Scintillation Characteristics across the GPS Frequency Band

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BIOGRAPHY

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ABSTRACT

We investigate the characteristics of ionospheric scintillation across the entire GPS frequency band spanning the L5–L1 carrier signals (1176 MHz–1575 MHz). Of particular interest is the intensity correlation between carrier pairs, since this dictates the extent to which frequency diversity may be leveraged to mitigate scintillation impacts on navigation accuracy. Since only a few satellites currently broadcast the L2C and L5 signals, a limited number of scintillation field measurements have been collected to date. We show recent scintillations observations on L2C in Brazil collected in April 2012. Since current solar conditions are less disturbed than during the previous two solar maximum periods, however, it is not possible to explore the full range of scintillation conditions with current measurements alone. Therefore, we have developed a high-fidelity simulation technique to infer the complex (amplitude and phase) fluctuations of the L2 and L5 carrier signals from complex fluctuations of the L1 carrier signal recorded during the previous solar maximum. We compare this technique with two stochastic approaches which use the $S_4$ index, rather than the raw complex data, as input. After demonstrating that the three techniques yield consistent results, we apply the simplest of these approaches to a database of L1 $S_4$ observations.
from the previous solar maximum to determine the climatology of intensity fluctuations on the L2 and L5 carriers and the cross-correlation of intensity fluctuations between them. As expected, we find a marked decrease in correlation between the GPS carrier signals as the scintillation strength increases. This suggests engineering approaches which utilize frequency diversity may be able to partially mitigate the effects of scintillation when it is most needed, e.g. when the scintillation is strong. Moreover, we find a simple relationship between the intensity correlations and the L1 $S_4$ index which holds irrespective of the propagation geometry. This relationship provides a simple way to predict the intensity correlations for use in scintillation trade studies.

1. INTRODUCTION

Ionospheric scintillations are fluctuations in the intensity and phase of satellite signals caused by scattering from irregularities in the distribution of electrons encountered along the radio propagation path. The occurrence morphology of ionospheric scintillation depends on longitude, local time, season, solar cycle, magnetic activity, and exhibits a high degree of night-to-night variability (Aarons 1982; Aarons 1993). A number of studies have characterized the climatology of scintillations on the GPS L1 carrier signal (Cervera and Thomas, 2006; Steenburgh et al., 2008; Béniguel et al., 2009; Akala et al., 2011; Paznukhov et al., 2012). Several investigations have quantified the detrimental impacts of scintillation on GPS tracking performance and positioning accuracy (Groves et al, 2000; Doherty et al., 2000; Carrano et al. 2005; Carrano et al., 2010; Seo et al., 2009; Seo et al; 2011; Akala et al., 2012).

The desire to exploit emerging new GPS signals for civilian applications has recently raised concerns about the characteristics of ionospheric scintillation effects across the entire GPS frequency band spanning the L5–L1 signals (1176 MHz–1575 MHz). The new signals feature more power and higher precision codes than previously available and their performance in the presence of moderate to strong scintillation is currently under investigation. A few recent studies have been conducted to monitor ionospheric scintillation with these new signals (Bougard, et al.; 2011; Crowley et al. 2011; Shanmugam et al, 2012). Gherm et al (2011) used phase screen techniques to investigate the correlation of phase measurements on L1 and L2 during scintillation activity. Sreeja et al. (2012) reported on the effects of scintillation on GNSS tracking performance for the GPS L1C/A, L2C, and GLONASS L1, L2 signals. Despite these useful studies, the frequency dependence of scintillation across the full L5-L1 bandwidth, a frequency span of 30% (relative to the L5 frequency), has not been examined in detail.

One approach to explore these issues is to collect new data, recording the desired signals under real-world scintillation conditions and analyzing the results. In this paper, we present some recent observations of scintillations on the L1C/A and L2C signals acquired in Brazil. The L2C signal is currently broadcast on PRNs 01, 05, 07, 12, 15, 17, 25, 29, and 31. The L5 signal is currently broadcast only on PRNs 01 and 25. Because the new signals are currently available on a limited number of satellites, and the solar flux is currently below historical peak levels, it is not possible to explore the full range of scintillation conditions with current measurements.

In this paper, we study scintillation characteristics across the GPS frequency band by utilizing data sets collected during the last solar maximum period. This data consists of 50 Hz observations of the GPS L1 signal collected during peak scintillation activity periods at Ascension Island in 2002. This data provides ample statistics for the whole spectrum of scintillation activity anticipated during a healthy solar cycle. Of course, neither the L2C or L5 signals were available at that time, and the semi-codeless receivers used at the time were frequently unable to track the L2 signal during strong scintillation. To leverage these past observations, a method is needed to infer the scintillation characteristics on L2 and L5 from scintillation measurements on L1.

A formal theory for investigating the frequency dependence of intensity scintillations has been available for some time (Yeh and Liu, 1982; Bhattacharyya and Yeh, 1988). The two-frequency single-point intensity correlation function may be determined by solving the stochastic differential
equation governing the 4th moment of the electric field. We apply this technique to the L1-L5 GPS carrier signals in this paper. The procedure is outlined in the Appendix. The disadvantages of this approach are as follows. First, the statistical characteristics of the ionospheric irregularities must be modeled. Second, this approach provides only the ensemble average of the correlation functions and not individual realizations of the complex amplitude on the carrier signals. These realizations, and not just the average behavior, are needed to experiment with different metrics of simultaneous fading on the different carriers.

To remedy these shortcomings, we developed a high-fidelity modeling approach to infer the L2 and L5 signal characteristics from the measured complex (amplitude and phase) L1 signal fluctuations. In this approach, we back-propagate the measured complex L1 signal up to ionospheric altitudes to determine an equivalent ionospheric phase screen, and then propagate simulated signals at the L1, L2 and L5 carrier-frequencies back down to the ground. Since the phase screen is determined from the L1 measurements, we refer to this technique as the deterministic phase screen method. The merits of this method are described by Carrano et al. (2012b). A principal advantage of the method is that few assumptions regarding the statistics of the ionospheric irregularities need be made. Instead, the ionospheric screen is derived directly from the scintillation measurements themselves. Another advantage is that this method is applicable to real-world scintillation measurements which may be statistically non-stationary, and hence unsuitable for traditional spectral analysis.

