Chapter 1
Introduction

Learning Objectives

1. Develop a general understanding of the management science/operations research approach to decision making.
2. Realize that quantitative applications begin with a problem situation.
3. Obtain a brief introduction to quantitative techniques and their frequency of use in practice.
4. Understand that managerial problem situations have both quantitative and qualitative considerations that are important in the decision making process.
5. Learn about models in terms of what they are and why they are useful (the emphasis is on mathematical models).
6. Identify the step-by-step procedure that is used in most quantitative approaches to decision making.
7. Learn about basic models of cost, revenue, and profit and be able to compute the break-even point.
8. Obtain an introduction to microcomputer software packages and their role in quantitative approaches to decision making.
9. Understand the following terms:
   - model
   - infeasible solution
   - objective function
   - management science
   - constraint
   - operations research
   - deterministic model
   - fixed cost
   - stochastic model
   - variable cost
   - feasible solution
   - break-even point
Solutions:

1. Management science and operations research, terms used almost interchangeably, are broad disciplines that employ scientific methodology in managerial decision making or problem solving. Drawing upon a variety of disciplines (behavioral, mathematical, etc.), management science and operations research combine quantitative and qualitative considerations in order to establish policies and decisions that are in the best interest of the organization.

2. Define the problem
   Identify the alternatives
   Determine the criteria
   Evaluate the alternatives
   Choose an alternative
   For further discussion see section 1.3

3. See section 1.2.

4. A quantitative approach should be considered because the problem is large, complex, important, new and repetitive.

5. Models usually have time, cost, and risk advantages over experimenting with actual situations.

6. Model (a) may be quicker to formulate, easier to solve, and/or more easily understood.

7. Let \( d = \text{distance} \)
   \( m = \text{miles per gallon} \)
   \( c = \text{cost per gallon} \),

   \[ \therefore \text{Total Cost} = \left( \frac{2d}{m} \right) c \]

   We must be willing to treat \( m \) and \( c \) as known and not subject to variation.

8. a. Maximize \( 10x + 5y \)
   s.t.
   \[ 5x + 2y \leq 40 \]
   \[ x \geq 0, \ y \geq 0 \]

   b. Controllable inputs: \( x \) and \( y \)
   Uncontrollable inputs: profit (10,5), labor hours (5,2) and labor-hour availability (40)
c. Profit:
   $10/unit for \( x \)
   $5/unit for \( y \)
Labor Hours:
   5/unit for \( x \)
   2/unit for \( y \)
40 labor-hour capacity

Uncontrollable Inputs

<table>
<thead>
<tr>
<th>Production Quantities ( x ) and ( y )</th>
<th>Max ( 10x + 5y ) s.t. ( 10x + 5y \leq 40 ) ( x \geq 0 ) ( y \geq 0 )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Controllable Input</th>
<th>Projected Profit and check on production time constraint</th>
</tr>
</thead>
</table>

Output

Mathematical Model

d. \( x = 0, y = 20 \) Profit = $100
(Solution by trial-and-error)

e. Deterministic - all uncontrollable inputs are fixed and known.

9. If \( a = 3, x = 13 \frac{1}{3} \) and profit = 133
   If \( a = 4, x = 10 \) and profit = 100
   If \( a = 5, x = 8 \) and profit = 80
   If \( a = 6, x = 6 \frac{2}{3} \) and profit = 67

Since \( a \) is unknown, the actual values of \( x \) and profit are not known with certainty.

10. a. Total Units Received = \( x + y \)

b. Total Cost = 0.20\( x \) + 0.25\( y \)

c. \( x + y = 5000 \)

d. \( x \leq 4000 \) Kansas City Constraint
   \( y \leq 3000 \) Minneapolis Constraint

e. Min \( 0.20x + 0.25y \) s.t.

\[
\begin{align*}
 x + y &= 5000 \\
 x &\leq 4000 \\
 y &\leq 3000 \\
 x, y &\geq 0
\end{align*}
\]
11. a. at $20 \; d = 800 - 10(20) = 600$
at $70 \; d = 800 - 10(70) = 100$
b. $TR = dp = (800 - 10p)p = 800p - 10p^2$
c. at $30 \; TR = 800(30) - 10(30)^2 = 15,000$
at $40 \; TR = 800(40) - 10(40)^2 = 16,000$
at $50 \; TR = 800(50) - 10(50)^2 = 15,000$
Total Revenue is maximized at the $40$ price.
d. $d = 800 - 10(40) = 400$ units
$TR = 16,000$

