from

$$\max_{x^f, x^m, z} \{ \mu(p/y) U^f(x^f) + U^m(x^m) \mid z = F(x^f + x^m), p'z = y \}$$

$h(p/y)$ identified from data on a household's purchases in many p/y regimes. More commonly, estimated from data on many household's purchases, assuming preferences are identical up to observable covariates and ignorable errors. Later we'll introduce unobserved heterogeneity into the model.

The difficulty is not identifying household's demand functions $z = h(p/y)$, rather, it is identifying individual household member's demand functions $x^j = h^j(p/y)$ that come from $\max_{x^j} \{ U^j(x^j) \mid px^j = y \}$.

Methods of Identifying Household Member's Demand Functions

Overly strong BCL Identify $h^j(p/y)$ from data on purchases by singles living alone.

How to weaken this assumption?

Model how individual's preferences change when they marry (not done).

Impose constraints on π , η to weaken data requirements on h , h^m , h^f .

Examples: π linear or Barten, η independent of y (at some y levels).

Impose restrictions on how h^m , h^f vary across people (later SAP).

Impose restrictions on how h^m , h^f vary across households of different types (later SAT).

Exploit combination of η independent of y with presence of distribution factors.

Simplifying the Model

Assume Barten technology: $a = 0$, A diagonal. Let

$w^k(y, p, A) =$ couple's budget share of good k .

$w_j^k(y, p) =$ budget share of good k from utility function $U^j(x^j)$, $j = m, f$.

BCL model then becomes, in budget share form

$$\begin{aligned}w^k(y, p, A) &= \eta w_f^k(\eta y, A'p) + (1 - \eta) w_m^k((1 - \eta)y, A'p) \\ \eta &= \eta(p, y, A)\end{aligned}$$

BCL estimate this model, with Quadratic Almost Ideal Demand System
QUAIDS model (Banks, Blundell, and Lewbel 1997) for singles.

Simplifying to Engel Curves

Drop prices, write the model in terms of Engel curves. Using data from just one price regime greatly reduces dimensionality and data requirements.

LP (2008), Bargain and Donni (2009) simplify by assuming Independence of Base (IB) (Lewbel 1989, Blackorby and Donaldson 1992), i.e., for each person j , there exists a D_j such that $V_j(A'p, y) = V_j[p, y/D_j(A, p)]$. Also assume η independent of y , makes household budget share Engel curves simplify to

$$w^k(x, A) = h^k(A) + \eta(A) w_f^k(y/I_f(A)) \\ + (1 - \eta(A)) w_m^k(y/I_m(A))$$

Where I_f and I_m are "indifference scales."

These papers simplify to Engel curves, but still use the BCL method of identifying the demand functions w_f and w_m from singles data.

Is η Independent of y reasonable?

Can resource shares η be independent of total expenditures y , as assumed by these "identification just from Engel curves" models? It generates testable restrictions, see DLP (2013).

Empirical evidence Menon, Pendakur, Perali (2012); Cherchye, De Rock, Lewbel and Vermeulen (2013).

Permits η to depend on prices, incomes of each member, wealth, distribution factors, etc. Only assuming η independent of y after conditioning on these other things.

Does not violate Samuelson (1956), who showed resource shares can't be constant for a large class of social welfare functions, since it permits shares to depend on prices.

DLP (2013, online appendix) provides an example of a sensible class of models of utility functions and Pareto weights that yield η independent of y : PIGL or PIGLOG (Muellbauer 1976) utility with weighted S-Gini (Donaldson and Weymark 1980) household social welfare functions.

Some Summary Empirical Results

Use the LP (2008) Engel curve simplification of BCL - singles and couples data. Four models (vary by demographics included and outlier handling). 1990-92 Canadian Family Expenditure Survey, 12 consumption goods, 419 single men, 450 single women, and 332 couples.