Because the back-propagation technique requires high rate complex data as input, it cannot be applied to the large extant datasets of scintillation measurements for which only the signal moments (e.g. $S_4$) are available. To address this issue, we also present an alternative phase screen technique that uses only $S_4$ (and the propagation geometry) as input. Since in this case the phase screen is derived from a stochastic model of the irregularities, we refer to this approach as the stochastic phase screen method (Carrano et al., 2012b). After demonstrating that the stochastic and deterministic phase screen approaches yield similar results for the intensity correlations, we analyze large amounts of data from the previous solar maximum. In this way, we are able to report on the climatology of scintillations on L2 and L5 and the correlation of intensity fluctuations on each pair of carrier signals. As anticipated, we find a marked decrease in correlation between the GPS carrier signals as the scintillation strength increases. This suggests engineering approaches which utilize frequency diversity may be able to partially mitigate the effects of scintillation when the scattering is strong. The reasoning is that the decorrelation of intensity with frequency might allow a receiver to maintain lock on at least one carrier for much of the time, even while other carriers experience deep fades.

We note that a recent study by Seo et al. (2011) investigated the potential for using the geometric diversity of the satellites to mitigate the scintillation impacts on aviation, depending on the correlation level of deep fades between satellites. In this work, we investigate the potential for using frequency diversity to mitigate scintillation impacts instead. In this case, we are interested in the correlation level of deep fades between the different frequency carrier signals broadcast from the same satellite.

2. SCINTILLATIONS OBSERVATIONS ON L1C/A AND L2C IN BRAZIL

With funding and support from the Federal Aviation Administration (FAA), Boston College and National Institute for Space Research (INPE) in Brazil have collaborated to collect scintillation observations on the GPS modernization signals L2C and L5. In April 2012, 50 Hz samples of L1C/A and L2C intensity and phase were collected using a Septentrio PolaRxS Pro GNSS receiver located at the INPE headquarters in São José dos Campos, Brazil (23.2°S, 45.9°W, 17.5°S dip latitude). São José is located near the southern crest of the equatorial anomaly, where the strongest scintillations are generally observed globally. We did not collect scintillation observations on the L5 signal during this time, because an L1/L2 only GPS antenna (the NovAtel 702GG model antenna) was inadvertently used to collect the data.

Figure 1 shows an example of moderately strong scintillations on L1C/A and L2C observed for GPS
satellite PRN01 on 1 April 2012 from São José dos Campos. The figure shows the level of signal intensity ($SI$) fluctuations, calculated from the post-correlator samples of in-phase ($I$) and quadrature ($Q$) components of the recorded signals, as $SI=I^2+Q^2$. The intensity of each signal has been detrended (normalized) by dividing it by a smoothed version of itself calculated using a sliding 60 second boxcar average. The resulting signals have unit mean intensity, e.g. $<SI>=1$.

Figure 1. Intensity fluctuations on L1 and L2C for PRN01 observed at São José on 1 April 2012.

The scintillation index, $S_4$, is calculated as the standard deviation of detrended power:

$$S_4 = \sqrt{\langle SI^2 \rangle - 1}$$

where it is reiterated that the signals have unit mean intensity so that $<SI>=1$. Next, we define the intensity correlation between frequencies $i$ and $j$ as the Pearson cross-correlation coefficient between the detrended intensity fluctuations observed on the two carrier signals:

$$Cor(i,j) = \langle SI_i \cdot SI_j \rangle$$

A 60 second time average was used in place of the ensemble average implied the $<\cdot>$ operator in (1)-(2).

Figures 2 and 3 show the intensity fluctuations on L1 and L2C for a 120 second period exhibiting moderate scintillations (L1 $S_4=0.48$) and strong scintillations (L1 $S_4=0.87$), respectively. When the scintillations are moderate (e.g. Figure 2) or weaker the L1 and L2C intensity fluctuations are highly (but not perfectly) correlated. In this example, the correlation between intensity fluctuations on L1 and L2C is 0.86. When the scintillations are strong (e.g. Figure 3), the intensity fluctuations decorrelate between the two frequencies. In this example, the cross-correlation between the intensity fluctuations on L1 and L2C is 0.56. Lower frequency signals experience stronger scintillations than do higher frequency signals passing through the same ionospheric irregularities. Therefore, L2C generally experiences deeper and more frequent fades than L1. However, some deep fades on L1 are not accompanied by similar fades on L2C. This decorrelation between fades on the two frequencies suggests that a receiver capable of tracking L1 and L2C independently might be able to maintain lock on at least one of these two signals for more of the time, thereby resulting in fewer GNSS navigation outages. The Septentrio receiver did lose lock on both L2C and L1 multiple times during the measurement period shown in Figure 1.

Figure 2. Moderate scintillations observed on L1 (red) and L2C (green) for PRN01 at São José.

The Seo et al. (2011) study identified some potential problems with the interpretation of the Pearson cross-
correlation coefficient of intensity between signals from different satellites. Specifically, they found that this definition of the correlation can be relatively insensitive to deep fading of the signals. We noted this in our investigations also. For example, consider the scintillation observations at São José dos Campos shown in Figure 4. During the first 60 seconds shown the scatter is weak \( L1 S_4 = 0.4 \), whereas during seconds 60-120 the scatter is strong \( L1 S_4 = 1.0 \). The strong scatter data clearly shows uncorrelated signal fading on L1 and L2C, which is precisely the effect we wish to quantify. The problem is that the Pearson correlation coefficient between the L1 and L2C intensities is actually higher on the strong scatter side (0.79) than it is on the weak scatter side (0.74). If we examine the intensity fluctuations in linear units, rather than dB (not shown), the explanation for this is simple. There are strong signal enhancements (due to focusing) that occur more or less simultaneously on both the L1 and L2C signals, and these enhancements dominate the correlation coefficient at the expense of the contribution from the deep fades.