12. a. $TC = 1000 + 30x$
b. $P = 40x - (1000 + 30x) = 10x - 1000$
c. Breakeven when $P = 0$
Thus $10x - 1000 = 0$
$10x = 1000$
x = 100

13. a. Total cost = $4800 + 60x$
b. Total profit = total revenue - total cost
   = $300x - (4800 + 60x)$
   = $240x - 4800$
c. Total profit = $240(30) - 4800 = 2400$
d. $240x - 4800 = 0$
   $x = 4800/240 = 20$
The breakeven point is approximately 20 students.

14. a. Profit = Revenue - Cost
   = $20x - (80,000 + 3x)$
   = $17x - 80,000$
Breakeven point
$17x - 80,000 = 0$
$17x = 80,000$
x = 4706
b. Loss with Profit = $17(4000) - 80,000 = -12,000$
c. Profit = $px - (80,000 + 3x)$
   = $4000p - (80,000 + 3(4000)) = 0$
   $4000p = 92,000$
   $p = 23$
d. Profit = $25.95 (4000) - (80,000 + 3 (4000))
   = $11,800

Probably go ahead with the project although the $11,800 is only a 12.8% return on the total cost of $92,000.

15. a. Profit = 100,000x - (1,500,000 + 50,000x)
           = 0
   50,000x = 1,500,000
   x = 30

   b. Build the luxury boxes.

   Profit = 100,000 (50) - (1,500,000 + 50,000 (50))
   = $1,000,000

16. a. Max  6x + 4y

   b.  50x + 30y ≤ 80,000
       50x ≤ 50,000
       30y ≤ 45,000
       x, y ≥ 0

17. a.  s_j = s_{j-1} + x_j - d_j
       
       or s_j - s_{j-1} - x_j + d_j = 0

   b.  x_j ≤ c_j

   c.  s_j ≥ I_j
Chapter 2
An Introduction to Linear Programming

Learning Objectives

1. Obtain an overview of the kinds of problems linear programming has been used to solve.
2. Learn how to develop linear programming models for simple problems.
3. Be able to identify the special features of a model that make it a linear programming model.
4. Learn how to solve two variable linear programming models by the graphical solution procedure.
5. Understand the importance of extreme points in obtaining the optimal solution.
6. Know the use and interpretation of slack and surplus variables.
7. Be able to interpret the computer solution of a linear programming problem.
8. Understand how alternative optimal solutions, infeasibility and unboundedness can occur in linear programming problems.
9. Understand the following terms:
   - problem formulation
   - constraint function
   - objective function
   - solution
   - optimal solution
   - nonnegativity constraints
   - mathematical model
   - linear program
   - linear functions
   - feasible solution
   - feasible region
   - slack variable
   - standard form
   - redundant constraint
   - extreme point
   - surplus variable
   - alternative optimal solutions
   - infeasibility
   - unbounded
Solutions:

1. a, b, and e, are acceptable linear programming relationships.

   c is not acceptable because of $-2x_2^2$

   d is not acceptable because of $3\sqrt{x_1}$

   f is not acceptable because of $1x_1x_2$

   c, d, and f could not be found in a linear programming model because they have the above nonlinear terms.

2. a.

   ![Diagram](image1)

   b.

   ![Diagram](image2)

   c.

   ![Diagram](image3)
3. a.

b.

c. Points on line are only feasible points
4. a.

Note: Point shown was used to locate position of the constraint line
5.

6. For $7x_1 + 10x_2$, slope = $-7/10$

For $6x_1 + 4x_2$, slope = $-6/4 = -3/2$

For $z = -4x_1 + 7x_2$, slope = $4/7$
7. [Graph showing a feasible region with lines and a shaded area.]

8. [Graph showing a line with a point (100,200) and a slope of 133 1/3.]
9.