Estimated resource share η for median women: 0.36 to 0.46. Small age and education effects (these affect both preferences and shares). Raising proportion of household's income she contributes up by .5 raises η by about .05.

Other estimates: scale-economy measure $p'z/p'x$ should lie between 1/2 (full sharing) and 1 (completely private). Estimated range 0.70 to 0.78. Indifference scales for women I_f around 1.53; for men I_m around 1.44. A person needs about two thirds, $1/1.53$ or $1/1.44$ of couple's income to reach the same indifference curve living alone that one attains living with a partner.

Dropping the use of Singles data

Bounds don't use singles data (does need prices). DLP (2013, 2016) get point identification without prices, without singles, and without the IB assumption. They use Engel curves, assume a private assignable good for each household member.

Further simplify estimation (and further relax data requirements and modeling constraints) by only using data on the one private assignable good for each household member, and without price data.

To maintain identification, keep the assumption that resource shares η are independent of total expenditures y (at least for low levels of y). In addition:

DLP (2013) Add SAP or SAT restrictions on demand functions, or,

DLP (2016) make use of distribution factors.

Focus on Assignable Goods, Bring in Children

Have s children, impose same utility function for each. Household does

$$\max\{\tilde{U} [U^f(x^f), U^m(x^m), U^c(x^c), p/y, s] \mid z = F(x^f + x^m + sx^s, s), p'z = y\}$$

Looking only at private assignable goods (say, clothing), household's budget shares are given by

$$W_{cs}(y) = s\eta_{cs} w_{cs}(\eta_{cs}y), \quad W_{ms}(y) = \eta_{ms} w_{ms}(\eta_{ms}y), \\ W_{fs}(y) = \eta_{fs} w_{fs}(\eta_{fs}y).$$

$W_{fs}(y)$ = fraction of y household spends on woman's clothes in a single price regime p . These can be estimated.

$w_{cs}(y)$ = fraction of y spent that would be spent on woman's clothes determined by $\max\{U^f(x^f) \mid \pi'_s z = y\}$, at shadow prices π_s given by Barten technology for household with s children.

η_{fs} = woman's resource share in house with s children.

Similar for man m and child c .

Identification Strategies

$$W_{cs}(y) = s\eta_{cs}w_{cs}(\eta_{cs}y), \quad W_{ms}(y) = \eta_{ms}w_{ms}(\eta_{ms}y), \\ W_{fs}(y) = \eta_{fs}w_{fs}(\eta_{fs}y).$$

We observe W functions, want to identify resource shares η .

LP (2008), Bargain and Donni (2009), extend BCL method by learning functions $w_{ms}(y)$ and $w_{fs}(y)$ from data on singles.

DLP (2013), place semiparametric restrictions (SAP) or (SAT) on the functions $w_{ms}(y)$, $w_{fs}(y)$, and $w_{cs}(y)$. Not restrictions on shape, but restrictions that make some feature of these functions be similar across people (SAP) or similar across household size/types (SAT).

DLP (2016), combine η not depending on y with distribution factors.

Similar Across People SAP Identification

SAP demand: $w_j(y, p) = h_j(p) + g\left(\frac{y}{G_j(p)}, p\right)$ for $y \leq y^*$, $j = m, f, c$.
Functions h_j , G_j , and g can be anything, but g is the same across people.

Paper gives SAP class of utility functions. SAP, which is similar to but weaker than shape invariance, need apply only to the assignable goods (clothing). Get

$$\begin{aligned}W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\tilde{g}_s(\eta_{cs}\gamma_{cs}y), \\W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\tilde{g}_s(\eta_{ms}\gamma_{ms}y) \\W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\tilde{g}_s(\eta_{fs}\gamma_{fs}y)\end{aligned}$$

Note $\gamma_{js} = G_j(\pi_s(p))$, shadow prices depend on household type/size s , which makes these functions vary by s in the Engle curves. Same for $\alpha_{js} = h_j(\pi_s(p))$ and $\tilde{g}_s(\cdot) = g(\cdot, \pi_s(p))$.