Figure 4. Weak and strong scintillations observed on L1 (red) and L2C (green) for PRN01 at São José.

Seo et al. (2011) proposed alternative metrics for the correlation of fades between satellite signals. Here, we employ a different approach and calculate the correlation between intensities on both linear and logarithmic (dB) scales. We define the logarithmic correlation between the intensities of signals \( i \) and \( j \) as follows:

\[
LogCor(i, j) = \left( 10 \log_{10} (SI_i) \cdot 10 \log_{10} (SI_j) \right)
\]  

In the example shown in Figure 4, the logarithmic correlation is smaller (0.69) on the strong scatter side (seconds 60-120) than it is (0.81) on the weak scatter side (seconds 0-60). The logarithmic correlation is more sensitive to deep signal fades than is the linear correlation.

Figure 5 shows how the correlation between L1 and L2C varies as a function of the \( S_4 \) on L1 during the period shown in Figure 1. When the L1 \( S_4 \) is low, the correlation coefficient is dominated by receiver noise, which (from Figure 5) appears to be uncorrelated between these two carriers. While there is considerable spread in the data, it is clear that the intensity correlation between L1 and L2C decreases as the scintillation strength (\( S_4 \)) increases. Figure 6 shows the logarithmic intensity correlation for L1 and L2C, the spread of which is somewhat less than when the correlation is calculated in linear scaling.

Figure 5. L1/L2 intensity correlation versus L1 \( S_4 \) for PRN01 at São José on 1 April 2012.

Figure 6. L1/L2 logarithmic intensity correlation versus L1 \( S_4 \) for PRN01 at São José on 1 April 2012.
3. SIMULATION USING DETERMINISTIC PHASE SCREENS

While it is currently possible to directly measure the intensity correlation between the L1, L2, and L5 carriers during scintillation activity, the strength of scintillations in the current solar cycle is not as strong as it was in the previous. Therefore, we will use measurements of signal fluctuations on L1 from the previous solar cycle to infer the fluctuations that would have been measured on L2C and L5, had these signals been broadcast at the time. To do this, we must make the following assumptions. First, we assume that refractive effects on the propagating wavefronts due to ionospheric irregularities can be adequately represented by a thin phase changing screen suitably located at ionospheric altitudes. Second, we assume that amplitude fluctuations which accumulate within the random ionospheric medium can be neglected. Third, we assume that the one dimensional phase structure captured in the measurement plane can be used as an equivalent 1D phase screen model. Lastly, we assume that we can neglect the signal fluctuations due to receiver noise.

The transmitted GPS carrier signal is approximated by the spherical wave $U_0(x,z) = A_0 \exp(i 2\pi R/\lambda)/R$, where $A_0$ is the amplitude (which we will take as unity), $\lambda=c/f$ is the radio wavelength, and $R$ is the distance from the transmitter (TX). The time dependence of the signal is omitted for clarity, but is implied. In the Fresnel approximation, the complex amplitude at the receiver after passage through the phase screen can be expressed as (Bernhardt et al., 2006; Carrano et al., 2012b):

$$U\left(\frac{d_1}{d_1+d_2} x, z_{RX}\right) = U_0\left(\frac{d_1}{d_1+d_2} x, z_{RX}\right) D(x)$$  (4)

where $D(x)$ is the ionospheric transfer function

$$D(x) = F^{-1}\left\{ \exp\left(\frac{i \kappa^2 d_s}{2k}\right) F[U_0(x)](\kappa) \right\}$$  (5)

In the above, $x$ is the horizontal distance (parallel to the ground), $z_{RX}$ is the vertical coordinate of the receiver, $d_1$ is the slant distance from TX to the IPP, $d_2$ is the distance from the IPP to RX, $U_0(x)$ is the complex amplitude of the wave at the phase screen (immediately after passage through it), $k=2\pi/\lambda$ is the signal wavenumber, and $F$ is the horizontal Fourier transform as a function of the horizontal wavenumber $\kappa$. The reduced propagation distance appearing in (5)

$$d_s = \frac{d_1 d_2}{d_1 + d_2}$$  (6)

accounts for the spherical spreading of the wavefronts as the radio wave propagates. It can be shown the result (4)-(5) is a solution of the parabolic wave equation in the region of free-space propagation beyond the phase screen.

A receiver on the ground measures a scaled version of the function $D(x)$. Equation (5) can be inverted to infer the complex amplitude of the wave at the altitude of phase screen:
\[ U_i(x) = F^{-1} \left\{ \exp \left[ -i \frac{k^2 d_R}{2k_i} \right] F \left[ D(x) \left( \frac{x}{k_i} \right) \right] \right\} \] \hspace{1cm} (7)

In general, the altitude of the screen (upon which \( d_R \) implicitly depends) is unknown. However, there is generally some altitude \( H_s^* \) for which the variance of \( |U_i| \) is minimized. We take \( H_s^* \) as the altitude of the screen, and retrieve the phase in the screen by taking the argument of \( U_i(x) \), i.e. \( \varphi(x) = \arg[U_i(x)] \). This procedure is referred to as back-propagation of the wave up to ionospheric heights (Bernhardt et al., 2006). It can be shown that if amplitude fluctuations vanish at the altitude of the screen, i.e. \( \text{Var}(|U_i|) = 0 \), then propagation through this equivalent phase screen produces the same diffraction pattern on the ground as the actual ionospheric irregularities do, even though the latter may be extended in altitude. In practice, the amplitude fluctuations in \( U_i \) do not vanish, however, because amplitude fluctuations do in fact develop while the radio wave propagates in the random medium. In any case, we must neglect these amplitude fluctuations in order to scale the phase screen from one signal frequency to another, as explained below.

For radio signals with frequencies at VHF and higher, the purely refractive phase change due to propagation through the ionosphere scales linearly with the wavelength of the signal. If the phase is \( \varphi_0(x) \) at some reference wavenumber \( k_0 \), then the phase at some other wavenumber \( k_i \) is given by:

\[ \varphi_i(x) = k_i \varphi_0(x) \] \hspace{1cm} (8)

We emphasize that (8) holds only for the refractive effects, and not for diffraction effects which do not scale linearly with the wavelength.

