Mathematical Problem Solving for Secondary Teachers
Spring 2014   MW3-4:15   235 Fulton

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Office Hours (Carney 365) TBA, or by appointment.

Prerequisites: Multivariable Calculus (MT202) and mathematical maturity of a junior mathematics major (MT216 or equivalent).

Web site: The web site for this course is www2.bc.edu/daniel-chambers then follow the link to MT455. It contains this syllabus, problem sets and solutions, plus handouts and other documents relating to the material we cover in class.

Text book: None. The problems which generate the mathematics in this course have been gathered from multiple sources (books, newspapers, internet, casual conversations,). In most cases, you will see the problems for the first time in class, and usually there will be a handout associated with the problem. As there is very little new "material"- the mathematics you'll be using is from the secondary level- we don't really need a text book. At the conclusion of each section of the class, a document called Class Notes will be posted on-line. These notes will serve in place of a text book, and you should print out a copy of those each week. That said, it would be useful for you to have a pre-calculus text book for reference at home- any one will do- which covers algebra, functions, graphing, trigonometry, etc. A calculus text might be similarly useful, although less often used than pre-calculus.

Goals of the Course: The primary goal of the course is to give you a deeper understanding of various topics in secondary mathematics, using problem solving and generalization as the tools. These topics include algebra, trigonometry, analytic geometry, probability, enumeration, pre-calculus (i.e. functions), and one-variable calculus. You will certainly get a review of these essential parts of high school math during the course, but our focus is on acquiring depth of knowledge by working through challenging (but fun) problems which, in almost every case, do not require mathematical ideas beyond the topics listed above. The review of basic secondary math will be passive- our active work is finding and extending solutions to mathematical problems, most of which will be more difficult than a typical problem you might give to your high school students- this course does count as an advanced elective for the mathematics major, after all! By working through harder problems and seeing how far they can be generalized, your understanding of the underlying mathematics, and your ability to call on that understanding while teaching the basics, will be enhanced.

Calculator: You should have a graphing calculator.

Class Format: A week's topic might have three sections, related to each other in some way as indicated by the title of the topic. Each section will involve solving one to three problems as well as traditional lecturing as well. Some portion of each class will consist of interactive problem solving with both group and individual work. For some problems, we will split up into groups, and you'll

work on it in your groups while I try to assist. In other situations, we'll work through the problems together with me at the board while you contribute in groups or individually.

As there is no text book, **attendance at class is mandatory**.

What to bring to class.
1. Notebook, as there will be some traditional blackboard lecturing.
2. Plenty of separate pieces of blank paper for group and individual problem solving in class.
3. Pens, pencils, some different colors might be useful but not necessary.

Note that this will be a handout-intensive course. I suggest you pick up a three ring binder and hole punch to keep notes and handouts organized. Date everything. The midterm and final exams will address work covered earlier in the semester, so it's important to have clear access to that.

Homework: Written homework will be assigned following each topic, except for the midterm and final as noted below. You are encouraged to work on the homework in groups of two or three, but you may, if you prefer, do your own individual homework assignment. If you work as a group, then you submit one assignment for the whole group. Just to be fair, most assignments will have a couple of extra parts of problems which are required for groups but not for individuals. You may switch groups for the next assignment, or do some assignments as individuals and others as a group, whatever works for you.

You may write the solutions by hand, or type them if that is easier for you. Whatever format you choose, please be sure that your homework is legible and logical, as most of the homework problems will require you to explain what you are doing or to explore a new situation. Also, most of the assignments will also contain a small number of Review Exercises, which are short problems like you'd find in a pre-calculus course. These are required problems, and they are designed each week to review some mathematics which will be used in the next topic.

You may use existing sources as you work on the homework- books, journals, on-line publications, whatever. This will be of limited use for the specific problems but will be useful for background information if you are stuck. Any use of existing materials, either printed or electronic, **should be noted** on your homework- you will not lose any credit for citing your sources. Some, but not all, of the problems will be graded.