10.
Value of Objective Function = $2\left(\frac{12}{7}\right) + 3\left(\frac{15}{7}\right) = \frac{69}{7}$

Optimal Solution
$x_1 = \frac{12}{7}, x_2 = \frac{15}{7}$

From (1), $x_1 = 6 - 2\left(\frac{15}{7}\right) = 6 - \frac{30}{7} = \frac{12}{7}$
11. Value of Objective Function = 750
   Optimal Solution
   \[ x_1 = 100, \ x_2 = 50 \]

12. a. Value of Objective Function = 13.5
   Optimal Solution
   \[ x_1 = 3, \ x_2 = 1.5 \]
b.

\[ x_2 \]

Optimal Solution
\[ x_1 = 0, \ x_2 = 3 \]

Value of Objective Function = 18

(0,0)

13. a.

\[ x_2 \]

Redundant Constraint

Optimal Solution
\[ x_1 = 2, \ x_2 = 2 \]

Value of objective function = 10

(0,0)

b. Yes, constraint 2.

The solution remains \( x_1 = 2, x_2 = 2 \) if constraint 2 is removed.
14. a. [Diagram of the feasible region consisting of a line segment only]

b. The extreme points are (5, 1) and (2, 4).

c. [Diagram of the optimal solution with the equations $x_1 + 2x_2 = 10$ and $x_1 = 2, x_2 = 4$ marked on the graph]
15. a. Similar to part (a): the same feasible region with a different objective function. The optimal solution occurs at (708, 0) with a profit of $20(708) + 9(0) = 14,160$.

c. The sewing constraint is redundant. Such a change would not change the optimal solution to the original problem.

16. a. A variety of objective functions with a slope greater than $-4/10$ (slope of I & P line) will make extreme point 5 the optimal solution. For example, one possibility is $3S + 9D$.

b. Optimal Solution is $S = 0$ and $D = 540$.

c. 

<table>
<thead>
<tr>
<th>Dept.</th>
<th>Hours Used</th>
<th>Max. Available</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>C &amp; D</td>
<td>$1(540) = 540$</td>
<td>630</td>
<td>90</td>
</tr>
<tr>
<td>S</td>
<td>$5/6(540) = 450$</td>
<td>600</td>
<td>150</td>
</tr>
<tr>
<td>F</td>
<td>$2/3(540) = 360$</td>
<td>708</td>
<td>348</td>
</tr>
<tr>
<td>I &amp; P</td>
<td>$1/4(540) = 135$</td>
<td>135</td>
<td>—</td>
</tr>
</tbody>
</table>

17. 

Max $5x_1 + 2x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$

s.t. 

$1x_1 - 2x_2 + 1/2x_3 + 1s_1 = 420$

$2x_1 + 3x_2 - 1x_3 + 1s_2 = 610$

$6x_1 - 1x_2 + 3x_3 + 1s_3 = 125$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$
18. a.  
\[ \begin{align*} 
\text{Max} & \quad 4x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 \\
\text{s.t.} & \quad 10x_1 + 2x_2 + s_1 = 30 \\
& \quad 3x_1 + 2x_2 + s_2 = 12 \\
& \quad 2x_1 + 2x_2 + s_3 = 10 \\
& \quad x_1, x_2, s_1, s_2, s_3 \geq 0 
\end{align*} \]

b.  
![Graph showing the optimal solution](image)

Optimal Solution  
\[ x_1 = \frac{18}{7}, \quad x_2 = \frac{15}{7}, \quad \text{Value} = \frac{87}{7} \]

c.  
\[ s_1 = 0, \quad s_2 = 0, \quad s_3 = \frac{4}{7} \]

19. a.  
\[ \begin{align*} 
\text{Max} & \quad 3x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 \\
\text{s.t.} & \quad -x_1 + 2x_2 + s_1 = 8 \quad (1) \\
& \quad x_1 + 2x_2 + s_2 = 12 \quad (2) \\
& \quad 2x_1 + x_2 + s_3 = 16 \quad (3) \\
& \quad x_1, x_2, s_1, s_2, s_3 \geq 0 
\end{align*} \]
b. Let \( E \) = number of units of the EZ-Rider produced
\( L \) = number of units of the Lady-Sport produced

Max \( 2400E + 1800L \)

s.t.

\[
\begin{align*}
6E & + 3L & \leq 2100 & \text{Engine time} \\
L & \leq 280 & \text{Lady-Sport maximum} \\
2E & + 2.5L & \leq 1000 & \text{Assembly and testing} \\
E, L & \geq 0
\end{align*}
\]

Optimal Solution
\( x_1 = 20/3 \), \( x_2 = 8/3 \)
Value = 30 2/3

\( s_1 = 8 + x_1 - 2x_2 = 8 + 20/3 - 16/3 = 28/3 \)

\( s_2 = 12 - x_1 - 2x_2 = 12 - 20/3 - 16/3 = 0 \)

\( s_3 = 16 - 2x_1 - x_2 = 16 - 40/3 - 8/3 = 0 \)
b. The binding constraints are the manufacturing time and the assembly and testing time.