Now suppose we take, as a reference, the wavenumber of the GPS L1 carrier signal at 1575.42 MHz. Then using equations (4)–(8) we can predict the complex amplitude of the signals at L1, L2, or L5 as follows:

\[ U_i(x) = F^{-1} \left\{ \exp \left[ i \frac{k^2 d_R}{2k_i} \right] F \left[ \exp \left[ i \frac{k_0}{k_i} \varphi_0(x) \right] \right] \right\} \] \hspace{1cm} (9)

where \( i = 1, 2, \) or 5 is used to indicate the L1, L2 or L5 carrier signal. In (9) we have neglected the amplitude modulation imposed by \( U_0 \), which is negligible for the short time series and long trans-ionospheric propagation distances considered here. We note that \( U_1 \) predicted by (9) is not identical to the measured L1 signal, in general, because we have neglected amplitude fluctuations in the back-propagated signal in order to scale the screen phase for each carrier frequency.

A receiver on the ground measures temporal fluctuations, rather than spatial ones, as the diffraction pattern implied by (4)-(5) sweeps past the receiver. Assuming the random medium is invariant over the measurement interval (this is the Taylor hypothesis of frozen-in flow), spatial fluctuations and temporal fluctuations can be related by a model-dependent effective scan velocity as \( \Delta x = v_{eff} \Delta t \) (Carrano et al., 2012a). In general, determination of \( v_{eff} \) is a rather involved process which requires knowledge of the IPP scan velocity, irregularity drift velocity, and magnetic field orientation (Rino et al., 1979). However, \( v_{eff} \) enters in (7) and (9) only through the horizontal wavenumber \( k \), which appears as a product with \( d_R \). Therefore, if we back-propagate the L1 signal measured on the ground to determine the ratio \( d_R/v_{eff}^2 \), and then forward-propagate signals at L1, L2, and L5 along the same path with this ratio fixed, the effective scan velocity need not be known separately from \( d_R \).

To demonstrate the back-propagation technique, we use 50 Hz measurements of the complex amplitude for the L1/C/A signal from PRN04 measured on the evening of 13 March 2002 at Ascension Island (7.98°S, 14.4°W, 15°S dip latitude). This dataset exhibited very strong scintillations typical of an equatorial anomaly site during solar maximum conditions. We constructed the ionospheric transfer function \( D(x) \) from post-processed measurements of the signal intensity \( S_{Im} \) and phase \( \varphi_m \) as

\[ D(x) = \sqrt{S_{Im}(x)} \exp \left[ i \varphi_m(x) \right] \] \hspace{1cm} (10)

The post-processing steps used to obtain \( S_{Im} \) and \( \varphi_m \) from the observations were as follows. The signal intensity was detrended by dividing it by a smoothed version of itself calculated using a sliding boxcar
average of length 60 sec. The phase was detrended by repairing cycle slips and then applying a 6th order Butterworth high-pass filter with 3dB cutoff at frequency 1/60 sec. The total electron content, calculated using the technique described by Carrano et al. (2009) and sampled at a rate of 60 seconds per sample, was added to the detrended phase. This last step was performed because it improved the accuracy of the phase screen calculations when the scatter was sufficiently strong that large scale structures in the screen generated small scale structures on the ground through focusing effects.

The scintillation index of the back-propagated wave is plotted as a function of the assumed altitude of the screen in Figure 8. A clear minimum is evident at the altitude $H_s^* = 356$ km. We take this as the altitude of the phase screen in our subsequent propagation calculations. The fact that the minimum value of $S^4 = 0.47$ achieved during the back-propagation step is greater than zero suggests that amplitude fluctuations developed while the radio wave traveled in the extended random medium. In any case, we discard these amplitude fluctuations and take $\arg[U_\delta(x)]$ as the screen phase. This results in some error in the simulated L1 intensity, as mentioned previously. Figure 9 shows the measured L1 intensity, the simulated L1 intensity, and the correlation between them. This correlation ranged from 0.86-0.99, which suggests the error incurred from neglecting the amplitude fluctuations in the screen is relatively small.

Figure 8. $S^4$ index of the back-propagated signal $U_\delta(x)$ from PRN04 at Ascension Island versus altitude of the screen. The vertical dotted line indicates the optimal screen altitude, $H_s^* = 356$ km.

Figure 9. Measured (top) and simulated (middle) L1 signal intensity for PRN04 at Ascension Island. Also shown is the correlation between them (bottom).

Figure 10 shows the simulated intensity fluctuations on L1, L2, and L5 for this case, which we will use to study the correlation of fading between the carrier signals. We note that the actual L2 signal for this case was recorded using semi-codeless techniques, and is not of sufficient quality to validate the simulation results. The L5 signal was not broadcast at the time these data were collected. The purpose of our simulation technique is that it allows us to estimate the correlation of intensity fluctuations between the L1, L2, and L5 carriers in the absence of actual scintillation measurements on each carrier.

Figure 10. Simulated intensity fluctuations on L1, L2, and L5 for PRN04 at Ascension Island.
Figure 11 shows the simulated intensity fluctuations on the L1, L2, and L5 carriers during a 30 second period when the scintillation conditions were moderate (L1 $S_4=0.61$, L2 $S_4=0.84$, L5 $S_4=0.86$). The intensity correlations between the carriers were $Cor(1,2)=0.84$, $Cor(1,5)=0.79$, and $Cor(2,5)=0.99$. Since these correlations are high in this case, the fading of all three carriers occurred nearly simultaneously, and we do not expect GPS processing techniques which leverage frequency diversity to provide much benefit.