IMPORTANT: You may not use bulletin boards nor any other interactive internet sites (e.g. Ask-the-Math-Geek sort of sites, on-line math help centers, problem solving forums, and the like), nor may you get human help other than others in our class. In other words, you may not inquire about the homework problems in any way that involves another human being responding to your inquiry. Since you have been given a lot of freedom on the homework to work in groups and to use existing sources, any violations of the no-interactive-help rule will be dealt with severely.

Midterm (take home): There will not be a regular homework assignment after the sixth topic-rather you will receive the take-home midterm that day. This will be strictly individual, no groups allowed, and no outside help other than printed material (i.e. no internet, period). It will cover the first six topics.

Final exam (take home). The take home final will be passed out at the last class, and will be individual like the mid-term, and all the same mid-term rules apply to the take-home final.

Grades: 10 homework assignments 60%
Take home Midterm 20%
Take home Final 20%

What this course is not: Since there are many courses in Mathematical Problem Solving out there, let me make a couple of distinctions about this course.

1. This is not a course in pedagogy. Although issues related to teaching secondary math will arise and will be noted, this is not a course about how to teach problem solving. Rather, our goal is mathematical content knowledge, which will make you a more confident and insightful secondary mathematics teacher.

2. This is not a course in strategies for solving math problems, nor it is practice for math competitions. Recognizing problem types, and learning particular approaches to take with them, will come up in our course, but the primary point is insight into secondary mathematics content rather than facility at certain types of problems. In MT455, classifying methods of attack will not play the central role that it does in a strategies kind of course.

3. The course will involve some traditional lecturing, plus a lot of problem solving in which we'll work as a class together under my direction, as well as some group work where you work together in class on your own. The most extensive opportunity for group exploration is on the homework, where you are strongly encouraged to work together. Time in class is very limited, so that while there will be some open-ended problem exploration in class, it is not something we can adopt as an exclusive mode of operation. Generalization: If there is one mathematical theme that connects most of the work we will do, it is generalization; in particular, much of the homework will involve generalizing the problems we solve in class. I chose that theme for three main reasons. First, there is no better way to gain insight into the solution of a particular problem than to (attempt to) generalize it- whether a generalization works or not, you will usually learn a lot by trying to find out, and your understanding of the original solution will be greatly enhanced either way. Second, generalization, together with the related notion of abstraction, is the motivation for much of modern theoretical mathematics which you have seen in so many college math courses, and it would be nice to see these concepts applied to problems from secondary mathematics. Finally, on a purely practical level, the ability to work with generalizations is a particularly useful skill for a mathematics teacher- for example, if you are writing a lesson plan, creating homework assignments and exams, or trying to find examples which are appropriate for students at different skill levels, generalizing existing examples and problems is an efficient and powerful tool.

The topics:
1. Dating Games
2. Reflecting on One's Problems
3. Stand-up Conics
4. A Thousand Points of Light
5. Tri, Tri Again
6. Thinking Globally
7. The Middle Class
8. Snoitcnuf
9. You Say You Want a Revolution
10. Worth the Weight
11. Worth Its Wait in Pennies
12. Evens, Odds, and Ends

Finally: The problems in this course will be quite varied- some are recreational while others are meat-and-potatoes secondary math, some involve applications and modeling while others are theoretical, some are short while others require many steps. Despite the variety, certain techniques and ideas, those at the heart of secondary mathematics, will come up repeatedly, such as algebraic manipulation, solving equations, graphs in the Cartesian plane, understanding variables vs. constants, and many others. You will get plenty of practice with these central notions while you solve the various problems.

I hope that this course will be challenging, useful, and fun. It is an original and unique course, so I will need your help in assessing which parts work well for you and which do not. So, even more than in every other course, I ask that you let me know what you find interesting vs. boring, hard vs. easy, useful vs. useless. And, please, have fun!