Similar Across People SAP Identification

SAP Identification overview: By SAP we have

$$\begin{aligned}W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\tilde{g}_s(\eta_{cs}\gamma_{cs}y), \\W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\tilde{g}_s(\eta_{ms}\gamma_{ms}y) \\W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\tilde{g}_s(\eta_{fs}\gamma_{fs}y)\end{aligned}$$

Look at derivatives with respect to y at $y = 0$:

$$\begin{aligned}W'_{fs}(0) &= \gamma_{fs}\eta_{fs}^2\tilde{g}'_s(0), & W''_{fs}(0) &= \gamma_{fs}^2\eta_{fs}^3\tilde{g}''_s(0), \\W'''_{fs}(0) &= \gamma_{fs}^3\eta_{fs}^4\tilde{g}'''_s(0)\end{aligned}$$

and same for m and c . Along with $\eta_{fs} + \eta_{ms} + s\eta_{cs} = 1$ gives 10 equations in 9 unknowns η_{fs} , η_{ms} , η_{cs} , γ_{fs} , γ_{ms} , γ_{cs} , $\tilde{g}'_s(0)$, $\tilde{g}''_s(0)$, and $\tilde{g}'''_s(0)$, for each household size s .

Identification only used derivatives at $y = 0$, so only needed restrictions to hold for small y .

Similar Across Types Identification

SAT demand: $w_j(y, p) = g_j\left(\frac{y}{G_t(p)}, \bar{p}\right)$ for $y \leq y^*$, $j = m, f, c$, where \bar{p} are prices only of private goods. Functions h_j , G_j , and g_j can be anything, but the g function only depends on prices through \bar{p} .

Again only need to hold for clothing. Get

$$\begin{aligned}W_{cs}(y) &= \alpha_{cs} + s\eta_{cs}\tilde{g}_c(\eta_{cs}\gamma_{cs}y) \\W_{ms}(y) &= \alpha_{ms} + \eta_{ms}\tilde{g}_m(\eta_{ms}\gamma_{ms}y) \\W_{fs}(y) &= \alpha_{fs} + \eta_{fs}\tilde{g}_c(\eta_{fs}\gamma_{fs}y)\end{aligned}$$

SAP made $\tilde{g}_s(\cdot) = g(\cdot, \pi_s(p))$ only have a type s subscript, while SAT makes $\tilde{g}_j(\cdot) = g_j(\cdot, \bar{p})$ only have a person j subscript.

For identification, look at same derivatives as in SAP, but now combine across a households of a few different types (difference sizes s), using that $\tilde{g}_j(0)$ doesn't vary by s , to get more equations than unknowns.

Some Estimates Using SAP or SAT - Malawi Data

The Malawi Integrated Household Survey, conducted in 2004-2005. from the National Statistics Office of the Government of Malawi with assistance from the International Food Policy Research Institute and the World Bank.

High quality data: enumerators were monitored; big cash bonuses were used as an incentive system; about 5 per cent of the original random sample in each year had to be resampled because dwellings were unoccupied; (only) 0.4 per cent of initial respondents refused to answer the survey.

We use 2794 households comprised of non-urban married couples with 1-4 children aged less than 15. Private assignable good is men's, women's and children's clothing (including footwear).

Demographics: region, children age summaries, fraction of girls, adult low and high age dummies, education levels of each spouse, distance to a road and to a market, dry season dummy, religion (christian, muslim, animist/other).

