Here are some number theory problems...

- Given a positive integer \( n \), let \( p(n) \) be the product of the nonzero digits of \( n \). Let
  \[
  S = p(1) + p(2) + \cdots + p(999).
  \]
  What’s the largest prime factor of \( S \)?

- If \( a \equiv b \pmod{n} \), show that \( a^n \equiv b^n \pmod{n^2} \). Is the converse true?

- Show that the sequence 1, 11, 111, \ldots contains an infinite subsequence whose terms are pairwise relatively prime.

- Show that for any fixed positive integer \( n \), the sequence
  \[
  2, 2^2, 2^{2^2}, 2^{2^{2^2}} \ldots \pmod{n}
  \]
  is eventually constant.

- **Putnam 1986 A2**
  What is the units (that is, rightmost) digit of
  \[
  \left\lfloor \frac{10^{20000}}{10^{100} + 3} \right\rfloor?
  \]
  Here \( \lfloor x \rfloor \) is the greatest integer less than or equal to \( x \).

- **Putnam 2011 B2**
  Let \( S \) be the set of all ordered triples \( (p, q, r) \) of prime numbers for which at least one rational number \( x \) satisfies \( px^2 + qx + r = 0 \). Which primes appear in seven or more elements of \( S \)?