Equivalence Scales Based on Collective Household Models

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December 2002

Abstract

Based on Lewbel, Chiappori and Browning (2002), this paper summarizes how the use of collective models of household behavior can overcome the identification problems associated with the construction and estimation of adult equivalence scales.

JEL codes: K1, D1, J17, C30, I31.

Keywords: Adult Equivalence Scales, Consumer Demand, Household Bargaining, Collective models, Insurance, poverty lines.

This research was supported in part by the NSF, grant SES-9905010.

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1 Introduction

Adult equivalence scales are defined as the income required by one household to be ”as well off” as another. For example, Muellbauer (1977) defines equivalence scales as ”budget deflators which are used to calculate the relative amounts of money two different types of households require to reach the same standard of living.” The early equivalence scale literature attempted to define this ratio of costs of living directly in terms of measurable quantities such as the costs of acquiring a required number of calories, but this was soon replaced by defining households to be equally well off if they attain an equal level of utility (see Lewbel 1997 and Pollak and Wales 1992 for surveys). Just as a true cost of living price index measures the ratio of costs of attaining the same utility level or indifference curve under different price regimes, equivalence scales are intended to measure the ratio of costs of attaining the same utility level or indifference curve under different household compositions. Numerous severe identification issues arise in the estimation of equivalence scales. See in particular Pollak and Wales (1979, 1992), Blundell and Lewbel (1991), and the surveys Lewbel (1997) and Slesnick (1998).

This paper summarizes a solution to the equivalence scale identification problem, due to Lewbel, Chiappori, and Browning (2002). The idea is that, rather than attempt to compare the standards of living of different households, equivalence scales are proposed that compare the indifference curves attained by the same or comparable individuals in two different settings, namely, living alone versus living with a spouse. This distinction is related to what Pollak and Wales (1979) call a situation comparison as opposed to a welfare comparison, and permits at least theoretical identification of equivalence scales given some assumptions about stability of preferences over goods.

This method of identifying equivalence scales depends on recovering the consumption demand functions of individuals within a houshold, and hence requires a collective household model. See, e.g., Bourguignon and Chiappori (1994) and Vermeulen (2000) for surveys of such models in empirical contexts. For constructing equivalence scales in this way it would be useful to observe the separate consumption behavior of individual household members, but with some additional behavioral assumptions, the model by Lewbel, Chiappori, and Browning (2002) can be applied to construct these equivalence scales using standard consumer demand data. An example empirically implementable model is provided.

Although the focus of this paper is equivalence scales, the same methodology could be employed to address other related issues, such as the calculation of appropriate levels of life insurance on wage earners, alimony calculations, and the calculation of net income in wrongful death legal cases (see Lewbel 2002). Potential direct applications of equivalences scales include social welfare analyses and the adjustment of poverty lines for households of different sizes and compositions.

2 Traditional Equivalence Scales

Let $U^i(x^i)$ denote the utility function describing the preferences of household i, where x^i is the vector of quantities of goods consumed by household i. If we think of i as indexing household composition, then U^i is what Pollak and Wales (1992) refer to as a "conditional" utility function, that is, it describes a household's preferences conditional on the household having a certain composition (e.g., number and age of members). This is in contrast to an unconditional utility function, which describes preferences over both goods and composition.

Let p be the vector of prices of goods, and let $i = c$ denote a reference household, in this case, a married couple. Let the couple's total expenditures be $y = p'x^c$. The traditional equivalence scale for some other household, say $i = f$ (denoting a single female) is defined as

$$\widetilde{s}^f = \min_{\widetilde{x}} \{ p'\widetilde{x}/y \mid U^f(\widetilde{x}) = U^c(x^c) \} \tag{1}$$

which equals the minimum expenditure level required by household (single) f to attain the same utility level as household c, divided by the total expenditures of household c. To use this equivalence scale, one would multiply a couple's income by $\widetilde{s}^f$ to obtain the income required to give the woman living alone the same level of utility as the couple.

There are many obstacles, both conceptual and practical, to implementing this procedure. Some sort of separability of tangibles x from intangibles must be assumed, and each utility function U^i must be interpreted as the utility that is only due to consumption, not intangibles. Preferences must be recovered from observed demands. We do not literally observe the demands of households in all possible price regimes, so demand functions must be estimated from survey data.