Estimated Levels of Resource Shares

		SAP		SAT		SAP&SAT	
		Est	<i>Std Err</i>	Est	<i>Std Err</i>	Est	StdErr
1 kid	man	0.443	<i>0.048</i>	0.378	<i>0.076</i>	0.400	0.045
	woman	0.308	<i>0.041</i>	0.368	<i>0.062</i>	0.373	0.042
	kids	0.249	<i>0.037</i>	0.254	<i>0.072</i>	0.227	0.036
	each kid	0.249	<i>0.037</i>	0.254	<i>0.072</i>	0.227	0.036
2 kids	man	0.423	<i>0.051</i>	0.436	<i>0.090</i>	0.462	0.051
	woman	0.222	<i>0.042</i>	0.212	<i>0.056</i>	0.221	0.043
	kids	0.355	<i>0.045</i>	0.352	<i>0.100</i>	0.317	0.045
	each kid	0.177	<i>0.022</i>	0.176	<i>0.050</i>	0.158	0.023
3 kids	man	0.427	<i>0.057</i>	0.437	<i>0.099</i>	0.466	0.053
	woman	0.185	<i>0.046</i>	0.166	<i>0.054</i>	0.176	0.044
	kids	0.388	<i>0.050</i>	0.397	<i>0.114</i>	0.358	0.050
	each kid	0.129	<i>0.017</i>	0.132	<i>0.038</i>	0.119	0.017
4 kids	man	0.318	<i>0.070</i>	0.352	<i>0.112</i>	0.384	0.063
	woman	0.214	<i>0.054</i>	0.168	<i>0.062</i>	0.182	0.052
	kids	0.468	<i>0.061</i>	0.479	<i>0.133</i>	0.434	0.059
	each kid	0.117	<i>0.015</i>	0.120	<i>0.033</i>	0.109	0.015

Summary of Results - SAP and SAT on Malawi Data

SAP and SAT accepted, no data on singles needed.

Can't reject constant Father's share of 40%.

Mother share decreases by 5.5% per child.

Girl's get about 90% of what boys get.

Mother education level at 90 percentile instead of median decreases Father's share to 30%, 2/3 of the gain goes to mother, 1/3 to children.

Amartya Sen observed the importance of policies that empower women in developing countries. Benefits for children are assumed.

This methodology provides a way to quantify the benefits to women and to children of proposed policies that affect intrahousehold power.

Another SAP and SAT application - Missing Women in India

Anderson and Ray (2010) estimate that in India, 1.7 million woman over age 45 “missing:” died at younger than expected ages.

The number missing increases with age from 45 to 70.

No good theory why. Poverty kills, but household poverty rates do not correlate with women’s age.

Calvi, R. (2016), "Why Are Older Women Missing in India? The Age Profile of Bargaining Power and Poverty" Boston College Working paper.

Another SAP and SAT application - Missing Women in India - continued

Calvi first looks at a an inheritance law that increases women's bargaining power.

Changes in the law varied by state, time, and religious affiliation.

Finds corresponding improvements in health outcomes where and when the law changed.

Calvi then uses SAP and SAT to estimate women's resource shares. Finds they decrease with age after 45.

Use the shares and household income to calculates the ratio of women in poverty to men in poverty by age.

The correlation of estimated poverty ratio to Anderson and Ray's estimate of missing women is amazing: .96

Point Identification by Distribution Factors

DLP (2016): No preference restrictions. η depends on distribution factors d , not on y , we have

$$W_{cs}(y, d) = s\eta_{cs}(d) w_{cs}[\eta_{cs}(d) y]$$

$$W_{ms}(y, d) = \eta_{ms}(d) w_{ms}[\eta_{ms}(d) y]$$

$$W_{fs}(y, d) = \eta_{fs}(d) w_{fs}[\eta_{fs}(d) y].$$

We observe W functions, want to identify resource shares η .

Identification (intuition, real proof doesn't need $y = 0$): Again look at derivatives wrt y at $y = 0$:

$$W'_{fs}(0) = [\eta_{fs}(d)]^2 w'_{fs}(0)$$

and similar for m and c . For each value d takes on, this along with $s\eta_{cs}(d) + \eta_{ms}(d) + \eta_{cs}(d) = 1$ gives 4 equations. If d takes on at least 3 different values then we get 12 equations in the 12 unknowns $w'_{cs}(0)$, $w'_{ms}(0)$, $w'_{fs}(0)$, and, for each of 3 values of d , $\eta_{cs}(d)$, $\eta_{ms}(d)$, and $\eta_{fs}(d)$.

