Borders, Trade, and Political Geography*

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Abstract

Since the Age of Discovery, the world has become economically integrated, while remaining politically disintegrated by national borders. We build a general equilibrium model of international trade and national borders across the world. Over a long time period, declining trade costs alter trade volumes across states but also incentivize states to redraw borders, generating political geography endogenously. Our model has significant implications for the global economy and politics, including trade patterns, geopolitics, and state-size distribution. The assumptions and findings of our model are consistent with digitized map data.

Keywords: nation-state, endogenous borders, trade costs, gravity model

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1 Introduction

National borders studied in international trade theories are usually synonymous with trade costs. In the existing trade theories, if national borders are replaced by other cost shifters, the related economic machinery remains unchanged. On the one hand, this is liberating. By abstracting from what borders refer to, trade theories deliver generalizable insights on trade costs. On the other, modeling borders as trade costs restricts the usefulness of the theories. Drawing borders are political decisions and drawn borders make geographic presence. Equating borders with trade costs alone deprives trade theories of an ability to rationalize political geography.

We make a step forward by endogenizing both international trade and political geography. In this paper, we build a general equilibrium model where trade and borders are jointly determined. In our model, national borders tax trade and thus reduce economic welfare, but they nonetheless exist because political governance mandates limited nation sizes. Local economies, termed *locales*, collectively optimize their national borders according to their geographic locations. The national borders endogenously chosen by locales partition the world into countries (interchangeably, nation-states or *states* for simplicity). Through this model, global trade, borders, and political geography are consolidated into a unified analytical framework.

The crux of our model is differential locational advantages owned by individual locales. We model all locales in the world as a line. Our "world line" follows the tradition in economics of using one dimensional space to differentiate economic agents.[1] In our context, the use of the world line reduces the dimensions of the world. The partitioning of a landmass (two-dimensional space), as drawn in world maps, is essentially using two-dimensional dividers (lines) to obtain states (polygons). By removing one dimension, we use one-dimensional dividers (points) to divide a one-dimensional landmass (world line) to obtain states (intervals). This reduction in dimensionality makes modeling borders possible. Along a line, borders as points have tractable coordinates, a technical feature lacked by higher dimensionalities.

The intuition of our model is as follows. By design, every locale in the world trades with every other locale. For every local economy (locale) in the world, a larger country (state) size is economically attractive because foreign trade is more costly than domestic trade per unit of distance, but a larger country size is politically unattractive because accommodating more locales in one country causes more internal conflicts and thus a higher cost of governance.

[1] For example, Hotelling (1929) on spatial competition, Dornbusch, Fischer, and Samuelson (1977) on comparative advantage, Black (1948) and Downs (1957) on majority-rule voting, and Ogawa and Fujita (1980) on urban structures.

The tradeoff between the two considerations, economical and political, differs across locales depending on their locations along the world line. The geometric center of the world line, namely its midpoint, has the smallest total distance from the rest of the world. As a result, locales have a centripetal tendency when choosing their neighbors to form countries. This universal centripetal tendency ensures that the model is solvable. We prove that there exists a unique partition of the world into different countries (Section 2).

Our model provides a framework where either international politics or international trade can be analyzed using the other as its backdrop (Section 3). On the political front, our model illustrates the political sensitivity of the regions geographically close to the rest of the world in globalization. According to our model, national borders with the highest proximity to the rest of the world are the most pressured to change when there is a worldwide reduction in unit trade cost (i.e. trade cost per unit of distance). This turns out to be in agreement with the tenet in geopolitics that locations close to the geographical center of the world are strategically crucial. Such geopolitical arguments have been known to lack rigorous analytical foundations, as the medium voter theorem cannot be simply applied to world geography or politics. To our knowledge, this is the first study that links the political sensitivity of national borders to international trade.

On the economic front, our model demonstrates that the relationship between bilateral trade and trade costs is more sophisticated than expected. Trade volume is known to rise when trade costs decline. We show that this is just one of three effects. The two missing effects are as follows. When unit trade cost decreases, two non-contiguous countries tend to trade less with each other because their economic sizes shrink, while at the same time, they tend to trade more with each other because the countries between them also shrink in size to bring them closer together. The emergence of the two additional effects results from endogenizing borders. They can be concisely depicted by our *long-term* gravity equation, as opposed to the gravity equation used in the international trade literature where national borders (and thus countries) are fixed.[2]

Although stylized, our model reconciles well with the actual distribution of national borders in the world. We digitized four world maps that correspond to the 18th century, the 19th century, the early 20th century and the modern era, respectively. Using these world maps, we estimate the location of the world geometric center for each time period. We find three data patterns that are consistent with our world-line model (Section 4). Although world geography is not linear in reality, the irregular shapes of global landmasses engender locational (dis)advantages across the world. So long as the locational (dis)advantages

[2]See Anderson (2011) and Head and Mayer (2014) for reviews of the gravity model in the international trade literature.

are in place, our model provides a reasonable approximation of the resulting international economics and politics.

This study is related to the literature on the efficient size of states (Alesina and Spolaore, 1997, 2005, 2006; Alesina, Spolaore, and Wacziarg, 2000, 2005; Brennan and Buchanan, 1980; Desmet, Le Breton, Ortuño-Ortín, and Weber, 2011; Friedman, 1977). In particular, the tradeoff between trade and governability builds on the pioneering model by Alesina et al. (2000). In this vein, country sizes are generally not solvable because all border decisions are interdependent and thus lead to numerous possibilities of country numbers and compositions. We depart from this literature by incorporating a world-line geography, which makes the model solvable in spite of general equilibrium complications. The world-line assumption ensures the tractability of the problem. It enables us to assess every locale's common interests with every other locale, with their own country, and with their contiguous countries.[3]

It is perhaps surprising that endogenizing borders and countries is a rare practice in the international trade literature. Borders form the demarcation between domestic and foreign trade, and countries (nation states) are both analytical and administrative units of international trade. The existing studies have examined the connections between international trade and various domestic institutions. The domestic institutions found to be influenced by trade range from check and balance (Acemoglu, Johnson, and Robinson, 2005) to parliamentary operations (Puga and Trefler, 2014), military operations (Acemoglu and Yared, 2010; Bonfatti and O'Rourke, 2014; Martin, Mayer, and Thoenig, 2008; Skaperdas and Syropoulos, 2001), contract enforcement (Anderson, 2009; Ranjan and Lee, 2007) and institutional structure (Greif, 1994). Meanwhile, there also exist extensive studies on the relationship between international trade and international institutions, primarily referring to economic integration and trade agreements (Baier and Bergstrand, 2002, 2004; Egger, Larch, Staub, and Winkelmann, 2011; Guiso, Herrera, and Morelli, 2016; Krishna, 2003; Keller and Shiue, 2014; Shiue, 2005). Notice that all these modern institutions, either domestic or foreign, build on nation states as their fundamental units. It was the emergence of nation states that ended the political dominance of feudalism and the church and initiated the modern era of state sovereignty, capacities, duty collections, and international relations. In this regard, our study serves as a theory of the nation-state system in light of international trade.

Our model also speaks to the studies on gravity models in the international trade literature. The new generation of gravity models emphasizes the importance of including

[3] Lan and Li (2015) analyze different levels of nationalism across regions within a state. They find that regions that receive globalization shocks endorse the existing state configuration less, because they share fewer (respectively, more) common interests with their domestic peer regions (respectively, the rest of the world).

remoteness-related terms into the gravity equation to formulate "structural gravity" (Anderson and van Wincoop, 2003; Head and Mayer, 2014; Allen, Arkolakis, and Takahashi, 2018). The remoteness-related terms (known as "multilateral resistance") capture worldwide general equilibrium effects that impact every bilateral trade relationship. We show that with linear world geography assumed, a structural gravity equation can be written without the remoteness-related terms while still incorporating the general equilibrium effects.

The rest of the paper is organized as follows. In Section 2, we present our theoretical model. In Section 3, we discuss the political and economic implications of our model. In Section 4, we present three groups of facts from digitized map data that are consistent with our model. In Section 5, we conclude.

2 Theory

Consider a *world* represented by a continuum of *locales*, indexed by $t \in [-1, 1]$. This world can be partitioned into different *states* (interchangeably, nation-states or countries) using *borders*. A partition of the world is characterized by a collection of borders:

$$\{b_n\} \equiv \{b_{-N}, ..., b_{-1}, b_{-0}, b_0, b_1, ..., b_N\}, \tag{1}$$

where $-1 \leq b_{-N} < b_N \leq 1$ and the total number of states is $2N + 1$.[4] Here, $N \geq 0$. When $N = 0$, the world is borderless and all its locales belong to one single global state.

For convenience, we index the states in equation (1) as

$$\{\text{state } -N, ..., \text{ state } -1, \text{ state } 0, \text{ state } 1, ..., \text{ state } N\}. \tag{2}$$

Here, state 0 refers to the state constituted by locales $[b_{-0}, b_0]$. On the right (left) side of state 0, state n (respectively, state $-n$), $1 \leq n \leq N$, refers to the state constituted by locales $(b_{n-1}, b_n]$ (respectively, $[b_{-n}, b_{-n+1})$). For all states (except state 0), we let the proximal-side (distal-side) borders be open (closed) interval endpoints.

Our model is built to illustrate how borders endogenously behave according to both economic and political machineries. The terms *world*, *states*, and *borders*, when interpreted literally, allude to the nation-state system. However, these terms do not have to be interpreted literally. They can instead represent other political structures. For example, if the "world" is a metropolis that consists of multiple districts, then a "state" refers to districts within the metropolis. So long as cross-district business costs are higher than within-district

[4]The state allegiance of locales $[-1, b_{-N}) \cup (b_N, 1]$ will be discussed at the end of Subsection 2.3.

business costs, the economic and political machineries discussed below will apply.

2.1 Environment

Economic setup All locales in the world $\{t : t \in [-1, 1]\}$ have the same quantities of land z and initial labor l^0, both inelastically supplied to produce locale-specific differentiated goods. Locales use equally efficient technologies, represented by

$$y(t) = z(t)^{\alpha} l(t)^{1-\alpha}, \tag{3}$$

where $0 < \alpha \leq 1$, $z(t)$ represents the land at locale t, and $l(t)$ the labor at locale t. The land $z(t)$ is immobile, meaning fixed at its locale t, owned by the lord of the locale. Labor can freely move across locales within a state (detailed later), owned by labor itself. In other words, by construction, $z(t)$ always equals z at any locale t, though $l(t)$ does not necessarily equal l^0. Firms within every locale compete perfectly.

Both lords and labor are consumers. Every consumer at locale t consumes goods made locally and elsewhere according to

$$C(t) \equiv \exp\{\int_{-1}^{1} \ln c(t, s) ds\}, \tag{4}$$

where $c(t, s)$ is the quantity of the good made by locale s and consumed at locale t. We let trade costs be incurred and paid by consumers. Trade is costless if the producer of the good is domestic, but has an iceberg cost if the producer is foreign. That is, only one unit of the good reaches the "consumer locale" t if $d(t, s) \geq 1$ units are shipped by the "producer locale" s, where

$$d(t, s) = \begin{cases} 1, & \text{if } s \in n_t, \\ \inf_{t \in n_t} \exp\{\tau \left| s - t \right|\}, & \text{if } s \notin n_t. \end{cases} \tag{5}$$

n_t is the state where locale t is located. The limit inferior $\inf_{t \in n_t}$ reminds us of the fact that domestic trade is costless such that the trade costs apply starting from the national border (namely, the farthest domestic locale from locale t in its state) and beyond. The parameter $\tau > 0$ sets the (foreign) trade cost per unit of distance.[5] The zero domestic trade cost is not essential in our context. A positive domestic trade cost does not alter the mechanism of our model as long as it is smaller than the foreign trade cost per unit of distance (discussed in Subsection 2.5).

[5]The exponential function in equation (5) results from aggregating incremental iceberg costs as the distance between the increments tends to zero (see Allen and Arkolakis (2014)).

Suppose that the factory-gate price of the good made at locale s is $p(s)$. Now $y(t,s)$ units of the good are shipped to locale t, then $c(t,s) = y(t,s)/d(t,s)$ units are delivered at locale t, where consumers pay the price $p(t,s) = d(t,s)p(s)$ per unit. In other words, given the factory-gate price $p(s)$, firms at locale s feel indifferent across sales destinations. In this example, the good made by locale s has the market clearing condition

$$\int_{-1}^{1} y(t,s)dt = y(s), \tag{6}$$

where $y(s)$ is locale s's total output. Apparently, the market clearing condition (6) is invariant across different origin s's, which is ensured by the Cobb-Douglas consumption structure (4) and the fact that consumers pay trade costs. This fact is important for understanding the supply side of our model.

Political setup The lord (land owner) and labor at locale t have different political roles. Each lord works with her neighboring lords to decide their state size. As a result, their lands become the territories of the state, and the labor initially on their lands becomes the labor force of the state. Labor in the state does not decide their state size but can migrate freely across locales within the state.

How do lords calculate their optimal state sizes? Recall the previous equations (4) and (5), then it becomes clear that a larger state size boosts consumption since it saves trade costs and thus reduces the prices paid by consumers, including lords and labor. As the lords have to coordinate with each other to configure the states, a larger state is less governable as a larger size involves more internal conflicts of interests. To formulate this tradeoff, we let the utility function of the lord at locale t take the form

$$U(t) = \frac{1}{1-\gamma} C^z(t)^{1-\gamma} - hS(t), \tag{7}$$

where $\gamma > 1$ and $h > 0$ represents a constant marginal disutility h from its state's size $S(t)$. Here $C^z(t)$ is just the $C(t)$ (as in equation (4)) of the lord. The term $-hS(t)$ in the lord's utility, following Alesina, Spolaore, and Wacziarg (2000, 2005), keeps state sizes limited.[6]

In comparison, the labor at locale t, who does not decide their state size, has the utility

[6] There are several interpretations of the disutility term $-hS(t)$. For example, one can interpret it as a "cost of heterogeneity" as in Alesina et al. (2000), which arises because a larger state means that more heterogeneous people (in terms of ethnicities, races, origins, etc.) have to conform to uniform state institutions. An alternative interpretation is to think of h as the cost of expanding borders for the locale per unit of distance. The cost is paid by local property tax and thus is written into the utility function of the lords.

function

$$V(t) = \frac{\psi}{1-\gamma} C^l(t)^{1-\gamma}, \tag{8}$$

where $C^l(t)$ is the $C(t)$ of the labor and $\psi > 0$ is a free scalar that allows a potential difference in marginal utility of consumption between the two types of consumers.

We are now ready to define the equilibrium of the model.

2.2 The Definition of Equilibrium

The timing of events is as follows. On date 1, lords in the world selectively join their neighboring lords to form states. As noted earlier, their lands become the territories of their states and the labor initially on those lands becomes the labor force of their states. On date 2, labor freely moves across locales within a state to join immobile local lands to produce local goods. On the same date, all local goods are traded and consumed.