The intensity correlation between the carrier pairs in Figure 12 are $Cor(1,2)=0.45$, $Cor(1,5)=0.37$, and $Cor(2,5)=0.93$. The first two correlations are significantly lower than in the case of moderate scintillation (Figure 11), while the correlation between L2 and L5 remains high, even during strong scintillation, by virtue of the relatively small frequency separation between these carriers. From Figure 12, it is clear that there are fades on L1 which are not accompanied by fades on L2 and vice-versa. In this case, we do expect that a receiver with independent tracking of each carrier may provide a more accurate navigation solution, since there is high probability that at least one carrier may have high signal strength while others are experiencing deep fades.

Figures 13 shows how the intensity correlations between carriers vary as a function of the $S_4$ index measured on the L1 carrier for PRN04 at Ascension Island. Figure 14 shows how these correlations vary as a function of $C_iL$, which is the vertically integrated irregularity strength at the 1 km scale (Carrano et al., 2012a; Carrano et al. 2012b). In section 4 we explain how we infer $C_iL$ from the measurements. Characterizing the strength of scatter using $C_iL$, rather than $S_4$, can be advantageous because the former is a property of the random medium alone, whereas the latter depends on the random medium, propagation geometry, and frequency of the radio wave. The trend shown in Figures 13 and 14 is clear; the intensity correlation between carriers decreases as the scintillation increases. It is possible to show on theoretical grounds that the limiting value of the correlations (when the scatter is weak) is independent of the perturbation strength and depends only on the frequency separation, propagation distance, and ionospheric slab thickness (Bhattacharyya and Yeh, 1988).
results, and both reproduce the average behavior of the correlations from the deterministic phase screen model (shown using asterisks). The deterministic phase screen results do show considerable spread about the average behavior, however.

4. SIMULATION USING STOCHASTIC PHASE SCREENS

In the deterministic phase screen technique, raw (50 Hz) measurements of intensity and phase on L1 were used to infer an equivalent phase screen via the back-propagation technique. While this method is expected to yield the most accurate simulation results, here we present an alternative technique that uses only the scintillation statistics (e.g. $S_4$) as input, rather than the raw measurements. Since this approach uses the statistics, and not the raw measurements, to infer an equivalent phase screen, we refer it as the stochastic phase screen technique. After demonstrating that the stochastic and deterministic techniques yield similar results for the intensity correlations, we proceed to analyze extensive datasets for which the raw measurements are not available. In this way, we are able to infer the climatology of scintillations on L2 and L5, and also the climatology of the intensity correlations, by leveraging our extensive database of L1 $S_4$ observations.

Equation (9) is quite general; it gives the complex amplitude of the wave on the ground after passage through an arbitrary phase screen $\phi_0(x)$. In this section, rather than using a deterministic screen derived from back-propagation, we use stochastically generated phase screens, which are realizations of the following one-dimensional power-law spectrum with an outer-scale, $L_0$:

$$\phi(\kappa) = \frac{T}{\left(\kappa_0^2 + \kappa^2\right)^{\nu/2}}$$ \hspace{1cm} (11)

In (11) the strength of the phase spectrum, $T$, is given by

$$T = \frac{1}{2} \pi^{3/2} r_e^2 \lambda^2 GC_e L \sec \theta \left(\frac{2\pi}{1000}\right)^{\nu+1} \frac{\Gamma(e)}{\Gamma(e + \frac{1}{2})}$$ \hspace{1cm} (12)

In (11) and (12), $r_e$ is the classical electron radius ($2.8179 \times 10^{-15}$ m), $C_L$ is the vertically integrated

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Figure 13. L1 $S_4$ versus L1/L2 intensity correlation (red), L1/L5 intensity correlation (green), and L2/L5 intensity correlation (blue) for PRN04. Correlations from the deterministic phase screen model are shown with asterisks. Correlations from the stochastic phase screen model are shown as dotted lines, while the two-frequency correlations determined from the $4^{th}$ moment solution are shown as solid lines.

Figure 14. $C_L$ versus L1/L2 intensity correlation (red), L1/L5 intensity correlation (green), and L2/L5 intensity correlation (blue) for PRN04. Correlations from the deterministic phase screen model are shown with asterisks. Correlations from the stochastic phase screen model are shown as dotted lines, while the two-frequency correlations determined from the $4^{th}$ moment solution are shown as solid lines.

The solid curves shown in Figures 13 and 14 are two-frequency single point correlations determined by solving the stochastic differential equation governing the $4^{th}$ moment of the field, as described in the Appendix. The dotted curves show the results of a stochastic phase screen simulation which will be discussed in the next section. Both stochastic simulations approaches produce essentially the same
strength of the irregularities at the 1 km scale, $p$ is the phase spectral index, $\kappa_0=2\pi/L_0$ is the outer scale wavenumber, $\Gamma$ is Euler’s gamma function, and $G$ is the phase enhancement factor (Rino, 1979). In the calculations described in this and the next section, we use $p=2.5$ and $L_0=10$ km. Furthermore, we assume the screen is located at 350 km in altitude and that the axial ratio of the irregularities along and across magnetic field lines is 50:1.

We note that spectral form given by (11) and (12) appears in (Rino, 1979) as the result of a 1-D scan through a fully 3-D structure and propagation model. Here, we use this result to define a 1-D phase screen and perform a 2-D propagation calculation which is not mathematically equivalent to the fully 3-D treatment given by Rino.

Figure 15. Simulated intensity fluctuations at L1, L2, and L5 for PRN04 using the stochastic phase screen method.

Figure 15 shows an example of intensity fluctuations at L1, L2, and L5 for PRN04 simulated using the stochastic phase screen method. The propagation and geometry parameters are identical to the simulation using back-propagation (Figures 9-10), except that a fixed value of $C_L=4\times10^{13}$ is used. Note that the scintillation time series produced by this technique are stationary, unlike the scintillation observations shown in Figures 9 (black curve). This stationarity is a consequence of imposing a finite outer scale on the turbulence. Nevertheless, an advantage of this stationarity is that we can generate arbitrarily smooth statistics by increasing the length of the phase screen.