Unobserved Distribution Factors

It's unrealistic to think we can observe all or even most of what affects household's resource distributions.

Suggests modeling resource shares as random variables.

Will also want to consider individual preferences varying across households (random utility parameters).

Chiappori and Kim (2013): have random resource shares, don't identify levels of shares.

DLP (2016): Assume resource shares conditionally independent of y , distribution factors that can take on 3 values, private assignables that are normal goods, differentiable in y . Then the entire joint distribution function of random resource shares η_{CS} , η_{MS} , and η_{CS} is identified.

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Proof sketch: Look in expenditures form $Y_{fs} = yW_{fs} = \psi_{fs}(\eta_{fs}y)$. Have

$$F_{Y_{fs}}(\omega | y) = \Pr(Y_{fs} \leq \omega | y) = \Pr(\psi_{fs}[\eta_{fs}y] \leq \omega | y) \\ = \Pr\left(\eta_{fs} \leq \frac{\psi_{fs}^{-1}(\omega)}{y} | y\right) = F_{\eta_{fs}}\left(\frac{\psi_{fs}^{-1}(\omega)}{y} | y\right) = F_{\eta_{fs}}\left(\frac{\psi_{fs}^{-1}(\omega)}{y}\right).$$
 So

$$\exp\left(\int \frac{y \partial F_{Y_{fs}}(\omega|y) / \partial \omega}{\partial F_{Y_{fs}}(\omega|y) / \partial y} d\omega\right) = \exp\left(\int \frac{\partial \ln \psi_{fs}^{-1}(\omega)}{\partial \omega} d\omega\right) = \psi_{fs}^{-1}(\omega) \kappa_{fs}$$
 where

unknown κ_{fs} is exponential of the constant of integration. So ψ_{fs}^{-1} is identified up to κ_{fs} , similar for other household members. Since $\psi_{fs}^{-1}(\omega) = \eta_{fs}y$, this shows we can identify the joint distribution of η_{fs}/κ_{fs} , η_{ms}/κ_{ms} , η_{cs}/κ_{cs} . Apply previous identification by distribution factors to the means of these distributions to identify κ_{fs} , κ_{ms} , and κ_{cs} , so then entire joint distribution of η_{fs} , η_{ms} , η_{cs} is identified.

Extension: Show can identify both random resource shares and a random utility (preference) parameter.

Other Welfare Calculations - Equivalence Scales vs Indifference Scales

An equivalence scale E^f is the fraction of a household's income an individual woman f living alone needs to attain *the same utility level as the household*. Given indirect utility functions V^f for the woman, and household indirect utility function V , Equivalence scale E^f solves:

$$V^f \left(\frac{p}{E^f y} \right) = V \left(\frac{p}{y} \right)$$

Problem: Traditional Equivalence scales are fundamentally not identifiable. For any monotonic G , (relabeling the woman's indifference curves) get a different E^f from the observationally equivalent equation

$$G \left[V^f \left(\frac{p}{E^f y} \right) \right] = \tilde{V}^f \left(\frac{p}{E^f y} \right) = V \left(\frac{p}{y} \right)$$

Also, household may use a bargaining model that does not correspond to existence of a well defined household utility function V .

A replacement for Equivalence Scales: Indifference Scales

BCL define an *Indifference scale* I^f as the fraction of the household's income that woman f living alone would need to attain *the same indifference curve over goods that she had as a member of the household*.

Given indirect utility function V^f for f , household shadow prices π from sharing and woman's resource share η , I^f solves:

$$V^f \left(\frac{p}{I^f y} \right) = V^f \left(\frac{\pi}{\eta y} \right)$$

Unaffected by monotonic transformations of utility. Replacing $V^f(\cdot)$ with $G[V^f(\cdot)]$ leaves I^f unchanged.