The lord at locale t decides its state size $S(t)$ by choosing its two borders $b^L(t)$ and $b^R(t)$ (L is short for left and R for right). Take a locale in the right half of the world, for example: by definition, $b^L(t) \le t \le b^R(t)$ and $S(t) = b^R(t) - b^L(t)$. The lord at locale t solves the following problem:

$$\max_{b^L(t), b^R(t)} U(t), \tag{9}$$

where $U(t)$ refers to the utility function (7).

Importantly, every lord has its own optimal state size, but forming a state is a collective decision that involves multiple neighboring lords. In other words, a lord cannot turn its optimal state size and borders into reality unless her neighboring lords choose the same state size and borders. If excluding any locale from the state can improve the welfare of the remaining locales, then the state is unsustainable and thus not part of any equilibrium. If any locale in a state can improve its own welfare by leaving a state, the state is unsustainable and thus off the equilibrium path as well. Furthermore, even if all locales in a state agree on the state's size and borders, the state may not be part of any equilibrium because other locales may also want to join the state. If letting those locales in improves their welfare but does not harm any existing locale in the state, then they should be included. In short, Pareto efficiency is a necessary condition for a partition of the world to be an equilibrium partition. The Pareto-efficiency requirement has clear domestic and international political interpretations. If a border change can improve any locale's welfare without harming others, that change has no reason not to have occurred.

Following the above requirement, we define below the equilibrium partition of the world that characterizes every locale's state allegiance. Denote an equilibrium partition of the world, following the notation $\{b_n\}$ in equation (1), as

$$\{b_n^*\} \equiv \{b_{-N}^*, ..., b_{-1}^*, b_{-0}^*, b_0^*, b_1^*, ..., b_N^*\}. \tag{10}$$

$\{b_n^*\}$ is an equilibrium partition of the world if it satisfies the following criteria (i) to (ii):

$$\text{(i) } b^L(t) = \sup_{i \in \{b_n^*\}} \{b_i^* | b_i^* < t\} \text{ and } b^R(t) = \inf_{i \in \{b_n^*\}} \{b_i^* | b_i^* > t\} \text{ for any } t \in [-1, 1], \tag{11}$$

and

$$\begin{aligned} &\text{(ii) For any } \widehat{b}^L(t) \neq b^L(t) \text{ and } \widehat{b}^R(t) \neq b^R(t), \text{ if } U(t|\widehat{b}^L(t), \widehat{b}^R(t)) > U(t|b^L(t), b^R(t)), \\ &\quad \text{there must be at least one } t' \neq t \text{ such that } U(t'|\widehat{b}^L(t), \widehat{b}^R(t)) < U(t'|b^L(t), b^R(t)). \end{aligned} \tag{12}$$

Notice that criterion (ii) requires that no other locale t' is worse off if locale t chooses different borders $b^L(t)$ and $b^R(t)$ to improve its own welfare, namely the Pareto-efficiency noted earlier.

With the equilibrium partition $\{b_n^*\}$ defined above, we can now give a full definition of an equilibrium of the model. An equilibrium of the model takes the form of

$$\Omega \equiv \left(\forall t \in [-1, 1] : b^L(t), b^R(t), C^z(t), C^l(t), l(t), y(t)\right), \tag{13}$$

where[7]

$$\{b^L(t), b^R(t), \forall t\} = \{b_n^*\}. \tag{14}$$

Specifically, after borders are settled on date 1, the production, consumption, and trade follow on date 2.

Notice that locales within the same state share left and right borders. For example, for all locales in $\{t : t \in (b_0^*, b_1^*]\}$,

$$b^L(t) = b_0^*, \;\; b^R(t) = b_1^*. \tag{15}$$

That is, they all belong to *equilibrium state 1*. Rememer that the locale b_0^* itself belongs to state 0 rather than state 1, following the open-closed convention we defined at the beginning of Section 2. Its trade cost situation is the same as its peers in state 0, which is better (i.e. paying less trade costs in consumption) than that in state 1. But starting from b_0^* rightward, all locales till b_1^* (included) belong to state 1.

[7]When a state n is located in the right (respectively, left) half of the world, $b^L(\cdot)$ is the proximal (respectively, distal) border for locales in the state.

Denoting the set of equilibrium states by $\{n^*\}$, we can alternatively write the equilibrium Ω above as

$$\Omega = \left(\forall n \in \{n^*\} : C^z(\forall t \in n), C^l(\forall t \in n), l(\forall t \in n), y(\forall t \in n)\right). \tag{16}$$

Next, we will solve the model.

2.3 Solving the Equilibrium

We solve the model by backward induction. That is, we start with date 2 to solve the economic aspect of the model, conditional on the partition of the world decided already on date 1. Then we revert to date 1 to solve the political aspect of the model (i.e. the equilibrium partition of the world).

Date 2 ("economic date") On this date, production is conducted at every locale, and all lords and labor in the world as consumers purchase goods worldwide. To make consumption decisions, they maximize their utility (equations (7)-(8)) subject to their respective budget constraints. At locale t, the expenditure on the good made by locale s equals (see Appendix A.1.1 for derivation):

$$\kappa(t) \equiv p(t,s)c(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} + \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)}, \tag{17}$$

where $\lambda^z(t)$ and $\lambda^l(t)$ are the shadow prices (Lagrange multipliers) of the lord and labor, respectively. By taking the integral of equation (17) across destination locale t's, we obtain the nominal GDP of the good's origin locale s:

$$p(s)y(s) = \int_{-1}^{1} p(t,s)c(t,s)dt = \int_{-1}^{1} \kappa(t)dt, \tag{18}$$

where $p(s)$ and $y(s)$ are factor-gate price and total output, respectively, of the good (made by locale) s. Notice that the nominal GDP does not vary by s. This is because trade costs are all paid by consumers and thus the Cobb-Douglas consumption structure ensures that all locales face the same "global demand side." For convenience, we rewrite equation (18) in the form of a *locale-invariant* nominal GDP:

$$\kappa \equiv \int_{-1}^{1} \kappa(t)dt = p(s)y(s) \text{ for any locale } s \text{ in the world.} \tag{19}$$

Then the rental rate $r(s)$ for land and wage rate $w(s)$ for labor at any locale s follow. By equations (3) and (19):

$$r(s)z = \alpha p(s)y(s) = \alpha\kappa, \tag{20}$$

and

$$w(s)l(s) = (1-\alpha)p(s)y(s) = (1-\alpha)\kappa. \tag{21}$$

where $l(s)$ is the *ex post* (i.e. after domestic migration) labor supply at locale s. Again, this κ applies to any locale in the world, regardless of which state it belongs to.

The domestic migration is worth elaborating on at this point. Within any state (formed on date 1), there is a statewide labor market on date 2. In this labor market, the total labor supply equals the aggregate of the initial labor across locales of the state. The total labor demand equals the aggregate of the locale-specific labor demand $l(s)$ in equation (21) across locales of the state. Since land is immobile within a state, the resulting wage rate is equalized across locales within the state. That is, for any given state n, its initial labor will be distributed uniformly across locales in equilibrium. It follows that $l(s) = l(s')$ for any $s, s' \in n$, and that

$$\int_{s\in n} l(s)ds = \int_{s\in n} l^0(s)ds, \tag{22}$$

where the right hand side represents the total labor supply (aggregated initial labor endowment) in the state. Intuitively, any locale with a labor supply larger than its initial labor amount would have a lower wage rate, causing the "extra" labor to leave for other locales. Then we have

$$y(s) = z^\alpha {l^0}^{1-\alpha}, \text{ for any locale } s \text{ in the world}, \tag{23}$$

and

$$p(s)y(s) = r(s)z + w(s)l^0, \text{ for any locale } s \text{ in the world}. \tag{24}$$

Equations (20)-(24) are a full characterization of the equilibrium on date 2, conditional on the partition of the world determined on date 1. Since all locales in the world have the same amount of initial labor l^0, these equations give us the same $l(s)$ and $y(s)$ across the world in equilibrium, regardless of which state locale s belongs to.

The technicalities above might be obvious, but they deliver a sharp result — the nominal GDP, captured by κ in equation (19), is invariant across locales not only domestically but globally as well. This sharp result stems technically from the Cobb-Douglas production and consumption structures and we will discuss its generalization and limitation in Subsection 2.5. Nevertheless, it offers a vital step for us to set forth the key mechanism of our model on date 1 as explained next — the nominal side of the world economy is independent from the partition of the world determined on date 1. That is, regardless of how the world line is partitioned on date 1, state 2 will have the same nominal outcomes, including $l(t)$ and $y(t)$ in equilibrium description Ω (recall equation (13) or (16)). Everywhere in the world, the lord receives the share $r(s)z/(p(s)y(s)) = \alpha$, while the labor receives the share $w(s)l(s)/(p(s)y(s)) = 1 - \alpha$. This leaves the partitioning of the world to real-term considerations.

Remoteness at locale and state levels The real-term considerations are easy to characterize with the help of a new notation. Since there are three prices in the two-equation system (23) and (24), we can drop one of them by normalization. We normalize $p(t) = r(t)z/2$, such that the consumption of the lord at locale t, who makes the political decision for the locale, has a sufficient statistic (see Appendix A.1.2 for derivation):[8]

$$C^z(t) = 1/R(t). \tag{25}$$

where

$$R(t) \equiv \exp[\int_{-1}^{1} \ln d(t,s)ds]. \tag{26}$$

$R(t)$ as an aggregate of locale t's bilateral distance from the rest of the world, is a measure of locale t's "remoteness" from the rest of the world.

$R(t)$ can also be interpreted as the price index faced by locale t's lord.[9] Since all locales in the world have the same nominal income, the nominal income can be rescaled as the one in equation (25). Then $1/R(t)$ is equivalent to the lord's real income.

A useful property of $R(t)$ is that it is increasing in its state-level minimal distance from the midpoint of the world line (the world geometric center, or GC). Recall equation (5) which implies that all locales in the same state have the same bilateral trade cost with any locale outside the state. As a result, locale t's $R(t)$ applies to all locales in the same state, which is hereafter referred to as a state-level R_{n_t}:

$$R(t) = \exp\{\int_{-1}^{b_{n_t-1}} \tau(b_{n_t-1} - s)ds + \int_{b_{n_t}}^{1} \tau(s - b_{n_t})ds\} \tag{27}$$

$$= \exp\{\frac{\tau}{2}[(1 + b_{n_{t-1}-1})^2 + (1 - b_{n_t})^2]\} \equiv R_{n_t}. \tag{28}$$

[8] The profit maximization based on production function (3) implies $\frac{p(s)}{r(s)} = \frac{1}{\alpha(l(s)/z(s))^{1-\alpha}}$. Thus, in equilibrium, the $p(s)/r(s)$ ratio is equalized across locales within a state (otherwise labor would move to other domestic locales for a higher wage).

[9] The price index for labor (the group of consumers other than lords) has a similar expression.

In equation (28), the first (second) term corresponds to the remoteness to the rest of the world on its left (right).

To this end, we can ignore the locale index t in the subscript n_t of R_{n_t}. We consider, without loss of generality, the right half the world. Now state n refers to the n-th nearest state to the world GC in the right half of the world line. Its remoteness is R_n. Denote the left (respectively, right) border of state n by b_{n-1} (respectively, b_n) and the state size by $S_n \equiv b_n - b_{n-1}$. Then the following partial derivatives follow from equation (28):

$$\frac{\partial R_n}{\partial S_n} = -\tau(1 - b_{n-1} - S_n)R_n < 0, \tag{29}$$

$$\frac{\partial R_n}{\partial b_{n-1}} = \tau(2b_{n-1} + S_n)R_n > 0, \tag{30}$$

$$\frac{\partial R_n}{\partial \tau} = \frac{1}{2}[(1 + b_{n-1})^2 + (1 - b_{n-1} - S_n)^2]R_n > 0. \tag{31}$$

They imply that

1. R_n increases if state n increases in size by extending its two borders farther apart, as indicated by equation (29). Specifically, the expansion may take the form of (a) fixing the left border and pushing the right border away from the world GC (as directly indicated by equation (29)), (b) fixing the right border and pushing the left border towards the world GC (rewrite (29) as $\frac{\partial R_n}{\partial S_n} = -\tau(1-b_n)R_n$ by inserting $S_n \equiv b_n - b_{n-1}$), or (c) mixing (a) and (b).
2. R_n decreases if state n moves leftward with its size unchanged, as indicated by equation (30).
3. R_n decreases if no border changes but the foreign trade cost per unit of distance τ decreases, as indicated by equation (31).

These results serve as a preparation for our following analysis of date 1.

Date 1 ("political date") With the remoteness $R(t)$ defined, we can now revert to date 1 to solve the equilibrium partition of the world (i.e. $\{b_n^*\}$). On date 1, lords in the world choose their neighbors to form states, who all have perfect foresight about what will happen on date 2 (as previously solved). Since labor does not participate in the decisions, we use the two terms *choices made by the lord(s)* and *choices made by the locale(s)* interchangeably.

The main political consideration by the lord of locale t stems from the disutility term $-hS(t)$ in her utility function (7). A marginally larger state gives her disutility h, which will be compared by her against the gains from foreign trade cost saving $d\frac{C^z(t)^{1-\gamma}}{1-\gamma}$. Given $C^z(t) = 1/R(t)$, the incentive to expand its state stems from saving (foreign) trade costs and thus reducing $R(t)$. To reduce $R(t)$, a locale may alternatively keep its size unchanged but choose to move towards the world GC by joining its neighbors on the proximal (close to the world GC) side and leaving some of its neighbors on the distal (away from the world GC). Of course, a combination of the two changes works, as well.

The challenge here emerges that neighbor choices have to be mutual. That is, locale t cannot form a state with locale t' unless both choose each other as peers to form a state. Moreover, every single locale is atomless in the continuum such that a state as an interval has to be endorsed by every locale *in the world* in order to be a state in an equilibrium partition of the world, as defined earlier.

Below, we show the existence of an equilibrium of the model. Consider the locale $t = 0$, which is precisely at the world GC. It has the lowest possible remoteness, which can be verified by examining equation (28). Therefore, if locale $t = 0$ sets its borders to include any other locale in the world to be its peer locale in the same state, that locale will agree to whatever borders chosen by locale $t = 0$ because that locale unambiguously benefits from being in the same state with locale $t = 0$ and thereby enjoys the lowest possible $R(\cdot)$ in the world. In other words, it is the dominant strategy for any locale in the world to accept whatever borders chosen by locale $t = 0$. This privilege of locale $t = 0$ results from its greatest locational advantage in the world. The only restriction on its border choices is that it cannot skip over any locale but has to choose contiguous neighbors (that is, either immediate neighbors or immediate neighbors of chosen neighbors).

