

Section 10.3 Taylor Polynomials

2. We compute the values of the function and its first five derivatives as follows:

$$f\left(\frac{\pi}{2}\right) = 1$$

$$f'\left(\frac{\pi}{2}\right) = \cos(x)\Big|_{x=\pi/2} = 0$$

$$f''\left(\frac{\pi}{2}\right) = -\sin(x)\Big|_{x=\pi/2} = -1$$

$$f'''\left(\frac{\pi}{2}\right) = -\cos(x)\Big|_{x=\pi/2} = 0$$

$$f^{(4)}\left(\frac{\pi}{2}\right) = \sin(x)\Big|_{x=\pi/2} = 1$$

$$f^{(5)}\left(\frac{\pi}{2}\right) = \cos(x)\Big|_{x=\pi/2} = 0$$

$$P_5(x) = 1 - \frac{(x - \pi/2)^2}{2} + \frac{(x - \pi/2)^4}{24}.$$

4. We compute the values of the function and its first six derivatives as follows:

$$f(1) = e$$

$$f'(1) = f''(1) = f'''(1) = f^{(4)}(1) = f^{(5)}(1) = f^{(6)}(1) = e^x \Big|_{x=1} = e$$

$$P_6(x) = e + e(x-1) + e \frac{(x-1)^2}{2} + e \frac{(x-1)^3}{6} + e \frac{(x-1)^4}{24} + e \frac{(x-1)^5}{120} + e \frac{(x-1)^6}{720}.$$

5. We compute the values of the function and its first five derivatives as follows:

$$f(0) = 1$$

$$f'(0) = (10x + e^x) \Big|_{x=0} = 1$$

$$f''(0) = (10 + e^x) \Big|_{x=0} = 11$$

$$f'''(0) = f^{(4)}(0) = f^{(5)}(0) = e^x \Big|_{x=0} = 1$$

$$P_5(x) = 1 + x + \left(\frac{11}{2}\right)x^2 + \left(\frac{1}{6}\right)x^3 + \left(\frac{1}{24}\right)x^4 + \left(\frac{1}{120}\right)x^5.$$

6. We compute the values of the function and its first five derivatives as follows:

$$f(0) = 0$$

$$f'(0) = 3 \cos(3x) \Big|_{x=0} = 3$$

$$f''(0) = -9 \sin(3x) \Big|_{x=0} = 0$$

$$f'''(0) = -27 \cos(3x) \Big|_{x=0} = -27$$

$$f^{(4)}(0) = 81 \sin(3x) \Big|_{x=0} = 0$$

$$f^{(5)}(0) = 243 \cos(3x) \Big|_{x=0} = 243$$

$$P_5(x) = 3x - \frac{9x^3}{2} + \frac{81x^5}{40}.$$

23. We compute the values of the function and its first four derivatives at $x = 1$ as follows:

$$f(1) = \frac{1}{2}$$

$$f'(1) = -\frac{x^2 - 1}{(x^2 + 1)^2} \Big|_{x=1} = 0$$

$$f''(1) = 2x \frac{x^2 - 3}{(x^2 + 1)^3} \Big|_{x=1} = -\frac{1}{2}$$

$$f'''(1) = -6 \frac{x^4 - 6x^2 + 1}{(x^2 + 1)^4} \Big|_{x=1} = \frac{3}{2}$$

$$f^{(4)}(1) = 24x \frac{x^4 - 10x^2 + 5}{(x^2 + 1)^5} \Big|_{x=1} = -3$$

$$\text{Therefore } P_4(x) = \frac{1}{2} - \frac{(x-1)^2}{4} + \frac{(x-1)^3}{4} - \frac{(x-1)^4}{8}.$$

24. We compute the values of the function and its first three derivatives at $x = 0$ as follows:

$$f(0) = 0$$

$$f'(0) = e^x(1+x) \Big|_{x=0} = 1$$

$$f''(0) = e^x(2+x) \Big|_{x=0} = 2$$

$$f'''(0) = e^x(3+x) \Big|_{x=0} = 3$$

$$\text{Therefore } P_3(x) = x + x^2 + \frac{x^3}{2}.$$