21. a. Let $F =$ number of tons of fuel additive
    $S =$ number of tons of solvent base

    Max $40F + 30S$

    s.t.
    $2/5F + 1/2 S \leq 200$ Material 1
    $1/5 S \leq 5$ Material 2
    $3/5 F + 3/10 S \leq 21$ Material 3

    $F, S \geq 0$

    Optimal Solution $E = 250, L = 200$
    Profit = $960,000$
b. Let \( R \) = number of units of regular model.
\( C \) = number of units of catcher’s model.

Max \( 5R + 8C \)

s.t.

\[
\begin{align*}
1R + \frac{3}{2} C & \leq 900 \quad \text{Cutting and sewing} \\
\frac{1}{2} R + \frac{1}{3} C & \leq 300 \quad \text{Finishing} \\
\frac{1}{8} R + \frac{1}{4} C & \leq 100 \quad \text{Packing and Shipping}
\end{align*}
\]

\( R, C \geq 0 \)
c. \(5(500) + 8(150) = 3700\)

d. C & S \(1(500) + \frac{3}{2}(150) = 725\)

\[F \quad \frac{1}{2}(500) + \frac{1}{3}(150) = 300\]

\[P & S \quad \frac{1}{8}(500) + \frac{1}{4}(150) = 100\]

e.

<table>
<thead>
<tr>
<th>Department</th>
<th>Capacity</th>
<th>Usage</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>C &amp; S</td>
<td>900</td>
<td>725</td>
<td>175 hours</td>
</tr>
<tr>
<td>F</td>
<td>300</td>
<td>300</td>
<td>0 hours</td>
</tr>
<tr>
<td>P &amp; S</td>
<td>100</td>
<td>100</td>
<td>0 hours</td>
</tr>
</tbody>
</table>

23. a. Let \(B\) = percentage of funds invested in the bond fund

\(S\) = percentage of funds invested in the stock fund

\[
\text{Max} \quad 0.06B + 0.10S
\]

\[\text{s.t.} \quad B \geq 0.3 \quad \text{Bond fund minimum}\]

\[
0.06B + 0.10S \geq 0.075 \quad \text{Minimum return}\]

\[
B + S = 1 \quad \text{Percentage requirement}\]

b. Optimal solution: \(B = 0.3, S = 0.7\)

Value of optimal solution is 0.088 or 8.8%
24. a. Let \( N \) = amount spent on newspaper advertising
\( R \) = amount spent on radio advertising

Max \( \quad 50N + 80R \)

s.t.
\[
\begin{align*}
N + R &= 1000 & \text{Budget} \\
N &\geq 250 & \text{Newspaper min.} \\
R &\geq 250 & \text{Radio min.} \\
N &\geq 2R & \text{News \geq 2 Radio}
\end{align*}
\]

\( N, R \geq 0 \)

b.

\[
\begin{array}{c}
\text{R} \\
\hline
1000 \\
500 \\
0 \\
\end{array}
\]

**Optimal Solution**
\( N = 666.67, \quad R = 333.33 \)

\( Value = 60,000 \)

25. Let \( I \) = Internet fund investment in thousands
\( B \) = Blue Chip fund investment in thousands

Max \( \quad 0.12I + 0.09B \)

s.t.
\[
\begin{align*}
1I + 1B &\leq 50 & \text{Available investment funds} \\
1I &\leq 35 & \text{Maximum investment in the internet fund} \\
6I + 4B &\leq 240 & \text{Maximum risk for a moderate investor} \\
l, b &\geq 0
\end{align*}
\]
Introduction to LP

b. The third constraint for the aggressive investor becomes

\[ 6I + 4B \leq 320 \]