Formally, to choose borders, the lord of locale $t = 0$ conducts the optimization problem (9) and reaches the first-order condition through equations (7), (25), and (29):

$$\tau R_0^{\gamma-1}(1 - b_0^* - b_{-0}^*) = h, \tag{32}$$

where the state index n is now set to $n = 0$, referring to the fact that state 0 is the state at the middle of the world line with borders b_0^* and b_{-0}^*. The two borders are symmetric. $R_0 = R_{n=0}$ represents locale $t = 0$'s remoteness, which is now also the remoteness of the entire state including all locales within $[b_{-0}^*, b_0^*]$.

Now consider a locale t' on the right side of state 0 which is quite close to the right border of state 0. That is, $t' \to b_0^{*+}$. This locale is clearly excluded by state 0 though it wants to join state 0. Including it into state 0 would violate the first-order condition (32) and thus harm locale 0 and all its peers currently in state 0, because state 0 would be too large with locale t' included. Therefore, adding this locale t' to state 0 must not be part of any equilibrium.

It is noteworthy that some locales currently in state 0, especially those close to the right border of state 0, such as $t'' \to b_0^{*-}$, could join $t' \to b_0^{*+}$ to form a state if *they were not part of state 0*. The state formed by such t' and t'' would not be too large for them if they are sufficiently close to each other. But given that t'' is definitely part of state 0, such possibilities are off the equilibrium path.[10]

Up to this point, one can safely say $b_{-0}^* = b^L(t)$ and $b_0^* = b^R(t)$ for any locale t in state 0, referring to the equilibrium description Ω in equation (13).

What will that locale $t' \to b_0^{*+}$ do? It is close to but excluded from state 0. It will form a state on the right of state 0. This state starts from b_0^* and extends rightward (i.e. away from the world GC). Call it state 1. The locales in state 1 all have the following first-order condition

$$\tau R_1^{\gamma-1}(1 - b_0^* - S_1) = h, \tag{33}$$

where b_0^* is fixed by state 0. Notice that the last term in first-order condition (33) is S_1, rather than b_{-0}^* as in the previous first-order condition (32) for state 0. This difference in first-order condition between state 0 and state 1 provides an insight on state 0's size relative to state 1, which will be provided in Section 4 (see "Fact 3" there).

Also notice that the locale b_0^* itself is in state 0 rather than state 1. The decision here is the choice of b_1^* made by locales possibly in state 1 (then $S_1 = b_1^* - b_0^*$ follows). This time, all locales on the right of b_0^*, namely $(b_0^*, 1]$, want to join state 1 because that would reduce their remoteness infinitely close to R_1. R_1, strictly speaking, is the remoteness of the locale $t = b_0^*$ *if it were in state 1*. But locale $t = b_0^*$ is in state 0 such that R_1 is the lower bound of the remoteness that applies to all locales in state 1.[11]

As before, any locale not excluded by the first-order condition (33) will join state 1, and no other locale beyond b_1^* will be allowed in. This leads to $b_0^* = b^L(t)$ and $b_1^* = b^R(t)$ for any t in state 1. The same reasoning continues. Generally, for state $n \geq 1$, the first-order condition is

$$\tau R_n^{\gamma-1}(1 - b_{n-1}^* - S_n) = h, \tag{34}$$

where b_{n-1}^* is fixed by state $n-1$ and the locale precisely at that location is in state $n-1$. The decision here is the choice of b_n^* and then $S_n = b_n^* - b_{n-1}^*$ follows. This also applies to the left half of the world line. At the end, all borders in the world, namely $\{b^L(t), b^R(t), \forall t\} = \{b_n^*\}$, settle in equilibrium to complete Ω. The number of states equals $2N+1$ in equilibrium, with

[10] In this example, the three geographic locations, ordered from proximal to distal, are $(0 <)t'' < (t = b_0^*) < t'(< 1)$.

[11] Locales such as $t' \to b_0^{*+}$ can lower their remoteness infinitely close to R_1 but cannot attain precisely that level of remoteness.

$2N$ satisfying

$$2N = \{2n : \frac{S_0}{2} + \sum_{i=1}^{n} S_i \leq 1 \text{ and } \frac{S_0}{2} + \sum_{i=1}^{n+1} S_i > 1\}. \tag{35}$$

Notice that this "nation-state system" leaves very distal locales out. That is, locales in $[-1, b_{-N}) \cup (b_N, 1]$ are not accepted into their proximal side states. They have incentives to form their own states to reduce foreign trade costs, and the sizes of their states are smaller than optimal. We can count them either as states (then there will be $2N+3$ states in equilibrium) or leave them as semi-state territories.[12] These distal locales, although peripheral in analysis, serve nontrivial technical purposes as discussed later in Subsection 2.5.

The above characterization of the equilibrium may sound sequential — as it starts from state 0 to state 1, 2, ..., N — but it is not. Our narrative begins from locale $t = 0$ because that is the locale with the greatest locational advantage. All information in the model is public and all locales take actions simultaneously. There are neither informational updates nor sequential moves. It is the dominant strategy of all locales to join their proximal side neighbors. Therefore, the equilibrium is not sequential per se but is just presented in a narrative starting from the center of the world. In the next subsection, we will show that the equilibrium found here is the unique equilibrium of the model.

2.4 Uniqueness of the equilibrium

The equilibrium found above follows a simple reasoning: no locales choose to be in a different state than their proximal side neighbors unless they have no other options. Below, we use two-step *mathematical induction* to show that the equilibrium partition of the world as stated in equation (10) is the only possible equilibrium.

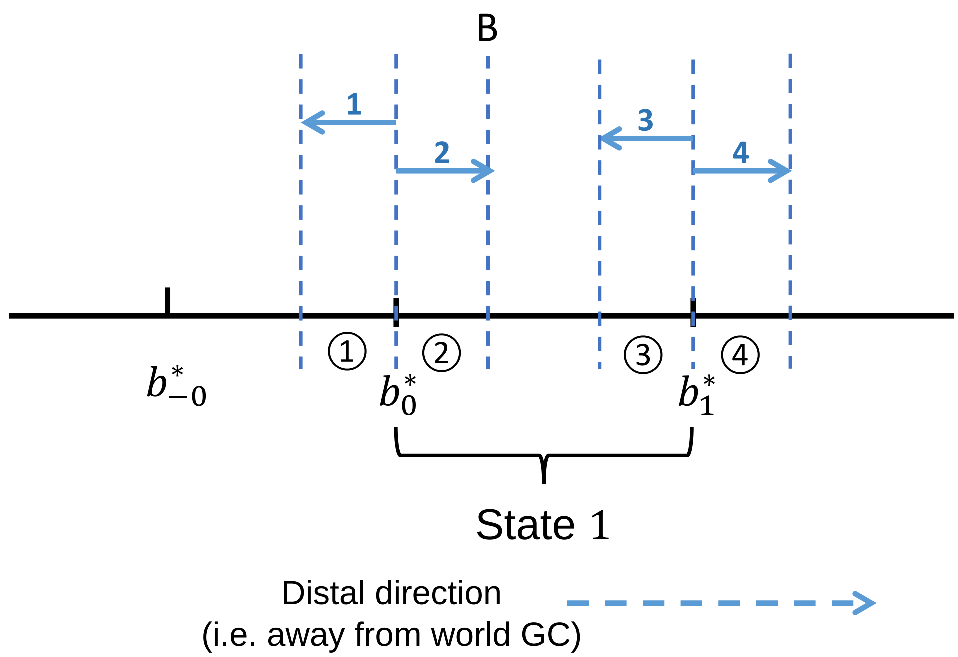
Consider an arbitrary state $n \geq 1$ in the previous equilibrium partition of the world. It is assumed, without loss of generality, to be on the right side of the world GC. Its left border is b^*_{n-1} and its right border is b^*_n.

First, suppose this state n is state 1 (i.e. $n = 1$). That is, its two borders are $b^*_{n-1} = b^*_0$ (open interval endpoint) and $b^*_n = b^*_1$ (closed interval endpoint). Now, a change to state 1 must take one out of the following four forms (labeled as arrows 1 to 4) in Panel (a) of Figure 1, or a combination of any two of them (e.g. an expansion of state 1 in both directions means cases 1+4). We now show that any single one of the four cases violates the equilibrium definition in Subsection 2.2 (therefore, a combination of any two in them violates

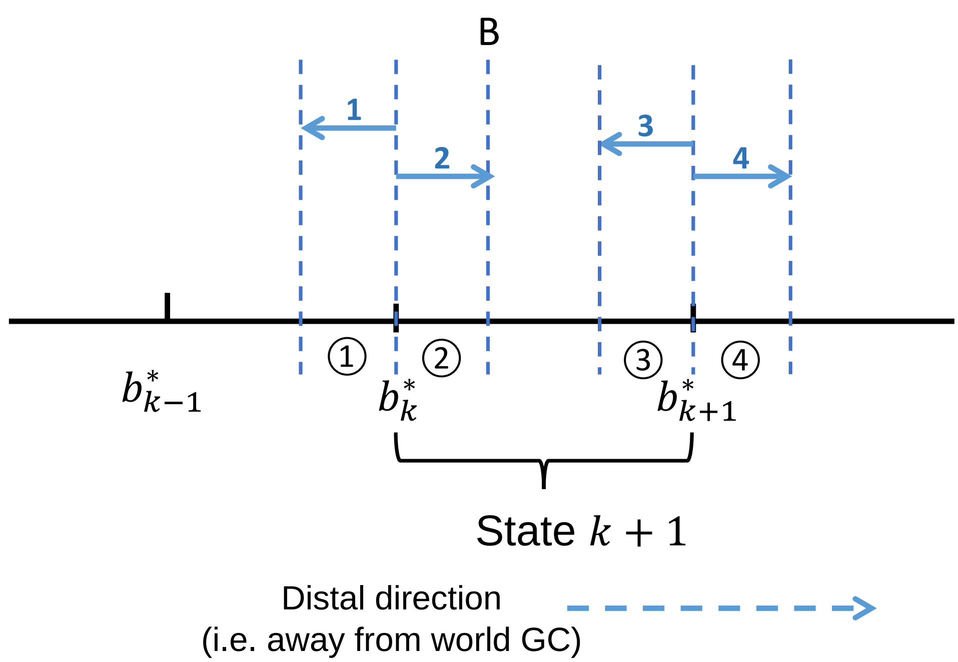
[12] In reality, non-state distant locales are usually taken by other full-fledged states as dependent territories for ad hoc reasons (such as serving as military bases).

Figure 1: Uniqueness of Equilibrium

Panel (a): the case state n=1



Panel (b): the case state n=k+1



that definition, as well). In brief, an equilibrium partition of the world requires that any change to any border would make some locale (lord) in the world worse off. Also, remember that on the *right* side of the world GC, any state's remoteness is calculated based on its *left* border (the proximal side border).

In case 1, the locales in interval ① that belong to state 0 will now be in state 1. This clearly causes those locales to be worse off because state 0 has the lowest possible remoteness in the world. So, case 1 will not be part of any equilibrium. Here, a special scenario that can keep the locales in interval ① from having a higher remoteness is to let the border extend further leftward to b^*_{-0}. In that case, the remoteness for the locales in interval ① remains as low as that for state 0. However, in that scenario, states 0 and 1 will merge into a new state 0, which is too large for any locale in the old state 0 because the size of the old state 0 is unambiguously its optimal size (determined by first-order condition (32)).

In case 2, all locales in state 1 except those in interval ② will be worse off, because the remoteness for them will then be calculated based on border "B" in the figure (notice that this border "B" will then be the point nearest to the world GC — and thus the point nearest to the rest of the world — within the new state 1). In case 3, locales in interval ③ will be worse off because they are excluded by state 1 and included instead in state 2. In case 4, the size of state 1 will be too large (violating the first-order condition (33)).

Similarly, no combinations of the above cases can make equilibrium. Cases 1+3 will make locales in interval ① worse off. Cases 1+4 will make state 1 too large. Cases 2+3 will make state 1 too small. Cases 2+3 will also raise the remoteness for state 1 since the remoteness of the state will then be calculated based on border "B." Cases 2+4 have the same problem.

As the next part of our mathematical induction, we need to show that, if some state k is part of the equilibrium, state $n = k + 1$ must also be part of the equilibrium. To show that, we just need to show that state $k + 1$'s two borders cannot be altered in any of the four fashions or any combination of them. As Panel (b) of Figure 1 illustrates, the analysis of the state $n = k + 1$ will repeat the reasoning above. The only difference is that the left side neighboring state is not state 0 anymore but is now state k which would stick to its borders (size) following the first-order condition (34).

Given the uniqueness of the equilibrium partition of the world, the uniqueness of the equilibrium in the model follows automatically because other variables in Ω either hinge on the partition (like $C^z(t)$ and $C^z(t)$) or are unrelated to the partitioning (like $l(t)$ and $y(t)$, uniquely solved from the system of equations (20)-(24)). This finishes the proof of the uniqueness.

2.5 Remarks

We would like to make a few remarks on the model before closing this section.

Number of states The equilibrium number of states in the world, namely $2N+1$ (or $2N+3$ with the two peripheral states counted in), leaves enough room for equilibrium stability and comparative statics. Every single border change in the model has general equilibrium effects on the whole rest of the world. Such effects could change the number of states in the world, but if they are not large enough, they will be absorbed into the very distal locales. Those locales are not full-fledged states and thus their flexible sizes absorb small changes to borders such that the total number of states does not change. This is why those very distal locales are technically useful. All analyses in this study assume an unchanged number of states in equilibrium. This creates no problem here because this study focuses on the relationship between trade and borders within a given time period.

Specific geography The necessity of using a specific geography in this study stems from the need to model the behavior of borders. Borders would be undefined without a specific geography.[13] In our case, the linear geography makes the equilibrium partition determinable by imposing the constraint that locales can only form states with their neighboring locales. This greatly simplifies the analysis as now every state must be an interval and the total mass of the states (intervals) automatically adds up to a constant (namely 2).

A circular geography may appear to be an alternative geography for our modeling purpose. But a ring (circle without interior) is by design symmetric, which renders the analysis of differential remoteness across locales infeasible. Using a disk (i.e. circle with interior) instead of a ring restores the differential remoteness — its center is its geometric center, just like the midpoint of a line. However, it remains unclear how to define borders within a disk.[14]

Cobb-Douglas preferences & technologies The two Cobb-Douglas structures give us the elegant sufficient statistic $1/R(t)$ in equation (25) that greatly simplifies the analysis of

[13] International trade theories, from traditional ones to the new trade theories, do not require specific geographies, because grouping different economies into conceptual countries suffices to let different parts of the world interact economically. In theory, borders could be shapeless. But without a specific geography, the set of states in the world has numerous possible cases. The locales $\{t\}$ in the world, if not anchored to a specific geography, can be partitioned arbitrarily into any $\{S_n\}$, where both composition and number of elements are endogenous. In that case, any locale's state allegiance depends on every other locale's state allegiance, rendering the equilibrium $\{S_n\}$ indeterminable.