For this simulation, L1 $S_4$=0.78, L2 $S_4$=0.92, L5 $S_4$=0.93. In this case, the L2 and L5 signals are close to saturation, while the L1 signal is not saturated. The intensity correlations between the carriers is Cor(1,2)=0.52, Cor(1,5)=0.44, and Cor(2,5)=0.86.

We can adapt the stochastic simulation method to match the level of scintillations observed if the data be used to specify $C_L$. Rino and Fremouw (1977) and Rino (1979) showed that for propagation through a thin layer of ionospheric irregularities in the Born approximation (single scatter), the $S_4$ index can be expressed as:

$$S_4^2 = \pi^2 \lambda^2 C_L \sec \theta \left( \frac{2\pi}{1000} \right)^{2v+1} \left( \frac{\lambda d_0}{4\pi} \right)^{v-\frac{1}{2}} \frac{\Gamma((2.5-v)/2)}{2\sqrt{\pi} \Gamma((v+0.5)/2)(v-0.5)}$$

(13)

where $\phi$ is a combined geometry and propagation factor given in equation 34 of Rino [1979]. The spectral slope, $\nu$, is defined such that $p=2\nu$ is the phase spectral index. Equation (13) is only applicable to weak scatter. No closed form solution exists to predict $S_4$ under strong scatter conditions. Nevertheless, one may apply an empirical correction to this weak scatter result, implied by Rice statistics, to approximately account for saturation effects caused by multiple scatter in the absence of strong focusing (Fremouw and Secan, 1984; Cervera and Thomas, 2006):

$$S_4^2 \approx 1 - \exp(-S_4^2)$$

(14)

where $S_{4w}$ is the weak scatter result given in (13) and $S_4$ is the measured value. We use (13) and (14) to infer the value of $C_L$ from measurements of $S_4$ on the L1 carrier. We note that a more robust, but significantly more computationally intensive, method to infer $C_L$ from the measurements is available (Carrano et al., 2012a). Evaluating $C_L$ from (13) and (14) is much simpler, but yields only approximate results since actual scintillation measurements deviate from Rice statistics. Nevertheless, the intensity correlations we derive using this approach agree reasonably well with the observations.

As mentioned earlier, the dashed lines in Figures 13 and 13 show the intensity correlations predicted using
the stochastic phase screen approach. To produce these results, we subdivided the data into 60 second intervals and calculated the value of $C_{kL}$ from the measured $S_4$ during each interval. For comparison, the solid lines in Figures 13 and 14 show the intensity correlations predicted by solving the stochastic differential equation for the 4th moment, as described in the Appendix. As is evident from the plots, there is considerable spread in the correlations predicted by the deterministic phase screen approach, but the two stochastic approaches do a reasonable job at reproducing the mean behavior. The stochastic approaches do appear to under-predict the L2/L5 correlation to some degree, however.

5. CLIMATOLOGY OF THE CORRELATIONS

In the previous section, we presented a methodology to characterize the intensity correlation over the GNSS band from $S_4$ measurements on L1 and the propagation geometry alone (i.e. without requiring high-rate samples of intensity and phase as input). Since the climatology of $S_4$ on L1 is relatively well established, we can now provide the climatology of $S_4$ on the L2 and L5 carriers and also the expected intensity correlations between the carrier pairs.

Figure 16 shows the percentile of occurrence for $C_{kL}$ shortly after sunset (between 21-23 UT) at Ascension Island in March 2002 when the monthly sunspot number was 98. The $C_{kL}$ values were calculated from the propagation geometry and measured L1 $S_4$ values during 60 second intervals, as described in Section 4. Only data for satellites above 30° in elevation were included in the analysis to minimize the influence of multipath on the $S_4$ measurements. The interpretation of the percentile plot shown in Figure 16, and in subsequent percentile plots, is as follows. The percentile shown gives the percentage of time for which the value was less than or equal to the value shown. For example, Figure 16 shows that 20% of the time $C_{kL}$ was $10^{-35}$ or less, and $C_{kL}$ was always less than $5\times10^{-36}$.

Figure 17 shows the percentile of occurrence for $S_4$ on the L1, L2, and L5 carrier signals predicted by the stochastic phase screen model between 21-23 UT in March 2002. As expected, L5 and L2 experience a given level of $S_4$ more frequently than does L1, because the scintillation intensity is stronger for lower frequency signals. At the 90th percentile, the scintillations on all three carriers are saturated.

Figure 18 shows the percentile of occurrence for the intensity correlations predicted by the stochastic phase screen model for March 2002. The solid lines show the linear correlations ($Cor$), while the dashed lines show the logarithmic correlations ($LogCor$). At the 40th percentile the intensity correlations are barely changed from their respective weak scatter values. The intensity correlations begin to decrease rapidly at after the 40th percentile, which suggests that scintillation mitigation schemes utilizing frequency diversity may be expected to provide benefit 40% of the time under similar scintillation conditions (e.g. similar local time, magnetic latitude, and sunspot number). As shown in Figure 18, 50% decorrelation
between the L1/L2, L1/L5, and L2/L5 carrier pairs occurs approximately 14%, 18%, and 8% of the time, respectively. The kink in the curves at the 10th percentile is due to uncertainty in the retrieval of $C_L$ when the L1 $S_4$ is close to saturation.

Figure 18. Percentile of occurrence for the L1/L2 (blue), L1/L5 (red), and L2/L5 (orange) intensity correlations. Solid and dashed lines indicate linear and logarithmic correlations, respectively.

Figure 19 shows a scatter plot of the L1/L2, L1/L5, and L2/L5 intensity correlations versus the irregularity strength $C_L$ for all data in March 2002 between 21-23 UT. The intensity correlations decline with increasing $C_L$ once the latter is sufficiently large to cause L-band scintillations, but considerable scatter is evident. We believe this scatter is principally caused by propagation geometry effects, which are not reflected in $C_L$ since it is a property of the random medium alone.

Figure 19. L1/L2 (blue), L1/L5 (red), and L2/L5 (orange) intensity correlations versus $C_L$.