The most serious obstacle to applying this standard equivalence scale methodology is that, by revealed preference theory, given demands for goods one can only recover indifference curves, not actual levels of utility. Consider two commodity space graphs, one consisting of indifference curves over bundles of goods according to the preferences of household f, and the other consisting of indifference curves over bundles of goods according to the preferences of household c.

The numbering of indifference curves in either of these graphs is arbitrary (equivalently, by ordinality preferences are unaffected by applying monotonic transformations to the functions U^i), but equating $U^f(x)$ to $U^c(x^c)$ requires that we know the unique one to one mapping that matches each indifference curve in one graph to an indifference curve in the other graph that delivers the same level of utility (see Pollak and Wales 1992, p. 86). No information about this mapping is identified from each household's separate demands, so equivalence scales themselves are not identified. Many schemes have been proposed to overcome this identification problem (see Lewbel 1997 for a survey), but ultimately all require extensive untestable and unobservable assumptions regarding comparability of preferences.

In some circumstances one might gain additional information from unconditional demands, by applying some form of revealed preference theory over household composition as well as over goods. This is difficult because, unlike goods, household composition is not directly priced.

Yet another difficulty with traditional equivalence scales is that a household may not possess a well defined utility function, but may instead use some kind of bargaining process to determine its purchases. In this case there may not exist a well defined utility level for the household, and hence no equivalence scale would exist by the traditional equation (1) definition.

3 Defining Identifiable Equivalence Scales

Traditional equivalence scales require a comparison of the utilities of different households. We propose instead that one compare the utility of the same individual in two different environments, namely, living alone versus with a spouse. This notion of an equivalence scale does not require utility comparisons across different individuals or groups, and so is potentially identifiable without untestable assumptions regarding comparability of utility across individuals.

To make this distinction concrete, consider for simplicity the case of a childless married couple having no joint or shared consumption. Let $U^f(x^f)$ and $U^m(x^m)$ be the utility functions of the female and male respectively, consuming bundles x^f and x^m. This couple together has total expenditures y, and chooses consumption bundles x^f and x^m by

$$\max_{x^f, x^m} \{\widetilde{U}\left[U^f(x^f), U^m(x^m), p/y\right] \mid (p/y)'(x^f + x^m) = 1\} \tag{2}$$

Where $\widetilde{U}$ is a social welfare function or a bargaining function that is increasing in U^f and U^m, and could itself depend on p/y or on other variables that affect the relative bargaining power of the husband and wife. In this simple example there is no joint or shared consumption, no public goods within the household, and hence no "economies of scale" to consumption. Generalizing this model to permit joint or shared consumption is discussed in the next section.

The function $\widetilde{U}$ may also embody intangible contributions to the couple's utility, such as those arising from consortium and companionship. Formally, the attained utility levels of the husband and wife may be functions of $U^f(x^f)$, $U^m(x^m)$, and various intangibles, and the bargaining or social welfare function for the household would in turn be functions of these attained utility levels and of variables that affect bargaining power.

Our new, identifiable, definition of the female's equivalence scale is then

$$s^f = \min_{x^*}\{p'x^*/y \mid U^f(x^*) = U^f(x^f)\} \tag{3}$$

Note in this definition that x^f is itself a function of p/y, since it is obtained from equation (2). This is the lowest cost way for the woman living alone to attain the same indifference curve she personally attained while living with a spouse, divided by the total expenditures of the couple. We call the equation (3) definition of s^f a collective model based, or intrahousehold based, equivalence scale.

This intrahousehold based definition of equivalence scales overcomes the principal source of nonidentification, because it only depends on the indifference curves of the individual f. To see this, observe that if the utility function $U^f(x^*)$ is replaced with an unobservable, arbitrary monotonic transformation $G^f[U^f(x^*)]$, then the numerical value of the traditional scale $\widetilde{s}^f$ given by equation (1) changes, but the value of s^f as defined by equation (3) is not changed.