Indifference scale don't require existence of a household utility function V , avoids issues of interpersonal comparability and differences in indifference curves, and is identified from revealed preference data given identification of π and η .

Constructing Indifference Scales

$V^f(p/y^f)$ is indirect utility of f . Apply Roy's to get demands $h^f(p/y^f)$.

In the household, f consumes equivalent bundle $x^f = h^f(\pi/(\eta y))$, same as if she were living alone and facing prices/income = $\pi/(\eta y)$.

The indifference scale is $I^f(p, y)$ defined by

$$V^f\left(\frac{p}{I^f(p, y)y}\right) = V^f\left(\frac{\pi(p/y)}{\eta(p/y)y}\right)$$

So $I^f(p, y)y$ is income that would be required by f living alone to attain same indifference curve over goods that f attains in the couple, consuming bundle $x^f = h^f(\pi/\eta)$.

$I^f(p, y)$ is fully identified and *ordinal*, not affected by choice of cardinalization for V^f .

Construct same for m , replacing f with m and η with $1 - \eta$.

Example Uses for Indifference Scales

If a couple has income y and husband dies, surviving f will need income $p' h^f(\pi / (\eta y))$ to buy the same bundle she consumed before, or $I^f y$ to be on the same indifference curve for goods as before (excludes loss of utility from companionship). Use for wrongful death lawsuits, life insurance, alimony.

$I^f y$ = fraction of a couple's income that a women living alone needs to be as well off as she'd be in the couple. Use for welfare comparisons.

Given singles poverty lines \bar{y}^f , \bar{y}^m , couple's poverty line is \bar{y} is the minimum value of y such that an η exist satisfying the inequalities

$$V^f \left(\frac{p}{\bar{y}^f} \right) \leq V^f \left(\frac{\pi (p/y)}{\eta y} \right), V^m \left(\frac{p}{\bar{y}^m} \right) \leq V^m \left(\frac{\pi (p/y)}{(1 - \eta) y} \right)$$

More Example Uses for Indifference Scales

Ratio I^f / I^m compares single's income requirements by looking at how much each needs alone to be as well off as they would be in the same household. I^f / I^m might not equal one even if f and m have same preferences, because of bargaining η .

If we know woman's outside option, i.e., the income \tilde{y}^f , she would have if lived alone, we can calculate how big her share $\tilde{\eta}$ would need to be to make her better off, in terms of goods consumption, in the household. Could be used for threat point bargaining calculations. This $\tilde{\eta}$ solves

$$V^f \left(\frac{p}{\tilde{y}^f} \right) = V^f \left(\frac{\pi(p/y)}{\tilde{\eta}(p, y, \tilde{y}^f)y} \right)$$

The model separately identifies tastes, bargaining, and sharing.

Conclusions - 1

Resource shares are a better measure of household resource allocation and power than Pareto weights (do not depend on a cardinalization of utility).

Many household welfare calculations depend on resource share levels, not just how they vary with distribution factors.

A variety of alternative identifying strategies are proposed to point identify or to bound resource shares.

Most recently, these include identifying distribution of resource shares across households allowing for unobserved distribution factors (unobserved variation in power) and unobserved heterogeneity in preferences.

Conclusions - 2

Among point identification strategies, most attractive may be: assume one private assignable good per individual type, resource shares independent of total expenditures, and an observable distribution factor. This permits complete identification of the joint distribution of random resource shares, with almost no restrictions on preferences, from just private goods demand functions, even without price variation.

An example of a welfare calculation based on resource sharing is the calculation of indifference scales. Unlike equivalence scales, indifference scales can be identified by revealed preference.

Other useful welfare calculations include bargaining power measures, measuring poverty rates for individuals (including children), extending regional inequality measures to the level of individuals instead of households, and measuring economies of scale arising from jointness of consumption.