Rank, Separability, and Conditional Demands

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April 2000, Revised October 2001

Abstract

Rank R demand systems that are either conditioned on some good (such as durables) or are separable from that good, are shown to be identical to unconditional demand systems having rank R or R+1. More generally, the rank of a conditional demand system or a separable subsystem is a lower bound on the rank of the corresponding demand system over all goods. This relationship is applied to reinterpret some existing empirical results regarding rank, conditioning goods, and separability.

This research was supported in part by the National Science Foundation through grant SES-9905010. I would like to thank Christopher Nicol for a lively debate that led to this paper, and for helpful comments. I'd also like to thank the editors of this journal and anonymous referees for useful comments and suggestions. Any errors are my own.

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Extending Gorman (1981), Lewbel (1991) defined the general concept of the rank of a demand system, and showed that rank has many important implications for specification, estimation, aggregation, and welfare calculations. The point of this note is to clarify some relationships between demand system rank, separability, and conditioning goods. These results help explain how differences in assumptions regarding either separability or the presence of conditioning goods may result in different estimates of demand system rank. In particular, it is shown here that, for any positive integer R, *any conditional or separable rank R demand system equals an unconditional demand system having rank equal to or greater than R.* The main implication of this result is that separability or the presence of conditioning goods can lead to underestimates of the true rank of demand systems.

Dropping household characteristics variables, a conditional rank R demand system can be written as

$$(1) \qquad q_i = \sum_{r=1}^{R} \alpha_{ri}(p_{(n)}) h_r(y_{(n)}, p_{(n)}, q_n), \qquad i = 1, ..., n-1.$$

for some functions α_{ri} and h_r for $r = 1, ..., R$, where q_i is the quantity of good i, n is the conditioning good, $p_{(n)}$ is the vector of prices of all goods except p_n, and $y_{(n)}$ is expenditures on all goods except the conditioning good. For example, if the conditioning good is durables then the goods $i = 1, ..., n-1$ would be various nondurables and services such as food, clothing, household operation, etc.,. In the conditional demand system of equation (1), demands are expressed as functions of prices excluding durables, total expenditures excluding durables, and on the quantity of durables.

The reason for estimating conditional demands is that the demand function for the conditioning good may be dynamic or otherwise quite complicated, or because appropriate price measures for the conditioning good are unavailable. Often q_n itself is not observed and must be proxied. for example, the correct q_n might be the flow of services resulting from the consumer's stock of durables, which is proxied in empirical conditional demand equations by a measure of the stock of durables, or by dummy variables indexing ownership of some durable goods like a house or car.

Another example could be measures of employment, proxying for leisure as a conditioning good. Demographic characteristics like age are not proxies for conditioning goods, so inclusion of demographic variables should more appropriately be interpreted as measures of preference heterogeneity.

Separability is the special case of conditional demands in which the functions h_r do not depend on q_n, so for example if the demands for nondurables and services are separable from the demands for durables, then in equation (1) the functions $h_r(y_{(n)}, p_{(n)}, q_n)$ would be replaced with $h_r^*(y_{(n)}, p_{(n)})$ for some functions h_r^*. Equivalently, separability implies that unconditional demands can be estimated for the subset of goods $i = 1, ..., n - 1$.

Let p_n be the price of the conditioning good n, let p be the vector of prices of all goods, and let y be expenditures on all goods, so by definition $y = y_{(n)} + p_n q_n$.

To demonstrate the main result regarding rank, let

$$(2) \qquad q_n = g_n(y, p)$$

be the unknown marshallian demand function for the conditioning good. Then the unconditional demand system corresponding to the exact same demands as equations (1) and (2) is the system

$$(3) \qquad q_i = \sum_{r=1}^{R+1} \beta_{ri}(p) g_r(y, p), \qquad i = 1, ..., n$$

To see the equality, for $i = 1, ..., n-1$ and $r = 1, ..., R$ define $g_r(y, p) = h_r[y - p_n g_n(y, p), p_{(n)}, g_n(y, p)]$ and $\beta_{ri}(p) = \alpha_{ri}(p_{(n)})$. Also define $g_{R+1}(y, p) = g_n(y, p)$, and let $\beta_{ri}(p) = 0$ whenever $r = R+1$ or $i = n$ except that $\beta_{R+1,n}(p) = 1$. With these definitions, equation (3) equals equation (1) for $i < n$ and equation (3) equals equation (2) for $i = n$. Equation (3) is the general form of a rank $R + 1$ demand system (see Lewbel 1991), but equation (3) will reduce to a rank R system if $g_{R+1}(y, p)$ equals $\sum_{r=1}^{R} \gamma_r(p) g_r(y, p)$ for some functions $\gamma_r(p)$.

Two examples will serve to illustrate these results. Banks, Blundell, and Lewbel (1997) estimate a demand system and find a rank of three. Their model does not include durables (assuming separability of nondurable goods and services from durables), so their results would be consistent with unconditional demands of rank three or rank four. Nicol (2001) estimates conditional demands, finding that some households have conditional rank two, which by the above result implies unconditional rank two or three. Nicol finds other household have conditional rank three, implying an unconditional rank of three or four. The only rank consistent with all these results is an unconditional of rank three.

It is worth noting that Gorman's (1981) theorem does not rule out the existence of rank four or higher systems, but rather only proves that such systems cannot be exactly aggregable. Indeed, Lewbel (2001) provides an example of a utility function that yields rank four demands, though that empirical application also found demands to be rank three, as did, e.g., Lewbel (1991) and Hausman, Newey, and Powell (1995).

The main result above extends to multiple conditioning goods. For example, if demands with two conditioning goods (or demands that are separable from two other goods) have rank R, then the rank of the underlying demand system for all goods could be as high as $R+2$. This has important implications for estimation when q_n is a composite good. For example, Nicol (2001) experiments with including different numbers of goods into the conditioning group q_n. The larger the fraction of expenditures that are included in q_n, the smaller is the resulting subsystem of goods that is estimated (equation 1). We can expect to find that the larger q_n is, the greater is the chance that the resulting conditional demand system will have low rank. Similarly, the smaller is the number of groups of goods n, the greater is the chance that the estimated rank will be low.

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