Another advantage of the s^f definition of equivalence scales is that it is directly relevant for answering various kinds of policy questions. For example, consider the question of determining an appropriate level of life insurance for a spouse. If the couple spends y dollars per year then for a nonworking wife to maintain the same standard of living after a working husband dies, she will need an insurance policy that pays enough to permit spending $s^f y$ dollars per year, not $\widetilde{s}^f y$ dollars, even if the latter could be identified. Similarly, in cases of wrongful death, juries are instructed to assess damages both to compensate for the loss in "standard of living," (i.e., $s^f y$) and, separately for "pain and suffering," which would be the noneconomic effects that are embodied in the function $\widetilde{U}$ (see Lewbel 2002). The calculations for appropriate levels of alimony are analogous.

4 Joint Consumption

Realistic models must allow for joint or shared consumption in a household. Following Lewbel, Chiappori, and Browning (2002), define a consumption technology function F that relates the bundle of goods consumed by the household, say z, to a vector of private good equivalents x. These private good equivalents are then divided up between the household members, with each member deriving utility from consuming their share of x. The proposed model of household consumption is then

$$\max_{x^f, x^m} \{\widetilde{U}\left[U^f(x^f), U^m(x^m)\right] \mid p' F(x^f + x^m) = y\} \tag{4}$$

where $z = F(x^f + x^m)$ is the bundle of goods that the household is observed purchasing. Because of joint consumption in the household, buying the bundle $z = F(x)$ with sharing is equivalent to buying the bundle $x = x^f + x^m$ without sharing.

This framework is similar to a Becker (1965) type household production model, except that instead of using market goods to produce commodities that contribute to utility, the household produces the equivalent of a greater quantity of market goods via sharing. This is essentially the motivation for Barton (1964) type equivalence scales and Gorman's (1976) linear household technologies, except that a collective model of the household is employed to account for the differences in preferences and consumption of the different household members.

The transformation from z to x embodied by the function F is intended to summarize all of the technological economies of scale and scope that result from living together. For a purely private good k for which there is no shared consumption, e.g., clothing, x_k could equal z_k. For a good k that is shared, e.g., automobile use, x_k might equal $f_k z_k$, where $f_k - 1$ represents the fraction of time that the good is consumed jointly. More generally f_k could be an arbitrary function of z, implying that the fraction of time that the car is consumed jointly depends on the total quantity of car use, and on the quantity of other goods, e.g., vacations and food consumed away from home.

5 An Example Model

Let $V^f(p/y^f)$ and $V^m(p/y^m)$ be the indirect utility functions corresponding to the female and male's direct utility functions $U^f(x^f)$ and $U^m(x^m)$ when living alone as singles. By applying Roy's identity to V^f and V^m, we may obtain in the usual way $w_k^f = \omega_k^f(p/y^f)$ and $w_k^m = \omega_k^m(p/y^m)$, which are the female's and male's budget shares of consumption good k when living as singles.

For couples, assume a Barten type technology function, defined as

$$z_k = A_k x_k \tag{5}$$

for each good k, so $z = F(x) = Ax$ where the matrix A is diagonal. Lewbel, Chiappori, and Browning (2002) show that, using the model of equation (4), with this technology the couple's budget shares for each good k from 1 to n will have the functional form

$$\omega_k(p) = \eta\omega_k^f\left(\frac{Ap}{\eta y}\right) + (1-\eta)\omega_k^m\left(\frac{Ap}{(1-\eta)y}\right) \tag{6}$$

where η, which lies between zero and one and could itself depend on p/y, is a parameter or function that is determined by $\widetilde{U}$ and represents the fraction of the couple's total expenditures that are devoted to the female's share of consumption.

