What you need to know from calculus for probability

January 22, 2010

1 One variable integration

Techniques of integration: substitution, parts, partial fractions, trigonometric substitutions. [include non-numerical problems] Improper integrals: unbounded integrands and infinite limits of integration. Second Fundamental Theorem of Calculus.

Find the following integrals.

a. $\int \frac{1}{x(1+\ln(x))^3}\, dx$

b. $\int \sin^5(x)\cos^2(x)dx$

c. $\int 4x\cos(x)dx$

d. $\int 3x^2e^{5x}dx$

e. $\int_0^\infty x\lambda e^{-\lambda x}dx$, where λ is a positive constant.

f. $\int_0^\infty x^2\lambda e^{-\lambda x}dx$

g. $\int_0^{\sqrt{\pi/2}} x\, \cos(x^2)\sin(x^2)dx$

h. $\int \frac{x+1}{3x^2+x^3}\, dx$

i. Find the following derivative: $\frac{d}{dx}\int_{x^2}^{x^3} e^{\cos(t)}\, dt$ (Do not try to find the integral first!)

j. Determine if the integral $\int_2^\infty \frac{1}{x\ln(x)}\, dx$ converges or diverges and show your work; if it converges, find its value.

k. Determine if the integral $\int_{-\infty}^\infty \frac{1}{1+x^2}\, dx$ converges or diverges and show your work; if it converges,

find its value.

l. Determine if the integral $\int_{-\infty}^{\infty} \frac{x}{1+x^2}\, dx$ converges or diverges and show your work; if it converges, find its value.

2 Series

Determine if the following series converge, **justifying your answers. Do not find the values of the series if they converge.** a. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

b. $\sum_{n=1}^{\infty} \frac{5n+3}{2n^4-6n}$

c. $\sum_{n=1}^{\infty} (5^{-n} + 4/\sqrt{n})$

Determine if the following series converge, **justifying your answers. Find the values of the series if they converge.**

d. $\sum_{n=0}^{\infty} (.8)^n$

e. $\sum_{n=3}^{\infty} (.8)^n$

f. $\sum_{n=2}^{\infty} \frac{4^{n-1}}{5^{n+2}}$

g. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

h. $\sum_{n=2}^{\infty} \frac{x^n}{n!}$

3 Multiple integration

a. Evaluate the following integral:

$$\int_0^1 \int_0^2 \frac{x+y}{8}\, dy\, dx$$

b. Evaluate the following integral:

$$\int_0^2 \int_0^{2-x} \frac{x+y}{8}\, dy\, dx$$

c. Evaluate the following integral after reversing the order of integration:

$$\int_0^4 \int_{\sqrt{y}}^2 \sin(x^3)\, dx\, dy.$$