[14] In a unit disk, any two straight lines have numerous possible combinations, thereby dividing the disk in numerous possible ways.

the equilibrium partitioning of the world. In general, the Cobb-Douglas preferences leave the partitioning to real-term considerations, as trade costs are paid by consumers such that Cobb-Douglas preferences keep all producers in the world from being affected by the partitioning of the world. The Cobb-Douglas technologies ensure the symmetry of labor forces across locales within each state, which is important for holding nominal income invariant across locales and thereby maintaining prices well-behaved within $R(t)$.

Relaxing the two Cobb-Douglas structures will not alter the key mechanism of our model, because the key mechanism of our model builds solely on differential locational advantages across the world line. Using more general functional forms does not alter the fact that locales have centripetal tendencies when choosing peers to form a state. That will, however, make the analysis less tractable. For example, a CES consumption structure will render nominal income depending on remoteness too. To that end, the findings from our model have little reason to change because the resulting higher remoteness of locales located far from the world GC now penalizes them twice, through both nominal income and price index, and therefore strengthens our results. But the resulting analysis will no longer be as tractable as partial derivatives (29)-(31) and first-order conditions (32)-(34).

Zero domestic trade cost Assuming zero domestic trade cost ensures centripetal tendencies as a dominant strategy for all locales in the world. The key mechanism of our model requires only trade cost per unit of distance to be lower in domestic trade than in foreign trade. Using positive domestic trade costs instead does not affect this key mechanism but creates additional forces and complicates the analysis. Specifically, within a state, locales close to its borders, such as the previously mentioned locale $t'' \to b_0^{*-}$ in state 0, may have incentives to bring foreign locales on the other side of the border, such as $t' \to b_0^{*+}$ in state 1, into state 0. Consequently, cooperative game theory is needed to solve the coalition formation. We leave that out of this study.

3 International Trade and Geopolitics

The parameter τ, which measures foreign trade cost per unit of distance in equation (5), can be considered as a measure of "economic disintegration." That is, a greater τ makes states with a given bilateral distance economically farther apart from each other and thus more self-reliant. The parameter h, which measures political frictions accruing with state size in equation (7), is conceivably a measure of "political disintegration." Namely, a greater h makes states with large sizes less governable, thereby partitioning the world line into more states.

As we show below, *each* of the two parameters affects *both* international trade and geopolitics. Specifically, economic disintegration τ affects not only international trade but also international geopolitics; likewise, political disintegration h also has remarkable implications on global trade. Although the convolutions between economics and politics are hardly surprising, the comparative statics of our previous model demonstrate how exactly economic and political disintegrations interact, in a way not yet explicated by the existing studies.

3.1 International Trade

Bilateral international trade is best characterized by the gravity equation, a fact that has long been known (Anderson, 2011; Head and Mayer, 2014). As the Newtonian analogy suggests, two states have a larger trade volume with each other if they are larger in size and/or closer to each other in distance. We derive a gravity equation from our model (see Appendix A.1.3 for detailed derivation):

$$X_{m,n} = \zeta S_m S_n \exp\{-\tau D_{m,n}\}, \tag{36}$$

where $X_{m,n}$ represents exports by state m to a nonadjacent state n, ζ is a positive scalar that applies to all pairs worldwide, S_m and S_n are the sizes of the two states as before,[15] and $D_{m,n}$ is the shortest distance between the two states. Remember that domestic trade is costless. Here we assume, without loss of generality, $n > m + 1 \geq 1$ — both states are in the right half of the world and nonadjacent, and state n is farther from the world GC than state m — such that $D_{m,n} = b_{n-1} - b_m$, which equals the total size of all the states between the two non-adjacent states.

Impact of $d\tau$ on trade Our gravity equation (36) prepares us for analyzing how economic disintegration τ and political disintegration h affect bilateral trade, respectively. Below, we use the $\hat{v} = dv/v$ to denote a percentage change in (any) variable v. First consider an exogenous reduction in τ: $d\tau < 0$. Its impact on the bilateral trade volume $X_{m,n}$ can be decomposed into three parts:

$$\underbrace{\hat{X}^{d\tau<0}_{m,n}}_{\lesseqgtr 0} = \underbrace{\hat{S}_m + \hat{S}_n}_{\text{size effect}<0} \underbrace{-D_{m,n} d\tau}_{\text{direct effect}>0} \underbrace{-\tau d D_{m,n}}_{\text{location effect}>0} . \tag{37}$$

[15] State size could be interpreted as economic size (GDP), population, or territorial area. In our context, locales are symmetric in production, income, and territory (atomless). Therefore, there is no difference between these interpretations of state size.

Among the three effects in equation (37), the *direct effect* is self-explanatory. The *size effect* refers to the fact that both states shrink in size when τ reduces. Intuitively, all states shrink in size when τ lowers because the resulting real-consumption boost can now sustain smaller states.[16] The net of these two effects, direct effect and size effect, has an ambiguous sign, depending on which of them is greater in magnitude. There is a third *location effect* that adds to the ambiguity. As reducing τ leads to smaller states worldwide, the size shrinkage of the states located between state m and state n brings the two states closer to each other.

To summarize, in the short term, state borders are fixed and thus the direct effect is the only effect. In the long term, state borders are endogenous such that the size and location effects emerge and oppose each other. As a result, the net effect of $d\tau < 0$ on trade volume is ambiguous.

Our model adds to the gravity literature in international trade in three ways. First, it illustrates two effects of $d\tau$, the size effect and the location effect, which are absent in the existing literature. The existing literature does not have these effects because they assume national borders to be fixed. This being said, equation (36) is a *long-term* gravity equation that allows borders to endogenously change, typically over a long time period. Second, the traditional gravity equation in the literature is isomorphic to equation (36), but is later found to be lacking because it does not account for differential remoteness of the two states within the world trade system. Gravity equations in the more recent literature, known as structural gravity equations following Anderson and van Wincoop (2003), include remoteness-related terms in the form of state m and state n fixed effects. Our gravity equation, due to its linear structure, has accounted for general equilibrium effects but keeps the traditional form of the gravity equation. Last, the so-called "remoteness" has a very concise geometrical form in our model: a state is remote from the rest of the world if it is far from the world GC.

The three additions to the literature listed above are interconnected. In fact, it is precisely the concise geometric form that sets our gravity equation (36) free from the additional remoteness terms and back to its traditional form. Technically, distance from the world GC covaries with S_m and S_n and thus having the size variables S_m and S_n suffices to incorporate remoteness into the gravity equation.

Notice that our gravity equation (36) still belongs to the family of gravity equations in the literature. If phrased using terms in the "universal gravity" proposed by Allen, Arkolakis, and Takahashi (2018), both "demand elasticity" (their ϕ) and supply elasticity (their ψ) are zero in our context.[17]

[16] Formally, this can be seen from equations (52) and (54) in Appendix A.1.4

[17] The demand elasticity is zero because of our Cobb-Douglas consumption structure (the elasticity of substitution, namely their σ, equals 1 such that $\sigma - 1 = 0$). The supply elasticity is zero because there is no

Impact of dh on trade We now move on to how political disintegration h impacts bilateral trade volume. Consider a marginal decrease in h (i.e. $dh < 0$) that occurs to gravity equation (36). It can be verified that

$$\underbrace{\hat{X}_{m,n}^{dh<0}}_{\lesseqgtr 0} = \underbrace{\hat{S}_m + \hat{S}_n}_{\text{size effects}>0} \underbrace{-D_{m,n} d\tau}_{=0} \underbrace{-\tau dD_{m,n}}_{\text{location effect}<0} . \tag{38}$$

Here the size effect is positive since state sizes grow when h decreases. The negative association between h and state size stems from the greater "tolerance" of each other among locales in all states — namely, less disutility from a larger state size — an effect first studied by Alesina and Spolaore (2005) and Alesina et al. (2000). The location effect is negative since m and n are farther apart owing to the expansion of the states between states m and n. Again, the net of the two effects is ambiguous. Put differently, if states become more integrated politically and thus larger in size, they still do not necessarily trade more with each other. This is reminiscent of the historical periods when large empires were pervasive but they did not necessarily trade more with each other because the trade routes between two empires were usually blocked by the other empires between them.

3.2 International Geopolitics

Unlike international trade, which is a mature field in economics, international geopolitics is a less developed social science. Geopolitical analysis, started by Huntington (1907), Mackinder (1904) and Fairgrieve (1917), does not comprise a well-defined discipline or sub-discipline, in spite of its significant influences on the work of historians (Braudel, 1949), human geographers (Diamond, 1999), and political scientists (Morgenthau, 1948; Kissinger, 1994, 2014; Brzezinski, 1997). The earliest geopolitical analysis dates back to Halford John Mackinder (1861-1947), who exaggeratively emphasized the geopolitical importance of Eastern Europe in world politics (Mackinder, 1904, 1919):

> Who rules Eastern Europe commands the Heartland;
> who rules the Heartland commands the World-Island;
> who rules the World-Island commands the world.

His heartland refers to the area ruled by the Russian Empire at that time, and his world-island refers to Eurasia and Africa. From today's perspective, his claims are oversimplistic and overreaching. However, they capture the fact that borders close to the geographical center of the world during his time were politically sensitive, a fact attested to by two

intermediate input in our model such that their ζ equals 1 here.

subsequent world wars and the Cold War. According to our estimates, the world GC during his time was in the Austro-Hungarian Empire, indicating that his thesis on the geopolitical importance of Eastern Europe is reasonably accurate (see Section 4 for details).

To this end, our model provides a formal illustration of the additional political sensitivity of the borders close to the world GC. Consider three borders in the right half of the world: b_{k-1}, b_k, and b_{k+1}. Locales $(b_{k-1}, b_k]$ are state k, and locales $(b_k, b_{k+1}]$ are state $k+1$. Holding borders b_{k-1} and b_{k+1} constant, we show below that a change in b_k affects state k more than it does state $k+1$. Recall $R_n = \exp\{\frac{1}{2}\tau[(1+b_{n-1})^2 + (1-b_{n-1}-S_n)^2]\}$. It follows that

$$-\frac{\partial R_n/\partial S_n}{R_n} = \tau(1-(b_n - b_{n-1})), \tag{39}$$

where $S_n \equiv b_n - b_{n-1}$. If $b_{k+1} - b_k > b_k - b_{k-1}$, the percentage change in the price index (thus welfare) is greater for state k than for state $k+1$. States in Eastern Europe are typically small, at least on average smaller than countries elsewhere (see Section 4 for details). This serves as a potential theoretical foundation for the previously mentioned geopolitical importance of Eastern Europe in world politics argued by geopolitical analysts.

Rationalizing the extant geopolitical premise is not our only goal here. Below, we analyze how economic disintegration τ and political disintegration h interact to generate new geopolitics.

$d\tau$ and dh in geopolitics In light of our model, economic disintegration τ impacts geopolitics by influencing the partitioning of the world, especially those close to the world GC. To see this influence, totally differentiate the first-order condition of state n (i.e. equation (34)) and consider a hypothetical interaction between τ and h (see Appendix A.1.4 for derivation):

$$\frac{dh}{d\tau} = R_n^{\gamma-1}(1-b_n)\{1 + \frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2 + (1-b_n)^2]\} > 0. \tag{40}$$

This hypothetical scenario leads to two observations. First, when the trade cost per unit of distance decreases, locales have to be more "tolerant" of each other (i.e. a smaller h, namely less disutility from living with more peers in the same state) in order to maintain the existing partition of the world. Otherwise, without such "tolerance compensation," the existing partition of the world will collapse and a finer (smaller-state) world partition will emerge. Second, the need for such a h-compensation is less for more remote states.[18] This is because the remoteness of a state is more sensitive to τ if the state is closer to the world

[18] When state n pertains to a farther state from the world GC, the effects through $(1+b_{n-1})^2$ and $(1-b_n)^2$ cancel each other, while the effect through $(1-b_n)$ at the front of equation (40) leads to this finding.

GC. Intuitively, for states far from the world GC (and thus far from the rest of the world), national borders are not as sensitive because their disadvantaged locations render a marginal change in the trade cost per unit of distance τ less influential to their welfare.

An alternative interpretation, which is more politically relevant than the one above, is that as τ lowers worldwide, h should decrease worldwide to keep borders in the world unchanged. If h happens to decrease (e.g. through bolstered nationalistic ideologies), the existing partition and states may be sustained. Otherwise, the existing partition will collapse and the most pressured national borders are those closest to the middle of the world (equivalently, nearest to the rest of the world).

Gravity and geopolitics The gravity equation in the trade literature, when microfounded by our model, also has geopolitical implications. A rearrangement of our gravity equation (37) illustrates how a reduction in trade cost, a force believed to promote economic integration, may instead affect the world's political geography. That is, given $d\tau < 0$, for any two states m and n in the world,

$$D_{m,n} d\tau (< 0) = \underbrace{-\hat{X}_{m,n}}_{\text{economic integration}} + \underbrace{\hat{S}_m + \hat{S}_n}_{\text{political disintegration}} \underbrace{-\tau dD_{m,n}}_{\text{enlarged middle-land}} . \tag{41}$$

Here, a reduction in trade costs $d\tau < 0$ is absorbed by three mutually exclusive margins in equation (41)

Economic integration: Trade volume rises (i.e. $\hat{X}_{m,n} > 0$);

Political disintegration: State sizes shrink (i.e. $\hat{S}_m < 0$ and/or $\hat{S}_n < 0$);

Enlarged middle-land: States become farther apart from each other (i.e. $dD_{m,n} > 0$).

Notice that here we are not discussing geopolitics between states m and n but how a change in τ potentially impacts the nation-state system of the world.

Equation (41) illustrates that a reduction in trade costs, an economic phenomenon, may lead to economic consequences and political consequences that compete against each other. Apparently, when τ decreases, one tends to think trade volume should increase, as shown by the "economic integration" term in equation (41). However, if borders are endogenous, the outcome could instead be "political disintegration," as shown by the second term in equation (41). As explained earlier, we find that declining trade costs may break existing states into smaller ones. Alesina et al. (2005) mention that the ease of trade was the reason that the city states of Italy and the Low Countries in Europe remained small. Alternatively, as trade costs decline, a pair of states may become farther apart because the state(s) between them become larger. This is also a force that can absorb the decline in trade costs. Fazal (2007) finds that "buffer states" (small states located between large states) have become less likely to break up in recent decades.

4 Historical Facts in Light of Our Model

We connect our theory with historical facts in this section. The following data analysis should not be taken as an empirical test of our theory. Given the large scale (the world) and long time period (three centuries), we are aware that it is hardly possible to find convincing exogenous variations. Instead, we choose to contrast our theoretical construct, including assumptions and implications, with what has actually occurred historically. That is, we retrieve elements from our model and examine to what extent they can explain historical events. We do not aim to refute other explanations.