For example, if single's have Deaton and Muellbauer's (1980) Almost Ideal demands, then, for $i = f$ and $i = m$,

$$V^i\left(\frac{p}{y^i}\right) = \frac{\ln\left(y^i\right) - c^i(p)}{b^i(p)} \tag{7}$$

$$\omega^i(p/y^i) = \alpha^i + \Gamma^i \ln p + \beta^i\left[\ln\left(y^i\right) - c^i(p)\right] \tag{8}$$

where $c^i(p)$ and $b^i(p)$ are price indices defined as

$$c^i(p) = (\ln p)'\alpha^i + \frac{1}{2}(\ln p)'\Gamma^i \ln p \tag{9}$$

$$\ln[b^i(p)] = (\ln p)'\beta^i. \tag{10}$$

Here α^i and β^i are n-vectors of parameters and Γ^i is a symmetric $n \times n$ matrix of parameters. The sum of the α^i parameters is one, and the sum of the β^i parameters and of each column of Γ^i is zero. These parameters may be estimated from observed single's budget share demands in the usual way. The couple's budget share demands are then

$$\begin{aligned}\omega_k(p) = \; & \eta\left(\alpha_k^f + \Gamma_k^f \ln(Ap) + \beta_k^f\left[\ln(y) + \ln(\eta) - c^f(Ap)\right]\right) \\ & + (1-\eta)\left(\alpha_k^m + \Gamma_k^m \ln(Ap) + \beta_k^m\left[\ln(y) + \ln(1-\eta) - c^m(Ap)\right]\right)\end{aligned} \tag{11}$$

One estimation method for couples would be to subsitute the parameters estimated from single's demands into the above budget share equation (11) for each good k, then estimate the remaining parameters A and η using the couple's demand data.

Many of the parameters of the model, in particular the intercept terms α_k^i and technology parameters A_k, should themselves be made functions of demographic composition variables such as age. Based on Browning, Bourguignon, Chiappori and Lechene (1994) and Browning and Chiappori (1998), a sensible model for the sharing rule η is

$$\eta = \eta_0 + \eta_1 \frac{Y^f}{Y^f + Y^m} + \eta_2 \ln\left(\frac{y}{P}\right) \tag{12}$$

where Y^i is the gross income of household member i and P is a Stone price index for the couple. This sharing rule depends on the wife's share of total gross income, which is a potential measure of bargaining power, and on the couple's total real expenditures.

Once the parameters are estimated, the collective model or intrahousehold based equivalence scale s^f defined by equation (3) equals the solution to the equation

$$V^f\left(\frac{p}{s^f y}\right) = V^f\left(\frac{Ap}{\eta y}\right). \tag{13}$$

With the above Almost Ideal specification for V^f, the solution to equation (13) is

$$\ln s^f = \left(\frac{b^f(p)}{b^f(Ap)} - 1\right)\ln(y) + \frac{b^f(p)}{b^f(Ap)}\left[\ln(\eta) - c^f(Ap)\right] + c^f(p) \tag{14}$$

In general, collective model equivalence scales are not independent of base (see Lewbel 1989), that is, they vary with income y. However, in the above Barten Almost Ideal collective model, $b^f(p)/b^f(Ap) = \exp\left(\sum_k \beta_k \ln A_k\right)$, so if $\sum_k \beta_k \ln A_k = 0$ and η is independent of y (e.g., if $\eta_2 = 0$ in equation 12), then the collective model equivalence scale s^f is independent of base.

Analogous to the above, the equivalence scale that makes the male as well off alone as he was in the couple is the solution to

$$V^m\left(\frac{p}{s^m y}\right) = V^m\left(\frac{Ap}{1-\eta}\right) \tag{15}$$

These scales depend on η. To separate issues of bargaining power from other considerations, we could evaluate these equivalence scales substituting $\eta = .5$ into equations (13) and (15). Similarly given poverty lines for single individuals, the model could be used to calculate the poverty line for a couple, defined as the minimum income y that is required by a couple that, by choosing η (or equivalently $\widetilde{U}$) optimally, can get each member's consumption $p'x^f$ and $p'x^m$ in (2), to equal their respective poverty lines.

6 Concluding Remarks

We propose collective or intrahousehold based equivalence scales, which overcome the main source of nonidentification of traditional equivalence scales. To illustrate this point, observe that replacing U^f with any monotontic transformation $G^f(U^f)$ is equivalent to replacing V^f with $G^f(V^f)$, and leaves all the demand functions and the resulting equivalence scale s^f unchanged.

As written, the above model does not allow for children. One way to incorporate children's welfare into the model would be to let U^f and all the associated demand functions and scales refer to the joint utility function of a woman and her children. The above model would then use data from single men, single mothers, and couples with children to calculate relevant scales.

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