Figure 20 shows a scatter plot of the L1/L2, L1/L5, and L2/L5 intensity correlations versus the $S_4$ index measured on the L1 carrier for all data in March 2002 between 21-23 UT. The relatively small scatter in the correlations about their mean values suggests they are essentially dictated by the value of L1 $S_4$, irrespective of the propagation geometry. This is result is very convenient, since the L1 $S_4$ is easy to measure.

We conjecture that the relationship between L1 $S_4$ and the intensity correlations is simple because both are controlled by the same physical parameters, namely the strength of scatter, the slant path distance through the irregularities, and the propagation distance past the screen. While the difference in frequency between L1 and L5 is substantial (30%), the difference in Fresnel scale $(2\lambda d_R)^{1/2}$ is only 15% since this is proportional to the square root of the wavelength. Note that the scatter in the L1/L5 correlation is generally larger than the scatter in the other two correlations, when plotted against the L1 $S_4$, presumably because the difference in Fresnel scale is largest in this case. While the intensity correlation vs. L1 $S_4$ curves are relatively insensitive to the propagation geometry, they may depend on our assumed form for the irregularity spectrum. When additional measurements become available on L2C and L5, we will verify the extent to which this simple relationship between the correlations and L1 $S_4$ gives the correct average behavior.

We note from Figure 20 that the L1 $S_4$ values corresponding to 50% intensity decorrelation in the

Figure 20. L1/L2 (blue), L1/L5 (red), and L2/L5 (orange) intensity correlations versus L1 $S_4$. 
L1/L2, L1/L5, and L2/L5 carrier pairs are equal to 0.76, 0.71, and 0.99, respectively. The L1 $S_4$ values corresponding to 50% decorrelation in the logarithmic intensity for the L1/L2, L1/L5, and L2/L5 carrier pairs are slightly less, namely 0.71, 0.66, and 0.97, respectively. These result suggests that the scintillation must be relatively strong (L1 $S_4$=0.71-0.76) before frequency diversity schemes are expected to benefit. Due to the close frequency separation between L2 and L5, decorrelation between them does not occur until scintillation levels on L1 approach saturation (L1 $S_4$=0.97-0.99).

Like the L1 $S_4$ index itself, the intensity correlations show a strong dependence on the level of solar activity. Figure 21 shows how the percentile of occurrence for the L1/L2 intensity correlation varies as a function of sunspot number. Figure 22 shows the corresponding result for the L1/L5 intensity correlation. During low sunspot number periods (e.g. October 2010) L1 scintillations are weak, and the L1/L2 and L1/L5 intensity correlations deviate significantly from their weak scatter values only about 1-2% of the time. Conversely, during high sunspot number periods (e.g. March 2000 and March 2002) the L1/L2 and L1/L5 intensity correlations deviate from their weak scatter values about 40% of the time.

These results suggest that scintillation mitigation schemes that utilize frequency diversity can be expected to provide significantly greater benefit during active portions of the solar cycle.

We note that a recent study by El-Arini et al. (2009) investigated the intensity correlation between L1 and L2 using a military dual-frequency GPS receiver (with access to the encrypted Y code on L2) at Thule, Greenland during the December solstice of 1989 (a solar maximum year). They determined a low correlation when L1 $S_4$ < 0.2, which we attribute to uncorrelated noise on the two carrier channels. For values of L1 $S_4$ exceeding 0.2, they found intensity correlation values close to 0.7, exhibiting little evidence of a dependence on L1 $S_4$ (once it exceeds this threshold value of 0.2). In light of our analysis, we interpret the correlation they found (0.7) be largely unchanged from its weak scatter value in the absence of contamination by receiver noise. These authors did not observe significant intensity decorrelation because the maximum L1 $S_4$ they observed was only 0.5. Figure 20 suggests that significant decorrelation only begins to occur for L1 $S_4$ values larger than this. A conclusion of their study is that frequency diversity techniques are unlikely to provide much help in mitigating scintillation due to the high latitude ionosphere because the correlation between L1 and L2 is high. We concur with this assessment regarding the high latitude ionosphere, since the scintillation must be stronger than is typical
for this region for frequency diversity techniques to provide benefit. Our results clearly show, however, that in the equatorial region, where the scintillation can be significantly more intense, substantial decorrelation between the carrier frequencies does occur. For receivers operating in the equatorial region during solar active periods, frequency diversity techniques could help mitigate the detrimental effects of scintillation on navigation accuracy.

6. CONCLUSIONS

We investigate the characteristics of ionospheric scintillation across the entire GPS frequency band spanning the L5–L1 carrier signals (1176 MHz–1575 MHz). Of particular interest is the intensity correlation between carrier pairs, since this dictates the extent to which frequency diversity may be leveraged to mitigate scintillation impacts on navigation accuracy.

All observations of scintillation on the new GPS modernization signals L2C and L5 have been made during the recent period of relatively modest solar flux. Therefore, it is not possible to explore the full range of scintillation conditions with current measurements. An extensive database of GPS scintillation observations from the previous solar maximum is available, but the tracking of L2 was generally performed using semi-codeless receivers which lose lock the L2 signal during strong scintillation. The L5 signal was not transmitted during the previous solar maximum period.

To remedy this problem, we developed simulation techniques to infer the L2 and L5 signal characteristics from scintillation observations on the L1 carrier collected during the previous solar maximum period. In the deterministic phase screen method, we back-propagate the complex signal observed on L1 to find an equivalent phase screen, and then propagate new waves through this screen at the L1, L2, and L5 carrier frequencies. In the stochastic phase screen method we use the $S_4$ index on the L1 carrier to predict L2, and L5 scintillations. Given realizations of the L1, L2, and L5 signal fluctuations obtained via simulation, we study the cross-correlations of intensity fluctuations between each pair of carriers. We show that the stochastic phase screen approach predicts the same intensity correlations as an alternative approach which solves the stochastic differential equations governing the 4th moment of the electric field. Moreover, we find that both stochastic approaches reproduce the average behavior of the intensity correlations predicted by the more computationally demanding deterministic phase screen approach. This enabled us to use the simpler stochastic phase screen approach to infer the climatology of the intensity correlations from our large extant database of L1 $S_4$ measurements collected during the previous solar maximum period at Ascension Island (7.98°S, 14.4°W, 15°S dip latitude).