We find three groups of historical facts that are consistent with our previous model. Below, for each group of facts, we first derive our model's implication and then present historical data. The first group of facts (henceforth, Fact 1) are concerned with the physical and economic geography of the real world, serving to assess the extent to which the real world geography can be approximated using a world line. Next we look into the distribution of state sizes within different time periods (Fact 2). Then we examine the distribution of state sizes that bear close proximity with the world geometric center (GC) across time periods (Fact 3).

Our major data source is digitized political maps of the world for different time periods. Data details, including summary statistics, are provided in Appendix A.2. Our benchmark map is the political world map of the year 1994. We refer to 1994 as the *modern* period, because no major border change has occurred in the world since then. We use digitized historical world maps to supplement the modern one. We successfully compiled three historical world maps, with base years 1750, 1815, and 1914-1938, respectively. The rationale behind the choices of those base years are discussed in Appendix A.2. For simplicity, we refer to them as the 18th century, 19th century, and early 20th century in the rest of the paper.

Preparation: Estimate the Location of World Geometric Center

A key concept in our theory is the world geometric center (GC), which refers to the midpoint of the line when a linear world geography is assumed. To estimate where the world GC is in the real world, we start with constructing locales in the world. A locale in the world is defined as an administrative division in the world map with a population of at least 15,000.

The population threshold is set moderately low to ensure that the landmass is used for permanent residence.[19]

To estimate the location of the world GC, we first calculated $Distance(t, t')$, which is the orthodromic distance between any two locales in the world (i.e. $t, t' \in W$), and then calculated every local t's total distance from all locales in the rest of the world.[20] The locale with the smallest total distance is designated as the world GC:

$$GC \equiv \arg\min_{t} \sum_{t' \in W} Distance(t, t'). \tag{42}$$

Table 1 reports the locations of world GCs over time. Its location is generally stable, reflecting the stability of human habitats in the world during recent centuries.

Table 1: Estimated Locatitons of the World Geometric Centers (GCs)

Modern period	*The 18th century*
Hradec Kralove, Czech Republic (50.21,15.83)*	Kisvarda, Austrian Empire (48.22,22.08)*
The 19th century	*Early 20th century*
Weißwasser, Germany (51.50,14.64)*	Hradec Kralove, Austro-Hungarian Empire (50.21,15.83)*

* Geographic coordinates in the parentheses are in the form (latitude, longitude).

Fact 1: Linear Approximation

We use a line to approximate the world geography in Section 2. The world-line assumption imposes geometric centrality on the world geography in that the midpoint is the point closest to the rest of the world. To see it, remember that point a in the line $[-1, 1]$ has a distance $|a|$ with the world line's GC at $a = 0$, and a total distance $a^2 + 1$ from the rest of the world line. In other words, point a's total distance from the rest of the world line is quadratically increasing in $|a|$ and minimized at the GC.

[19]A high threshold would limit the sample to industrial clusters, while too low a threshold would cause the locales with only temporary public projects, scattering periodic employers, or seasonal school enrollments to be over-represented. The value 15,000 is the lowest population requirement used by the US Census to determine central cities of metropolitan statistical areas. Lowering that population threshold to zero is equivalent to treating every state as a polygon. We use that in our robustness checks.

[20]Orthodromic distance (great-circle distance) is the shortest distance between two points on the surface of the earth. It is measured along the surface rather than through the interior of the earth.

We subject this idea to data. Denote locale t's distance from its contemporary world GC (see Table 1) by

$$D(t) = Distance(t, \text{world GC}), \tag{43}$$

where $Distance(\cdot,\cdot)$ represents orthodromic distance as before. Denote locale t's total distance from the rest of the world by

$$TD(t) \equiv \sum_{t' \in W} Distance(t, t'). \tag{44}$$

If the real-world geography exhibits geometric centrality, we should see that $TD(t)$ is quadratically increasing in $D(t)$: $TD(t) = \delta_0 D^2(t) + \delta_1$, where δ_0 and δ_1 are positive constants.

This is indeed what we find in the data. We regress $TD(t)$ on a constant term, a first-order $D(t)$,[21] and a second-order $D(t)^2$. The coefficient of the second-order term $D(t)^2$ is hypothesized to be positive. The constant term, expected to have a positive coefficient as well, corresponds to the 1 in $a^2 + 1$. Table 2 reports the regression results. The coefficients of the constant term, $D(t)$, and $D(t)^2$ are all positive and statistically significant, and the R^2 statistics are between 0.981 and 0.997. Coast and island dummy variables as well as continent fixed effects are included. Geometric centrality is evidently observed. Also, when still higher orders of $D(t)$ are incrementally added into the regression, the fitness shows little improvement. When too many high-order terms are included, all coefficients except that of the constant term disappear as expected.

This is an interesting finding, considering that the earth is actually a three-dimensional sphere. We think that the reasonably successful approximation is due to the fact that the inhabitable landmass on the surface of the earth is distributed into continents. As a result, some locations are closer to others. If the landmass were uniformly distributed across the surface of the earth, we would not find such geometric centrality.

We next investigate whether proximity to the world GC, namely $D(t)$, has any economic relevance. We specify the following gravity regression following the literature:

$$\ln T(n, n') = \mu \ln Distance(n, n') + \bar{\vartheta} \cdot \begin{bmatrix} \ln Size(n) \\ \ln Size(n') \end{bmatrix} + \bar{\omega} \cdot \begin{bmatrix} \ln D(n) \\ \ln D(n') \end{bmatrix} + \iota' Z_{nn'} + \epsilon_{nn'}, \tag{45}$$

where $T(n, n')$ is the trade volume (imports) between states (countries) n and n', $Distance(n, n')$ is the distance between the two states, $Size(n)$ and $Size(n')$ are their sizes (either population or area), $Z_{nn'}$ are control variables, and $\epsilon_{nn'}$ is the error term. We added two novel terms

[21] The first-order term is added as the real world is unlikely to be axisymmetric.

Table 2: Geometric Centrality

Dependent variable is ln(total distance from the rest of the world (TD))				
Constant term	1.187e+08***	1.234e+08***	1.248e+08***	1.234e+08***
	(1810979.989)	(647,497.479)	(662,080.449)	(1029453.236)
Distance from the world GC	5,206.437***	2,382.739***	-992.376	2,427.142
	(818.453)	(509.458)	(1,845.611)	(2,854.955)
Distance from the world GC^2	0.412***	0.732***	2.452**	0.063
	(0.082)	(0.160)	(0.973)	(3.050)
Distance from the world GC^3		0.000	-0.000	0.000
		(0.000)	(0.000)	(0.001)
Distance from the world GC^4		-0.000***	0.000	-0.000
		(0.000)	(0.000)	(0.000)
Distance from the world GC^5			-0.000	0.000
			(0.000)	(0.000)
Distance from the world GC^6			0.000	0.000
			(0.000)	(0.000)
Distance from the world GC^7				-0.000
				(0.000)
Distance from the world GC^8				0.000
				(0.000)
Coast and island dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
R-squared	0.981	0.997	0.997	0.997

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05.

$D(n) \equiv \min_{t\in n} D(t)$ and $D(n') \equiv \min_{t\in n'} D(t)$, measuring the shortest distance between each state and the world GC. They are our variables of interest as they capture whether proximity with the world GC has any trade implication. We hypothesize that their coefficients are negative, such that states farther from the world GC have locational disadvantages in their trade with *every* trade partner.

A natural question arises as to how the regression specification (45) reconciles with the long-term gravity equation (36) that we derived earlier. In that long-term gravity equation, terms $D(n)$ and $D(n')$ are absent because their roles have been incorporated implicitly into the size variables (i.e. $\ln Size(n)$ and $\ln Size(n')$ here). We have terms $D(n)$ and $D(n')$ in the regression because we will use data from the current time period to run the gravity regression. At a given point of time, borders are predetermined and therefore have inertia (fixed costs) that resists changes. When borders are allowed to optimize simultaneously with trade, terms $D(n)$ and $D(n')$ will disappear (i.e. statistically insignificant in regression (45)).

Table 3: **Economic Relevance of Proximity to the World GC**

Panel A: Dep. variable is ln(Trade volume)				
	Size=population		*Size=area*	
ln(Size of exporter)	0.518***	0.499***	0.323***	0.313***
	(0.009)	(0.009)	(0.009)	(0.009)
ln(Size of importer)	0.444***	0.426***	0.260***	0.251***
	(0.009)	(0.009)	(0.009)	(0.008)
ln(Bilateral distance)	-0.466***	-0.253***	-0.404***	-0.222***
	(0.021)	(0.023)	(0.022)	(0.025)
ln(Exporter's distance from the world GC)	-0.305***	-0.331***	-0.404***	-0.408***
	(0.014)	(0.014)	(0.015)	(0.016)
ln(Importer's distance from the world GC)	-0.255***	-0.281***	-0.332***	-0.335***
	(0.014)	(0.014)	(0.016)	(0.016)
Other control variables+	No	Yes	No	Yes
Observations	18,839	18,839	19,019	19,019
Panel B: Dep. variable is estimated fixed effect in the structural gravity model				
ln(Distance from the world GC)		-0.230**		
		(0.112)		
Coast and island dummies		Yes		
Observations		155		

Notes: The data are for the year 1994 in both panels. + Control variables include dummies for being in the same regional trade agreement(s), sharing legal origins, sharing currency, sharing border(s), sharing official language, dummy for being a GATT member (each side), dummy for selling to colony, dummy for buying from a colony. Robust standard errors in parentheses. *** p<0.01, ** p<0.05.

This being said, the long-term gravity equation is a theoretical tool for analyzing economic and political tensions over a long time horizon rather than an empirical tool applicable to a given period of time.

Regression (45), if without terms $D(n)$ and $D(n')$, is the traditional gravity regression. It can alternatively be estimated with two state fixed effects. The two fixed effects have a theoretical interpretation — they capture the inverse of each state's "remoteness" to the rest of the world. Following this reasoning, we hypothesize that the remoteness is increasing in our estimated $\ln D(n)$. To implement this idea, we run regression (45) and extract the importer fixed effect. A smaller fixed effect suggests that the corresponding state is more remote from the rest of the world. In Panel B of Table 3, we regress these estimated fixed effects on $\ln D(n)$. We find a negative correlation between them, indicating that a larger $\ln D(n)$ is associated with a greater remoteness from the rest of the world in trade.

Table 3 demonstrates that a shorter distance from the world GC explains *some* of a state's locational advantage in global trade.[22] Tables 2 and 3 together illustrate that the world-line assumption does not deviate far from reality, as both physical geography (in Table 2) and economic geography (in Table 3) demonstrate varying locational advantages across places, both negatively correlated with the distance from the world GC.[23]

Fact 2: Static State Size Distribution

Our theory in Section 2 implies that within any time period, states farther from the world GC are greater in size (unless τ is very low, see Appendix A.1.5 for details). Intuitively, as long as τ is not too small to matter, locales farther from the world GC join more neighboring locales to form larger states, in order to keep their price levels low.[24] To trace such forces in the data, we collect data on territorial areas of states from world maps, denoted by $\ln Area(n)$. Figure 2 shows the distribution of $\ln Area(n)$ across different time periods. Next, we regress $\ln Area(n)$ on the previous $\ln D(n)$ (i.e. state n's shortest distance from the world GC). The results are reported in Table 4, where a positive and statistically significant correlation between $\ln Area(n)$ and $\ln D(n)$ is found. We include continent fixed effects in all regressions.

In Panel A of Table 4, we limit control variables to geographic characteristics: a coast dummy and an island dummy. Column (1) of Panel A corresponds to the modern period. Since states have different numbers of locales, the state-level minimum distance from the world GC, as a sample statistic, may cause heteroskedasticity in the regression. We experiment with weighting regressions using numbers of locales at the state level to address potential heteroskedasticity. The results turn out to be similar. Here we minimize the use of control variables to maximize sample sizes. In column (1) of Panel B, we control for military expenses, iron and steel production, and primary energy consumption. With national powers controlled for, our sample size shrinks slightly (from 162 to 156). The coefficient of $\ln Dist(n)$ remains positive and statistically significant, either unweighted or weighted. In later tables, we report only unweighted results to save space.[25]

World geography in the benchmark model is a continuous landmass, whereas the land-

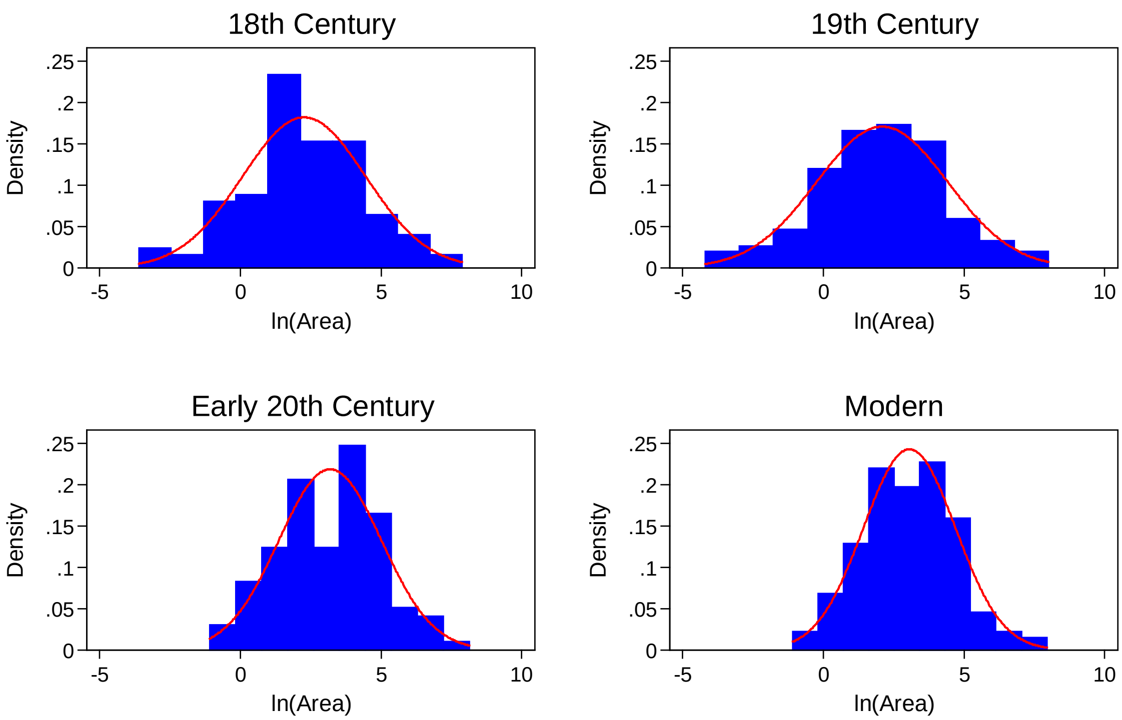
[22] Here we are not proposing a new D-based approach to estimate gravity models. The existing gravity estimation methods do not rely on any specific geography, and thus can account for any specific geography including but not limited to a linear world geography.

[23] Zeros in trade volumes are excluded (for 155 states, the full sample size in the form of state pairs should be 23,870 rather than 18,839 as in Table 3), but restoring them does not change our findings.

