A COMPACT STRONG-SCATTER SCINTILLATION MODEL

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Summary. This document describes numerical solutions to the 4th-order moment equation for scintillation under strong scatter conditions. It is well known that scintillation depends on the parameters that describe the structure size distribution, and the Fresnel radius. However, in an unconstrained fixed power-law environment, the theory can be formulated in terms of a single parameter. In effect, different combinations of perturbation strength, propagation distance, and frequency produce exactly the same results. This paper exploits this relation and extends it to two-component power-law spectra.

1 INTRODUCTION

The strong-scatter theory of scintillation characterizes the moments of fields that have propagated through a disturbed region followed by an extended region of free space. For Beacon satellite signals that traverse the ionosphere at moderate propagation angles, the disturbed region can be replaced by a single phase screen. Under these conditions, the fourth-order moment equation, which characterizes the intensity spectral density function (SDF) and the scintillation index, can be reduced to a particularly simple form for computation[1,2]. Complete developments of the results as presented here can be found in[3,4]:

$$\Phi_I(\boldsymbol{\kappa}) = \iint \exp\{-g(\boldsymbol{\eta}, \boldsymbol{\kappa}\rho_F^2)\} \exp\{-i\boldsymbol{\kappa} \cdot \boldsymbol{\eta}\} d\boldsymbol{\eta} \tag{1}$$

where

$$\rho_F = \sqrt{x/k} \tag{2}$$

is the Fresnel radius. The g(.,.) is defined by a separate integration

$$g(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2) = 4k^2 l_p \iint \Phi_{\delta n}(0, \boldsymbol{\kappa}) \sin^2(\boldsymbol{\kappa} \cdot \boldsymbol{\alpha}_1/2) \sin^2(\boldsymbol{\kappa} \cdot \boldsymbol{\alpha}_2/2) d\boldsymbol{\kappa}/(2\pi)^2 \tag{3}$$

The intensity SDF depends on the SDF that characterizes the ionospheric irregularity structure. To accommodate the full range of reported structure configurations consider the generalized power-law SDF

$$\varphi(\kappa) = C_s \begin{cases} \kappa^{-p_1} & \kappa < q_0 \\ \kappa^{-p_2} q_0^{(p_2-p_1)} & \kappa \geq q_0 \end{cases} \tag{4}$$

Note that if p_1=0, the break scale q_0 is functionally an outer scale. If q_0 is arbitrarily large, only the leading power-law contributes. By substituting (4) into (3) the defining relation can be rewritten as

$$\Phi_I(\boldsymbol{\kappa}; \rho_F) = \iint \exp\{-\gamma(\boldsymbol{\eta}/\rho_F, \boldsymbol{\kappa}\rho_F)\} \exp\{-i\boldsymbol{\kappa} \cdot \boldsymbol{\eta}\} d\boldsymbol{\eta} \tag{5}$$

where

$$\gamma(\boldsymbol{\alpha}, \boldsymbol{\beta}) = 4 \iint \begin{cases} U_1 \chi^{-p_1} & \chi < (q_0 \rho_F) \\ U_2 \chi^{-p_2} & \chi \geq (q_0 \rho_F) \end{cases} \times \sin^2(\boldsymbol{\chi} \cdot \boldsymbol{\alpha}/2) \sin^2(\boldsymbol{\chi} \cdot \boldsymbol{\beta}/2) d\boldsymbol{\chi}/(2\pi)^2 \tag{6}$$

and

$$U_1 = (k^2 C_s l_p \rho_F^{p_1-2}) \tag{7}$$

$$U_2 = \left(k^2 C_s l_p q_0^{(p_2-p_1)} \rho_F^{p_2-2}\right) \tag{8}$$

The scintillation index is computed from the integration of (5) over all wavenumbers. It can be shown that that integration is independent of the Fresnel radius, aside from (7) and (8). It follows that complete behavior of the strong scatter theory for two-component power-law structures is encapsulated in (7) and (8).

2 APPLICATIONS

Critical details are hidden in the development as presented in Section 1. Field-aligned anisotropy of the structure imposes a strong dependence on the angle between the propagation direction and the magnetic field direction. At a minimum, to map that structure into the model, the magnitudes of the spatial wavenumbers are derived from the square root of quadratic forms. In effect there are hidden parameters in (6). One way around the problem is to assume that the structure in the cross-field direction dominates and is captured by a one-dimensional phase screen constructed along the effective scan direction. This one-dimensional disturbance model has been used successfully by Carrano et al. to interpret and extrapolate strong scatter data.[5]

3 Summary

This paper will demonstrate the applications of the compact form of the strong-scatter theory to predict scintillation structure in two-component power-law environments. One of the more interesting results is the mitigation of strong focusing by two-component spectra. The two-component power-law was originally discovered from measurements, but the result was supported by simulations performance by Franke[6] that showed that it could explain the frequency dependence of equatorial scintillation. A well-known theorem shows that if the Fresnel radius increases indefinitely, the scintillation index will converge to unity. However, depending primarily on the steepness of the power-law index, strong focusing can produce highly non-gaussian structures with SI indices well in excess of unity. As lower frequencies come out of strong saturation, higher frequencies might well be in saturation with scintillation indices greater than their lower-frequency counterparts. Two-component power-law spectra afford some freedom in manipulating where these transitions occur. The theoretical results are reinforced with simulations.

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