As expected, we find that the intensity decorrelates with frequency as the scintillation strength on L1 increases. The L1 $S_4$ values corresponding to 50% decorrelation in the L1/L2, L1/L5, and L2/L5 carrier pairs were determined to be 0.76, 0.71, and 0.99, respectively. During low sunspot number periods (2010) L1 scintillations are weak, and the L1/L2 and L1/L5 intensity correlations deviate from their weak scatter values only about 1-2% of the time (between 21-23 UT) at Ascension Island. Conversely, during high sunspot number periods, the L1/L2 and L1/L5 intensity correlations deviate from their weak scatter values about 40% of the time (between 21-23 UT). Therefore, scintillation mitigation techniques which leverage frequency diversity can be expected to provide significantly greater benefit during active portions of the solar cycle.

These results suggest that engineering approaches which utilize frequency diversity may be able to partially mitigate the effects of scintillation when it is most needed, e.g. when the scintillation is strong ($L1 S_4 > 0.5$). The decorrelation of intensity with frequency might allow a receiver with independent tracking capabilities to maintain lock on at least one carrier for a larger percentage of time, resulting in improved GNSS navigation during scintillation activity. Moreover, we find that a simple relationship exists between the intensity correlations and the L1 $S_4$ index, irrespective of the propagation geometry. This relationship provides a straightforward way to predict the intensity correlations for use in scintillation trade studies.
We anticipate collecting additional scintillation observations on L2C and L5 later this year. These data will be used to validate the deterministic phase screen simulation technique and its predictions for the intensity correlations between carrier pairs. We will test the empirical relationship between the correlations and the $L_1 S_4$ index to establish the extent to which it can be applied universally (i.e. independently of the propagation geometry). We will also explore alternative metrics for describing the correlation of signal fades, such as those developed by Seo et al. (2011). We will also evaluate the intensity correlations for receivers located at the dip equator and at other longitudes with the goal of establishing the global climatology of GPS intensity correlations. We note that the techniques introduced in this paper can be easily applied to other GNSS satellite systems as well.

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APPENDIX

The 4th moment of the electric field describes the statistics of the complex wave amplitude, $U$, at four positions $(\xi)$ in the transverse plane:

$$
\Gamma_{2,2} = <U(\xi_1, z, k_1)U(\xi_2, z, k_2)U(\xi_3, z, k_3)U(\xi_4, z, k_4)> 
$$

(15)

For the case of plane wave incidence on a thin phase-changing screen with homogenous statistics, a change of variable can be used to write the following differential equation for 4th moment (Bhattacharyya and Yeh, 1988):

$$
\frac{\partial}{\partial d} \Gamma_{2,2}(x_1, x_2, z) = -i \frac{\partial^2}{\partial x_1 \partial x_2} \Gamma_{2,2}(x_1, x_2, z)
$$

$$
-\frac{1}{L} G(x_1, x_2) \Gamma_{2,2}(x_1, x_2, z) 
$$

(16)

where

$$
G(x_1, x_2) = (1 + r^2) R_{xy}(0) - R_{xy}(x_2) - r^2 R_{xy}(r x_2)
$$

$$
+ r [R_{xy}(x_1 + \frac{r}{2} x_2) - R_{xy}(x_1 + \frac{r}{2} x_2)]
$$

$$
- R_{xy}(x_1 - \frac{r}{2} x_2) + R_{xy}(x_1 - \frac{r}{2} x_2)]
$$

(17)

In (16), $L$ is the thickness of the scattering layer, and $d$ is the slant distance past the phase screen. In (17), $R_{xy}$ is the correlation function of phase fluctuations in the screen, and $r$ is the ratio of the wavenumbers of the two carrier signals, $r=k_1/k_2$.

Equation (16) can be solved via Fourier transform methods to give the 4th moment on the ground (Carrano and Rino, 2011):

$$
\Gamma_{2,2}(x_1, x_2) = \frac{1}{2 \pi} \left| \frac{1}{k_1} \right| K_{v-1/2}(\kappa_0 x) \frac{1}{2 \pi} \Gamma((p+1)/2)
$$

(18)

where $F$ denotes the 2D Fourier transform with respect to the transverse spatial separations $x_1$ and $x_2$. The reduced propagation distance, $d_R$, is used in (18) to account for the spherical spreading of the wavefronts. Note that the thickness of the scattering layer has canceled and does not appear in (18). The phase correlation function, consistent with the stochastic modeling approach described in Section 4, can be obtained by Fourier transformation of the phase power spectral density in (11) and (12):

$$
R_{xy}(x) = r^2 \lambda^2 GC_x L \sec \theta \left(\frac{2 \pi}{1000}\right)^{p+1}.
$$

(19)

where $K_{v-1/2}$ is the modified Bessel function of order $v-1/2$. Once $\Gamma_{2,2}$ is known the two-frequency, two-point intensity correlation function $R(x)$ can be calculated as:

$$
R(x) = \Gamma_{2,2}(x, 0, z) - 1
$$

(20)

The correlation in (20) is not normalized. We can compute the normalized intensity correlation for zero spatial separation by dividing $R(0)$ by the $S_4$ index for each signal, as follows:

$$
B(k_1, k_2) = R(0) /[S_4(k_1) S_4(k_2)]
$$

(21)

where the $S_4$ index for each frequency is determined by a separate evaluation of (18) at each frequency with $k_1=k_2=k$ and

$$
S_4^2(k) = \Gamma_{2,2}(0, 0, z) - 1
$$

(22)

Calculating the normalized intensity correlation coefficients $B(k_1,k_2)$ for the L1/L2, L1/L5, and L2/L5 carrier pairs requires 6 evaluations of (18), i.e. once for each frequency pair and once for each frequency alone to obtain the normalizations required. $B(k_1,k_2)$ can be compared directly with the Pearson correlation coefficients $Cor(i,j)$ described in the paper.