[24] If τ is very low, trade cost as a state-expanding force has little influence.

[25] Weighted results are available upon request. We are in favor of the unweighted specification because the application of weighted regressions to non-survey data is controversial. Weighting regressions may aggravate rather than mitigate heteroskedasticity (Solon, Haider, and Wooldridge, 2015).

Figure 2: Dispersion of Territorial Area across States in the World



mass on the earth is divided by oceans into different continents. Among all continents, the geography of Eurasia fits our theoretical construct best. We rerun the regressions in Table 4 using the subsamples of Eurasian and Non-Eurasian states in each period. The results are reported in Table 5. Both subsamples display patterns similar to those above.

In Appendix A.3, we provide two additional explorations. First, we experiment with using the rank value of $D(n)$ instead of $\ln D(n)$ as the main explanatory variable. Also, we use the centroid of every state (i.e. the arithmetic mean position of all the points in the state as a polygon) as the state's GC to rerun the results. The findings are similar.

Fact 3: Dynamic State Locations

The dynamic state locations in this subsection refer to *repetitions of the static equilibrium over time* rather than any intertemporal optimization in state formation. The previous static pattern on static state-size distribution has an exception — state 0. Recall that state 0 and other states solve different optimization problems (equation (32) vs. equation (34)). State 0, which solves both its borders, does not have to be smaller than state 1 (or -1). We provide a formal derivative in Appendix A.1.6. Intuitively, state 0 could be large in size because when it sets its two borders in two different directions, the disutility from extending borders

Table 4: Fact 2 (Static State Size Distribution)

Dependent variable is ln(Area)	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: Full sample				
ln(Distance from the world GC)	0.628***	0.760***	0.651***	0.383***
	(0.196)	(0.204)	(0.122)	(0.130)
Coast dummy	1.745**	-0.116	0.704***	0.456*
	(0.703)	(0.359)	(0.266)	(0.275)
Island dummy	-2.089***	-1.038**	-1.439***	-1.376***
	(0.598)	(0.401)	(0.467)	(0.371)
If weights are used:#				
ln(Distance from the world GC)	0.607***	0.701***	0.628***	0.639***
	(0.153)	(0.234)	(0.196)	(0.102)
Continent FE	Yes	Yes	Yes	Yes
Observations	162	121	137	174
Panel B: With national power controls				
ln(Distance from the world GC)	0.522***		1.937***	0.850***
	(0.110)		(0.643)	(0.248)
Coast dummy	-0.400*		0.939**	0.012
	(0.223)		(0.406)	(0.452)
Island dummy	-1.025***		-2.474*	-1.006***
	(0.328)		(1.273)	(0.349)
ln(Military expenses)	0.003		-0.068	0.037
	(0.130)		(0.290)	(0.127)
ln(Iron & steel production)	0.027		0.449*	0.001
	(0.056)		(0.254)	(0.099)
ln(Primary energy consumption)	0.487***		-0.116	0.255***
	(0.103)		(0.206)	(0.068)
If weights are used:#				
ln(Distance from the world GC)	0.774***		2.239***	1.511***
	(0.129)		(0.768)	(0.254)
Continent FE	Yes		Yes	Yes
Observations	156		51	75

Notes: # In both panels, regressions are rerun under the same specification but with weights (number of locales), with only the coefficient of ln(Distance from the world GC) reported as a separate row (other coefficients available upon request). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

spreads across its two fronts. This effect applies to none of the other states.

This claim concerning state 0, arguing for an indeterminate size relative to its neighbors, reminds us of empires in history. Note that in Table 1, the world GCs were in large

Table 5: Fact 2, Continued
(Static State Distribution, Eurasia and non-Eurasia)

Dependent variable is ln(Area)	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: The Eurasian subsample				
ln(Distance from the world GC)	0.410***	0.868***	0.620***	0.356**
	(0.135)	(0.229)	(0.132)	(0.136)
Island and coast dummies	Yes	Yes	Yes	Yes
Observations	82	67	81	90
Panel B: The Non-Eurasian subsample				
ln(Distance from the world GC)	1.033**	1.554***	1.673***	1.027**
	(0.427)	(0.451)	(0.270)	(0.445)
Island and coast dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	80	54	56	84

Notes: Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$.

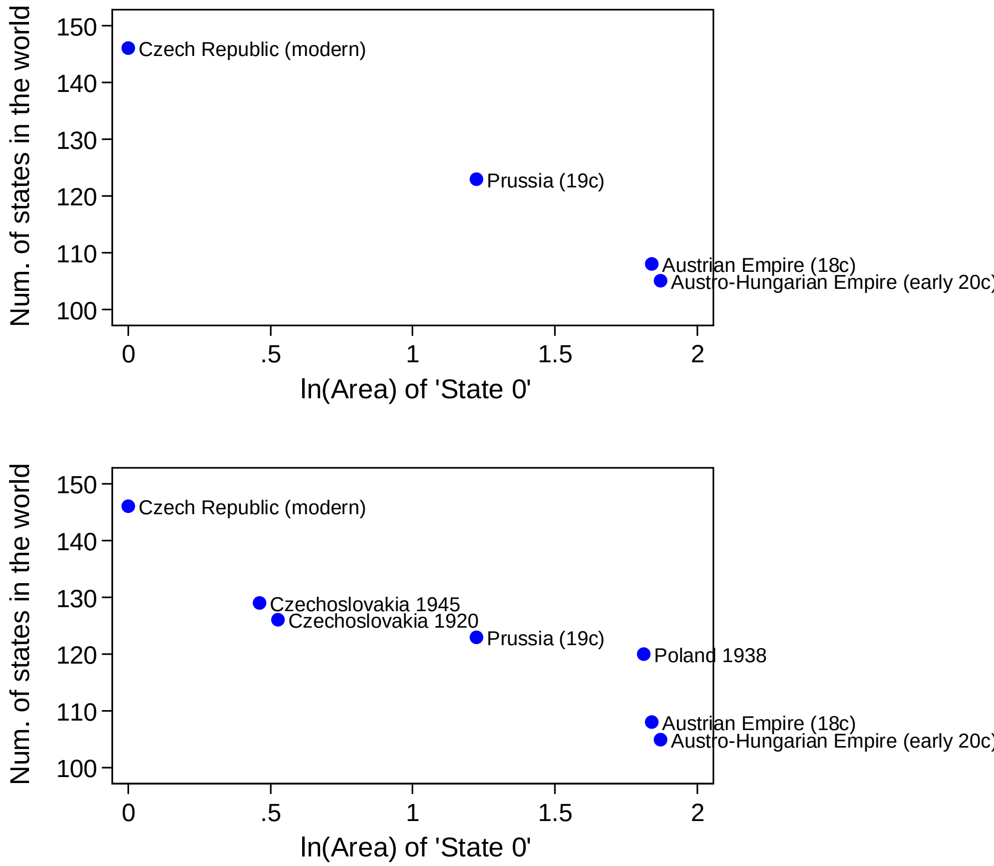
states in three out of the four periods (Austrian Empire, Germany, and Austro-Hungarian Empire, respectively). The modern state 0, the Czech Republic, has the smallest size in comparison with its historical counterparts. In contrast, state 0 in the 18th century, the Austrian Empire, had quite a large territory. Meanwhile, states in the world shrink in size over time, a fact that has already been seen in the previous Figure 2 where the dispersion of $\ln Area(n)$ is presented for every period.

When state 0 is larger in size, the number of states in the world decreases (see Appendix A.1.7 for derivation). Figure 3 demonstrates the negative correlation between state 0's area and the number of states in the world over different time periods. A negative association between the two variables is evident. In the lower panel, we add a post-war observation (Czechoslovakia in 1920), an interwar observation (Poland in 1938), and another post-war observation (Czechoslovakia in 1945). The negative correlation remains and actually becomes more pronounced.[26]

The number of observations in Figure 3 is admittedly small. In addition, all states

[26] A possible concern is that the total inhabitable area in the world increases over time, though that works against finding a negative correlation between the two variables.

Figure 3: Number of States and 'State 0'



Notes: c is the abbreviation of century. The lower panel includes three additional observations related to the two world wars, which are excluded by the upper panel. Czech Republic (modern) has the smallest area among all state 0's. We normalize it to one (zero in log). For all other periods, the ln(Area) of state 0 refers to the difference between actual ln(Area) and the ln(Area) of Czech Republic (modern). This normalization is in order to keep the horizontal axis short.

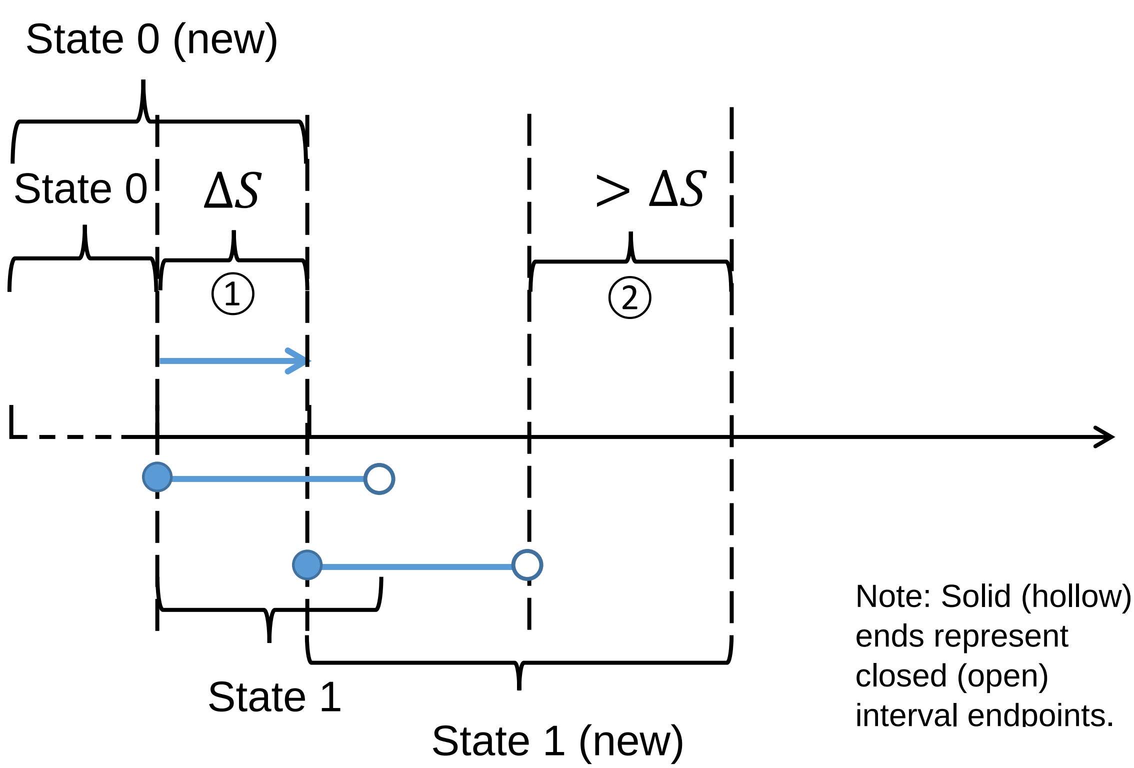
(including state 0) in a world with a larger number of states are "mechanically" smaller. That is, if the world's area is randomly cut into states, a smaller state 0 might simply be driven by more and finer "cuts" of the earth's surface. So we pursue a regression analysis as follows.

Considering such empires as "state-0 shocks" over time, we hypothesize that every state n is farther from the world GC when its contemporary state 0 is larger, an effect that is increasing in the index value n. More formally (see A.1.8 for derivation):

$$\frac{\partial^2 b_n}{\partial b_{n-1} \partial b_0} > 0, \tag{46}$$

and it is increasing in the index n. The rationale is as follows. Given a larger state 0, all other states are "pushed away" from the world GC. When pushed away, those states have to be larger in size, as the new locales gained by them have locations less advantageous than those lost by them.[27] Figure 4 illustrates the mechanism using state $n = 1$ as an example. Suppose state 0 expands its right-side border rightward by a distance of ΔS for exogenous reasons. Locales in region ① with a measure of ΔS, which belonged to state 1, are now in state 0. Consequently, the composition of state 1 will now include region ②, with a measure greater than ΔS. This size increase is owing to the fact that the "gained" territories (region ②) are locationally worse than the "lost" territories (region ①).

Figure 4: Foundation of Fact 3



[27]This size expansion is the reason why the total number of states in the world decreases, as shown in Figure 3.

The state indexes here depend only on their proximity to the world GC, regardless of national identities. That is, state n simply refers to the n-th nearest state to the world GC, to whichever state that index pertains in every time period. The effect represented by equation (46) is increasing in the index n, meaning that a greater n is associated with an even larger $|b_n - b_{n-1}|$ increase. This is because the farther a state is from the world GC relative to other states, the more it has to expand in size to compensate for its even less advantageous location.

We limit the state index n to 1-30, 1-50, and 1-70, respectively, in our data analysis. We do not consider $n > 70$ because states with very large indexes do not exist in every period. Equation (46) informs the following regression:

$$\ln D_{pr}(n) = \eta_0 \times I(n) + \eta_1 \times State0Area_{pr} + \eta_2 \times I(n) \times State0Area_{pr} + \bar{\xi}' X_{n,pr} + \epsilon_{n,pr}, \quad (47)$$

where $\ln D_{pr}(n)$ is the shortest distance between state $n = 1, 2, ..., 30/50/70$ and the world GC in period pr, $I(n)$ is the state index normalized between 0 and 1. That is, $I(n) = 0$ (respectively, $I(n) = 1$) if state n is the nearest to (farthest from) the world GC within the sample. Its coefficient η_0 is expected to be positive. $State0Area_{pr}$ is the area of state 0 in period pr, and $X_{n,pr}$ is a vector of control variables. η_1, expected to be positive, captures the mechanical fact that a larger state 0 means that all other states are farther from the world GC. What interests us is η_2, which is expected to be positive. As an alternative to including $State0Area_{pr}$ in the regression, we can use a more inclusive period fixed effect to absorb its own variation, with the interaction term $I(n) \times State0Area_{pr}$ unchanged.

The results are reported in Table 6. The sample used in Panel A is states 1-30 in each of the four periods, so that the full sample size is 120. We use $State0Area_{pr}$ in columns (1) and (2) and use period fixed effects instead in columns (3) and (4). We include no national power control variables in columns (1) and (3), so that their numbers of observations are both 120. In columns (2) and (4), we include national power control variables, which are unavailable for all states in the 18th century and for some states in later periods. Therefore, the sample size shrinks to 78 in these two columns. The coefficient of the interaction term, namely $\hat{\eta}_2$, is positive and statistically significant in all columns. The specifications in Panels B and C are the same as in Panel A, except that their samples include states 1-50 and states 1-70, respectively. Very similar findings are obtained from them.

Table 6: Fact 3 (State Locations Over Time)

Dependent variable is ln(Distance from the (contemporary) world GC)				
	(1)	(2)	(3)	(4)
Panel A: 30 Nearest States to the World GC				
State index normalized¶	4.052***	4.940***	3.573***	5.025***
	(0.852)	(0.886)	(0.554)	(0.887)
Area of State 0	-0.382	1.363**		
	(0.486)	(0.532)		
State index normalized × Area of State 0	12.458***	9.034**	15.163***	9.420***
	(3.413)	(3.449)	(2.722)	(3.516)
Period FE	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island and cost dummies, and continent FEs	Yes	Yes	Yes	Yes
Observations	120	78	120	78
Panel B: 50 Nearest States to the World GC				
State index normalized¶	4.361***	5.263***	4.254***	5.297***
	(0.402)	(0.471)	(0.270)	(0.475)
Area of State 0	0.049	1.552***		
	(0.346)	(0.380)		
State index normalized × Area of State 0	8.295***	5.694***	9.124***	6.078***
	(1.408)	(1.725)	(1.193)	(1.742)
Period Fixed Effect	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island and cost dummies, and continent FEs	Yes	Yes	Yes	Yes
Observations	200	121	200	121
Panel C: 70 Nearest States to the World GC				
State index normalized¶	5.220***	5.322***	5.082***	5.363***
	(0.203)	(0.418)	(0.215)	(0.419)
Area of State 0	0.706**	1.757***		
	(0.274)	(0.358)		
State index normalized × Area of State 0	3.538***	3.333**	4.006***	3.508**
	(0.826)	(1.501)	(0.803)	(1.516)
Period Fixed Effect	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island and cost dummies, and continent FEs	Yes	Yes	Yes	Yes
Observations	280	151	280	151

Notes: ¶ The normalized state index equals 0 (respectively, 1) for the state with the shortest (longest) distance to its contemporary world GC. ¥ National power controls include military expenses, iron & steel production, and primary energy consumption (all in log terms). Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$.

5 Concluding Remarks

Linearization is a common modeling technique in economics, and we apply it to the world geography to rationalize the interactions between national borders and international trade. It proves very useful for our purpose because it makes the modeling of endogenous borders possible. Building on a linear world, our general equilibrium model offers a political geography of the world created by international trade. Our model bridges local economies with the world economy, local welfare with foreign welfare, and national borders with the worldwide nation-state system. We also find solid facts in historical maps that are consistent with the assumption and conclusions of our theory.

The limitations of this study are threefold, each providing an avenue for future research. First, on the theoretical front, the downside of using a linear world geography stems from the loss of interplay between states with the same distance from the world geometric center. Advancements in this direction mandate a two-dimensional world geography, thus facing the challenge of characterizing arbitrary one-dimensional borders in a two-dimensional geography. We did not find a satisfactory mathematical tool to address this challenge, and speculate that differential geometry may provide a solution. Second, on the empirical front, we did not find worldwide bilateral trade data dating back to the 18th century. If found, such data would be valuable for evaluating how trade volumes and nation-states influence each other over time. Such data are scarce, although they have started to become accessible for certain regions, such as Western Europe and East Asia. Third, colonization is not studied here, but our model provides a framework for studying that process. A full general equilibrium of colonization is expected to be complicated, as it involves international migration, international trade, and national borders on both sides (empires and colonies). The world map in the era of colonization was closer to linearity (Eurocentric, having only few Pacific routes) than in later eras. Thus, our linear world model offers a promising way to model the general equilibrium of colonization.

References

Acemoglu, Daron, Simon Johnson, and James A. Robinson (2005), "The Rise of Europe: Atlantic Trade, Institutional Change, and Economic Growth." *American Economic Review*, 95, 546–579.

Acemoglu, Daron and Pierre Yared (2010), "Political limits to globalization." *American Economic Review: Papers and Proceedings*, 100, 83–88.

Alesina, Alberto and Enrico Spolaore (1997), "On the Number and Size of Nations." *Quarterly Journal of Economics*, 112, 1027–56.

Alesina, Alberto and Enrico Spolaore (2005), "War, peace, and the size of countries." *Journal of Public Economics*, 89, 1333–1354.

Alesina, Alberto and Enrico Spolaore (2006), "Conflict, defense spending, and the number of nations." *European Economic Review*, 50, 91–120.

Alesina, Alberto, Enrico Spolaore, and Romain Wacziarg (2000), "Economic integration and political disintegration." *American Economic Review*, 90, 1276–1296.

Alesina, Alberto, Enrico Spolaore, and Romain Wacziarg (2005), "Trade, Growth and the Size of Countries." In *Handbook of Economic Growth (vol. 5)* (Philippe Aghion and Steven Durlauf, eds.), 1499–1542, Elsevier.

Allen, Treb and Costas Arkolakis (2014), "Trade and the Topography of the Spatial Economy." *Quarterly Journal of Economics*, 129, 1085–1140.

Allen, Treb, Costas Arkolakis, and Yuta Takahashi (2018), "Universal Gravity." Working paper.

Anderson, James (2009), "Does trade foster contract enforcement?" *Economic Theory*, 41, 105–130.

Anderson, James E. (2011), "The gravity model." *Annual Review of Economics*, 3, 1–668.

Anderson, James E. and Eric van Wincoop (2003), "Gravity with gravitas: A solution to the border puzzle." *American Economic Review*, 93, 170–192.

Baier, Scott L. and Jeffrey H. Bergstrand (2002), "On the endogeneity of international trade flows and free trade agreements." Working paper.

Baier, Scott L. and Jeffrey H. Bergstrand (2004), "Economic determinants of free trade agreements." *Journal of International Economics*, 64, 29–63.

Barraclough, Geoffrey (1994), *The times atlas of world History (4th ed.)*. BCA London.

Black, Duncan (1948), "On the rationale of group decision-making." *Journal of Political Economy*, 56, 23–24.

Bonfatti, Roberto and Kevin Hjortshøj O'Rourke (2014), "Growth, import dependence and war." NBER Working Paper 20326.

Braudel, Fernand (1949), *The Mediterranean and the Mediterranean World in the Age of Philip II*. University of California Press (1995).

Brennan, Geoffrey and James M. Buchanan (1980), *The Power to Tax: Analytic Foundations of a Fiscal Constitution*. Cambridge University Press.

Brzezinski, Zbigniew (1997), *The Grand Chessboard: American Primacy And Its Geostrategic Imperatives*. Basic Books.

Desmet, Klaus, Michel Le Breton, Ignacio Ortuño-Ortín, and Shlomo Weber (2011), "The stability and breakup of nations: a quantitative analysis." *Journal of Economic Growth*, 16, 183–213.

Diamond, Jared M. (1999), *Guns, Germs, and Steel: The Fates of Human Societies*. W. W. Norton & Company.

Dornbusch, Rudiger, Stanley Fischer, and Paul A Samuelson (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods." *American Economic Review*, 67, 823–39.

Downs, Anthony (1957), *An Economic Theory of Democracy*. Harper Collins.

Egger, Peter, Mario Larch, Kevin E. Staub, and Rainer Winkelmann (2011), "The Trade Effects of Endogenous Preferential Trade Agreements." *American Economic Journal: Economic Policy*, 3, 113–43.

Fairgrieve, James (1917), *Geography and world power*. E. P. Dutton & Company.

Fazal, Tanisha M. (2007), *State Death: The Politics and Geography of Conquest, Occupation, and Annexation*. Princeton University Press.

Friedman, David (1977), "A theory of the size and shape of nations." *Journal of Political Economy*, 85, 59–77.

Greif, Avner (1994), "Cultural beliefs and the organization of society: A historical and theoretical reflection on collectivist and individualist societies." *Journal of Political Economy*, 102(5), 912950.

Guiso, Luigi, Helios Herrera, and Massimo Morelli (2016), "Cultural Differences and Institutional Integration." *Journal of International Economics*, 99, S97–S113.

Head, Keith and Thierry Mayer (2014), "Gravity equations: Workhorse, toolkit, and cookbook." In *Handbook of International Economics* (Elhanan Helpman, Kenneth Rogoff, and Gita Gopinath, eds.), 131–195, Oxford.

Head, Keith, Thierry Mayer, and John Ries (2010), "The erosion of colonial trade linkages after independence." *Journal of International Economics*, 81, 1–14.

Hotelling, Harold (1929), "Stability in competition." *Economic Journal*, 39, 41–57.

Huntington, Ellsworth (1907), *The Pulse of Asia, a Journey in Central Asia Illustrating the Geographic Basis of History*. Houghton, Mifflin and Company.

Keller, Wolfgang and Carol H. Shiue (2014), "Endogenous Formation of Free Trade Agreements: Evidence from the Zollverein's Impact on Market Integration." *Journal of Economic History*, 74, 1168–1204.

Kissinger, Henry (1994), *Diplomacy*. Simon & Schuster.

Kissinger, Henry (2014), *World Order*. Penguin Press.

Krishna, Pravin (2003), "Are Regional Trading Partners "Natural"?" *Journal of Political Economy*, 111, 202–231.

Lan, Xiaohuan and Ben G. Li (2015), "The Economics of Nationalism." *American Economic Journal: Economic Policy*, 7, 294–325.

Mackinder, H. J. (1904), "The geographical pivot of history." *Geographical Journal*, 23, 421–437.

Mackinder, H.J. (1919), *Democratic Ideals and Reality: A Study in the Politics of Reconstruction*. Holt.

Martin, Philippe, Thierry Mayer, and Mathias Thoenig (2008), "Civil Wars and International Trade." *Journal of the European Economic Association*, 6, 541–550.

Morgenthau, Hans J. (1948), *Politics Among Nations*. Alfred A. Knopf.

Ogawa, Hideaki and Masahisa Fujita (1980), "Equilibrium land use patterns in a nonmonocentric city." *Journal of Regional Science*, 20, 455–475.

Overy, Richard (2010), *The Times Complete History of the World*. Times Books.

Puga, Diego and Daniel Trefler (2014), "International Trade and Institutional Change: Medieval Venice's Response to Globalization." *Quarterly Journal of Economics*, 129, 753–821.

Rand McNally, (1992), *Atlas of World History*.

Rand McNally, (2015), *New Historical Atlas of the World*.

Ranjan, Priya and Jae Young Lee (2007), "Contract Enforcement And International Trade." *Economics and Politics*, 19, 191–218.

Reba, Meredith, Femke Reitsma, and Karen C. Seto (2016), "Spatializing 6,000 years of global urbanization from 3700 bc to ad 2000." *Scientific Data*.

Shiue, Carol H. (2005), "From political fragmentation towards a customs union: Border effects of the German Zollverein, 1815 to 1855." *European Review of Economic History*, 9, 129–162.

Singer, J. David (1987), "Reconstructing the correlates of war dataset on material capabilities of states, 1816-1985." *International Interactions*, 14, 115–32.

Skaperdas, Stergios and Constantinos Syropoulos (2001), "Guns, butter, and openness: On the relationship between security and trade." *American Economic Review*, 91, 353–357.

Solon, Gary, Steven J. Haider, and Jeffrey M. Wooldridge (2015), "What are we weighting for?" *Journal of Human Resources*, 50, 301–316.

Appendices

A.1 Proofs and Derivations

A.1.1 Equation (17)

At locale t, the lord maximizes $U(t) = \frac{1}{1-\gamma} C^z(t)^{1-\gamma} - hS(t)$, where $C^z(t) \equiv \exp\{\int_{-1}^{1} \ln c^z(t,s) ds\}$, subject to the budget constraint $\int_{-1}^{1} p(t,s) c^z(t,s) ds = r(t)z$. Her first-order condition is

$$p(t,s)c^z(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} \equiv \kappa^z(t). \tag{48}$$

If plugging it back into the budget constraint, we obtain $\kappa^z(t) = r(t)z/2$.

At locale t, the labor maximizes $V(t) = \frac{\psi}{1-\gamma} C^l(t)^{1-\gamma}$, where $C^l(t) \equiv \exp\{\int_{-1}^{1} \ln c^l(t,s)ds\}$, subject to budget constraint $\int_{-1}^{1} p(t,s)c^l(t,s)ds = w(t)l(t)$. Her first-order condition is

$$p(t,s)c^l(t,s) = \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)} \equiv \kappa^l(t). \tag{49}$$

If plugging it back into the budget constraint, we obtain $\kappa^l(t) = w(t)l(t)/2$.

So, the aggregate first-order condition is the sum of equations (48) and (49):

$$p(t,s)c(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} + \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)} \equiv \kappa(t).$$

This is equation (17) in the text. The value of $\kappa(t)$ is

$$\kappa(t) = \kappa^z(t) + \kappa^l(t) = (r(t)z + w(t)l(t))/2.$$

Notice that the aggregate first-order condition is used to derive the aggregate expenditure on locale s's product at locale t, namely $p(t,s)c(t,s)$. The lord and labor solve their own utility maximization. Social welfare maximization is not involved here.

A.1.2 Equation (25)

By equation (48), we have $c^z(t,s) = \kappa^z(t)/p(t,s)$. By inserting the $c^z(t,s)$ into $C^z(t)$, we obtain

$$\begin{aligned}
C^z(t) &= \exp\{\int_{-1}^{1} (\ln \kappa^z(t) - \ln p(t,s))ds\} \\
&= \exp\{\int_{-1}^{1} (\ln \kappa^z(t)/p(t) - \ln d(t,s))ds\} \\
&= \exp\{2 \ln \kappa^z(t)/p(t) - \int_{-1}^{1} \ln d(t,s)ds\} \\
&= (\kappa^z(t)/p(t))^2 \exp\{-\int_{-1}^{1} \ln d(t,s)ds\} \\
&= (\frac{r(t)z}{2p(t)})^2 \exp\{-\int_{-1}^{1} \ln d(t,s)ds\} \\
&= (\frac{r(t)z}{2p(t)})^2 / R(t),
\end{aligned}$$

where $p(t)$ is the normalized factory-gate price $p(t) = r(t)z/2$ in the text. Thus, $C^z(t) = \frac{1}{R(t)}$, which is equation (25) in the text.

A.1.3 Derivation of Gravity Equation (equation (36))

We assume, without loss of generality, that $n > m + 1 \geq 1$ — both states are in the right half of the world and nonadjacent, and state n is farther from the world GC than state m — such that $D_{m,n} = b_{n-1} - b_m$. The export volume from state m to state n is

$$\begin{aligned} X_{m,n} &= S_m \int_{b_{n-1}}^{b_n} p(s)c(s,b_m)ds = \frac{S_m}{2}\int_{b_{n-1}}^{b_n} \kappa d(s,b_m)^{-1}ds \\ &= \frac{\kappa}{2\tau} S_m[\exp\{-\tau(b_{n-1}-b_m)\} - \exp\{-\tau(b_n - b_m)\}] \\ &= \frac{\kappa}{2\tau} S_m \exp\{-\tau D_{m,n}\} \times (1 - \exp\{-\tau S_n\}), \end{aligned}$$

where $D_{m,n} = b_{n-1} - b_m$. Here, the second equality stems from equation (18). Since states sizes are small compared with 1 (the total size of all states on each side is 1), $1 - \exp\{-\tau S_n\} = \tau S_n$. So, equation (36) is obtained:

$$X_{m,n} = \zeta S_m S_n \exp\{-\tau D_{m,n}\},$$

where $\zeta = \kappa/2$ applies to all pairs worldwide.

A.1.4 International Geopolitics (equation (40))

The first-order condition (34) for state n is equivalent to

$$F \equiv \tau R_n^{\gamma-1}(1 - b_{n-1} - S_n) - h = 0, \tag{50}$$

which implies the following partial derivatives:

$$\begin{aligned} F_h &= -1 < 0, && (51)\\ F_S &= -(\gamma-1)R_n^{\gamma-1}\tau^2(1-b_n)^2 - \tau R_n^{\gamma-1} < 0, && (52)\\ F_{b_{n-1}} &= (\gamma-1)R_n^{\gamma-1}\tau^2(2b_{n-1}+S_n)(1-b_{n-1}-S_n) - \tau R_n^{\gamma-1}, && (53)\\ F_\tau &= R_n^{\gamma-1}(1-b_n)\{1 + \frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2 + (1-b_n)^2]\} > 0. && (54) \end{aligned}$$

So,

$$\frac{dh}{d\tau} = -\frac{F_\tau}{F_h} = F_\tau = R_n^{\gamma-1}(1-b_n)\{1 + \frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2 + (1-b_n)^2]\} > 0.$$

A.1.5 Foundation of Fact 2

By equation (53) (located in the previous subsection A.1.4), $F_{b_{n-1}} > 0$ if

$$\tau > \frac{1}{(\gamma - 1)(b_0(1 - b_0))}. \tag{55}$$

By total differentiation, $\frac{\partial S_n}{\partial b_{n-1}} = -\frac{F_{b_{n-1}}}{F_S}$. Recall $F_S < 0$ in equation (52). Thus, $\frac{\partial S_n}{\partial b_{n-1}} > 0$ so long as inequality (55) holds.

A.1.6 State 0's Size

The first-order condition (32) for state 0 is equivalent to

$$\tau R_0^{\gamma-1}(1 - \frac{S_0}{2}) = h. \tag{56}$$

The first-order condition (33) for state 1 is equivalent to

$$\tau R_1^{\gamma-1}(1 - \frac{S_0}{2} - S_1) = h. \tag{57}$$

Recall $R_0 < R_1$ and $\gamma > 1$. The only requirement on the relative sizes of S_0 and S_1 is that $1 - \frac{S_0}{2}$ must be greater than $1 - \frac{S_0}{2} - S_1$. That always holds. So, S_0 could be greater than, less than, or equal to S_1. Similarly, the first-order condition for state n is

$$\tau R_n^{\gamma-1}(1 - [\frac{S_0}{2} + \sum_{k=1}^{n-1} S_k] - S_n) = h. \tag{58}$$

If n is very large, $\sum_{k=1}^{n-1} S_k + S_n$ would be so large that equations (56) and (58) could not hold simultaneously. Otherwise, state 0 could be larger than state n.

The possibility for state 0 to be smaller than state n is obvious.

A.1.7 State 0's Size and Number of States in the World

A simple manipulation of equation (34) shows

$$\frac{\partial b_n}{\partial b_{n-1}} = \frac{\partial(b_{n-1} + S_n)}{\partial b_{n-1}} = 1 + \frac{\partial S_n}{\partial b_{n-1}} > 0, \tag{59}$$

where $\frac{\partial S_n}{\partial b_{n-1}} > 0$ comes from Appendix A.1.5 (if τ is not too small). By equation (59),

$$\frac{\partial b_n}{\partial b_0} = \prod_{i=0}^{n-1} \frac{\partial b_{n-i}}{\partial b_{n-i-1}} > 0, \tag{60}$$

for any $n \geq 1$, and thus

$$\frac{\partial S_n}{\partial b_0} = \frac{\partial S_n}{\partial b_{n-1}} \frac{\partial b_{n-1}}{\partial b_0} > 0. \tag{61}$$

That is, a larger state 0 results in larger sizes of all states in the world, meaning a smaller number of states in the world.

A.1.8 Derivation of equation (46)

Since $\frac{\partial^2 b_n}{\partial b_{n-1} \partial b_0} = \frac{\partial^2 S_n}{\partial b_{n-1} \partial b_0} = \frac{\partial^2 S_n}{\partial b_0 \partial b_{n-1}}$, we can show instead that $\frac{\partial^2 S_n}{\partial b_0 \partial b_{n-1}} > 0$ and is increasing in n. Recall equation (61) above. Its first term is positive and increasing in n. Specifically, for a greater n (and thus $n-1$), b_n has to be extended further from b_{n-1}, resulting in a larger S_n.

Now move on to the second term in equation (61), which equals

$$\frac{\partial b_{n-1}}{\partial b_0} = \prod_{i=1}^{n-1} \frac{\partial b_{n-i}}{\partial b_{n-i-1}},$$

following equation (60). Here, every term inside the product is weakly greater than 1. They all equal 1 if all states from 1 to $n-1$ keep their original sizes but move outward. For a greater n (and thus $n-1$), the product has one more term in it. It will weakly increase. Notice that this result is independent from the change in b_n (and thus S_n).

To combine the two terms, one can see that $\frac{\partial^2 S_n}{\partial b_0 \partial b_{n-1}} > 0$ and is increasing in n.

A.2 Data details

Historical maps We used multiple historical atlases, including Barraclough (1994), Rand McNally (1992, 2015) and Overy (2010), as our data sources because digitized maps from historical atlases are usually provided for different region-time blocks. Combining different sources enabled us to compile a world map for different historical periods, each starting from a *base* year and extending to approximately 20-30 years later.

The selection of base years inevitably involves judgments, since a balance has to be struck between historical significance and map availability. In principle, we selected years that (i) follow major wars and (ii) precede relatively peaceful 20-30 year periods. World political geography in those base years resulted from the resolution of the power imbalances that triggered the wars, and was known for temporary regional stability afterwards. Specifically, the year 1750 followed the War of the Austrian Succession, and the year 1815 was the year when the Treaty of Paris was signed. It is difficult, by this principle, to find a qualified base year in the early 20th century, because the interwar years (1919-1938) were too short as a peaceful period. In this setting, choosing a single year would risk using a political map filled with persuasive regional tensions that changed borders soon. At the same time, the first half of the 20th century, as a notable period of struggle in modern history, should not be plainly excluded from this study. As a compromise, we pooled all states that existed in three separate base years — 1914, 1920, and 1938.[28]

Similar judgments were made when we determined what states in world maps to exclude. In principle, territories with ambiguous sovereignty statuses were excluded. By this principle, small island states were usually excluded because many of them were dependent territories. There are two exceptions to this principle. First, although colonies had ambiguous sovereignty statuses, they were good examples of border reshuffling and state formation. Thus, colonies were treated as independent states in their own periods *if they later transitioned to independent states.* Second, kingdoms in the 18th century were considered to be independent states if they were independent from neighboring states that had clear sovereignty statuses. Without making these two exceptions, states in historical periods would be quite small in number.

Locales in the world The information on within-country administrative divisions is obtained from the GeoNames database.[29] The GeoNames database reports geographic coordinates of administrative divisions across the globe. It also reports corresponding current population. For our modern time, there are 21,068 such divisions (our "locales"). There exist no GeoName data corresponding to historical periods. We used two methods to address the problem. First, we used the current GeoName population to proxy for historical population, since the GeoNames data represent the largest possible set of human habitats on the earth. Second, we used historical urban population compiled by Reba, Reitsma, and Seto (2016). The urban data compiled by Reba et al. (2016) are from historical records, but cover only a small number of cities (mostly megacities) in history. We later compared the maps of locales obtained using the two methods with each other, and also compared both

[28]If a state altered its name across the three base years, we treated it as a new state. If a state kept its old name, we treated it as a "steady state" and accordingly averaged its variables across the three base years.

[29]The database is accessible online at `www.geonames.org`, with both free and paid data services provided.

maps against historical maps that have estimated population density marked. They turn out to be highly consistent. We prefer the first method because of its large coverage and compatibility with other country-level control variables (see below).[30]

Other historical data Apart from using historical world maps, we extracted population, iron and steel production, military expenditures, and primary energy consumption from the National Material Capabilities Dataset (version 4) compiled by Singer (1987), which is now part of the Correlates of War (COW) project.[31] The dataset is regularly updated and thus extends beyond the year 1987, providing us with control variables that reflect every country's national power and industrialization level. Its coverage begins with the year 1815 and thus the data are unavailable for our 18th century sample. The data on world political geography and industrialization are the main variables in this study.

Summary statistics of all the variables described above are provided in Table A1.

[30] Notice that the location of the world GC does not remain the same because uncharted areas differ from period to period. Locales mapped to uncharted areas in a historical period are dropped from the collections of locales for that period. This is why the world GC estimated (Table 1) changes over time.

[31] The COW project is accessible online: `www.correlatesofwar.org`.

Table A1: Summary Statistics

Variable	Obs	Mean	STD	Min	Max	Obs	Mean	STD	Min	Max
	Panel A: Modern period					*Panel B: The 18th century*				
Distance from the world GC (km)	162	5365	3575	132.2	17968	121	4959	3609	364.7	17620
Area (square km)	162	86.41	274.7	0.338	2806	121	71.00	269.7	0.0269	2664
Coast dummy	162	0.753	0.433	0	1	121	0.752	0.434	0	1
Island dummy	162	0.123	0.330	0	1	121	0.182	0.387	0	1
Military expenditure#	156	3.548e+06	9.153e+06	4783	5.700e+07					
Iron and steel production (tons)	156	5054	19802	0	205259					
Primary energy consumption*	156	118773	308762	25.74	2.461e+06					
	Panel C: The 19th century					*Panel D: Early 20th century*				
Distance from the world GC (km)	137	4945	3867	110.9	17970	174	5606	3523	194.0	17968
Area (square km)	137	84.07	308.4	0.0148	2976	174	120.1	387.7	0.338	3401
Coast dummy	137	0.679	0.469	0	1	174	0.828	0.379	0	1
Island dummy	137	0.153	0.362	0	1	174	0.126	0.333	0	1
Military expenditure#	51	5146	4316	14.73	20687	75	745823	1.919e+06	0	9.970e+06
Iron and steel production (tons)	51	325.5	444.2	0	2806	75	1908	5953	0	45349
Primary energy consumption*	51	7100	9968	0	62639	75	30703	100490	0	809321

Notes: # Following the COW database, the unit is 1,000 US dollars (1,000 British Pounds) in Panels A and D (Panel C). * The unit is 1,000 of coal-ton equivalents.

Gravity variables We extracted the year 1994 from the CEPII gravity dataset to estimate the gravity regression (45). The CEPII data are widely used in international trade studies. It is accessible online: `www.cepii.fr`. For details, see Head, Mayer, and Ries (2010) and Head and Mayer (2014).

A.3 Additional Results

State rank instead of distance from the world GC (Fact 2) We experimented with using the rank value of $D(n)$ instead of $\ln D(n)$ as the main explanatory variable. The state which is the n-th nearest to the contemporary world GC has a rank value n. We normalized the rank value between 0 (nearest to the world GC) and 1 (farthest from the world GC) within every period, so that the rank value is unaffected by the different numbers of states across periods. In Table A2, the rank value is used instead of $\ln D(n)$ and the specifications are otherwise the same as in Table 4. It shows results that highly resemble those in Table 4. The shortcoming of the rank value is its lack of cardinal meaning. The variation in the rank value is ordinal and thus the differences among its values are difficult to interpret. It serves only as a robustness check here.

Centroid-based results (Fact 2) Locale-level data were used to construct the world GC and $D(n)$ in the previous Fact 2. Alternatively, we used the centroid of every state (i.e. the arithmetic mean position of all the points in the state as a polygon) as the state's GC, and the centroid of the world as the world GC. This approach can be easily implemented using GIS software. We find the centroid of the modern world to be at (40.52N, 34.34E), located in Yarımca, Uğurludağ, Çorum, Turkey. Based on these coordinates, we recalculated $D(n)$ and reran our study for the modern period. The centroid-based results are reported in Table A3, where both regression specification and sample states follow Table 4. As in Table 4, a positive and statistically significant correlation is found between $\ln Area(n)$ and $\ln D(n)$. The centroid approach serves only as a robustness check, since it overstates the importance of territories with low (including zero) population density.

Table A2: Fact 2 (Rank Instead of Distance)

Dependent variable is ln(Area)	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: Full sample				
Rank (Distance from the world GC)	0.007**	0.021***	0.017***	0.007**
	(0.003)	(0.005)	(0.004)	(0.003)
Coast dummy	0.202	-0.205	0.709**	0.528*
	(0.254)	(0.364)	(0.281)	(0.276)
Island dummy	-1.355***	-1.067***	-1.401***	-1.383***
	(0.371)	(0.403)	(0.458)	(0.356)
Continent FE	Yes	Yes	Yes	Yes
Observations	162	121	137	174
Panel B: With national power controls				
Rank (Distance from the world GC)	0.008***		0.058***	0.014**
	(0.003)		(0.018)	(0.005)
Coast dummy	-0.342		1.155***	0.192
	(0.230)		(0.427)	(0.483)
Island dummy	-0.965***		-2.403*	-0.885**
	(0.328)		(1.316)	(0.371)
ln(Military expenses)	0.022		-0.025	0.044
	(0.133)		(0.288)	(0.137)
ln(Iron & steel production)	-0.014		0.463	-0.038
	(0.054)		(0.297)	(0.098)
ln(Primary energy consumption)	0.512***		-0.235	0.267***
	(0.105)		(0.253)	(0.074)
Continent FE	Yes		Yes	Yes
Observations	156		51	75

Notes: This table is a robustness check for Table 4. All specifications here are the same as those in Table 4, except that the main regressor is Rank (Distance from the world GC) instead of ln(Distance from the world GC). Rank 0 (respectively, 1) means the shortest (longest) distance from the world GC. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table A3: Fact 2 (Based on Centroids)

Dependent variable is ln(Area)		
	(1)	(2)
ln(Distance from the world centroid)	0.554**	0.411**
	(0.236)	(0.172)
Coast dummy	0.226	-0.365
	(0.284)	(0.257)
Island dummy	-1.674***	-1.127***
	(0.448)	(0.407)
ln(Military expenses)		0.047
		(0.152)
ln(Iron & steel production)		-0.052
		(0.064)
ln(Primary energy consumption)		0.580***
		(0.127)
Observations	162	156

Notes: The data is based on the 1994 world map. The set of states is the same as in column (1) of Table 4. Robust standard errors in parentheses. *** p<0.01, ** p<0.05.