This constraint is redundant; the available funds and the maximum Internet fund investment constraints define the feasible region. The optimal solution is:

Internet fund $35,000
Blue Chip fund $15,000
Annual return $5,550

The aggressive investor places as much funds as possible in the high return but high risk Internet fund.

c. The third constraint for the conservative investor becomes

\[ 6I + 4B \leq 160 \]

This constraint becomes a binding constraint. The optimal solution is

Internet fund $0
Blue Chip fund $40,000
Annual return $3,600
The slack for constraint 1 is $10,000. This indicates that investing all $50,000 in the Blue Chip fund is still too risky for the conservative investor. $40,000 can be invested in the Blue Chip fund. The remaining $10,000 could be invested in low-risk bonds or certificates of deposit.

26. a. Let 
   \[ W = \text{number of jars of Western Foods Salsa produced} \]
   \[ M = \text{number of jars of Mexico City Salsa produced} \]

   Max \[ 1W + 1.25M \]
   s.t.
   \[ 5W + 7M \leq 4480 \quad \text{Whole tomatoes} \]
   \[ 3W + 1M \leq 2080 \quad \text{Tomato sauce} \]
   \[ 2W + 2M \leq 1600 \quad \text{Tomato paste} \]
   \[ W, M \geq 0 \]

   Note: units for constraints are ounces

   b. Optimal solution: \[ W = 560, \ M = 240 \]

   Value of optimal solution is 860

27. a. Let \[ B = \text{proportion of Buffalo's time used to produce component 1} \]
    \[ D = \text{proportion of Dayton's time used to produce component 1} \]

   **Maximum Daily Production**
   \[
   \begin{array}{c|cc}
   & \text{Component 1} & \text{Component 2} \\
   \hline
   \text{Buffalo} & 2000 & 1000 \\
   \text{Dayton} & 600 & 1400 \\
   \end{array}
   \]

   Number of units of component 1 produced: \[ 2000B + 600D \]

   Number of units of component 2 produced: \[ 1000(1 - B) + 600(1 - D) \]
For assembly of the ignition systems, the number of units of component 1 produced must equal the number of units of component 2 produced.

Therefore,

\[ 2000B + 600D = 1000(1 - B) + 1400(1 - D) \]
\[ 2000B + 600D = 1000 - 1000B + 1400 - 1400D \]
\[ 3000B + 2000D = 2400 \]

Note: Because every ignition system uses 1 unit of component 1 and 1 unit of component 2, we can maximize the number of electronic ignition systems produced by maximizing the number of units of subassembly 1 produced.

\[ \text{Max } 2000B + 600D \]

In addition, \( B \leq 1 \) and \( D \leq 1 \).

The linear programming model is:

\[ \text{Max } 2000B + 600D \]
\[ \text{s.t. } \]
\[ 3000B + 2000D = 2400 \]
\[ B \leq 1 \]
\[ D \leq 1 \]
\[ B, D \geq 0 \]

The graphical solution is shown below.
Optimal Solution: $B = .8, D = 0$

Optimal Production Plan

<table>
<thead>
<tr>
<th>Location</th>
<th>Component 1</th>
<th>Component 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buffalo</td>
<td>.8(2000) = 1600</td>
<td>.2(1000) = 200</td>
</tr>
<tr>
<td>Dayton</td>
<td>0(600) = 0</td>
<td>1(1400) = 1400</td>
</tr>
</tbody>
</table>

Total units of electronic ignition system = 1600 per day.

28. a. Let $E$ = number of shares of Eastern Cable
       $C$ = number of shares of ComSwitch

Max $15E + 18C$

s.t.

$40E + 25C \leq 50,000$  Maximum Investment
$40E \geq 15,000$  Eastern Cable Minimum
$25C \geq 10,000$  ComSwitch Minimum
$25C \leq 25,000$  ComSwitch Maximum

$E, C \geq 0$

b. [Graph showing the constraints on the number of shares of Eastern Cable (E) and ComSwitch (C)]
c. There are four extreme points: (375,400); (1000,400); (625,1000); (375,1000)

d. Optimal solution is $E = 625$, $C = 1000$
Total return = $27,375$
29.

Introduction to LP

Feasible Region

Optimal Solution
$x_1 = 3, x_2 = 1$

$3x_1 + 4x_2 = 13$

Objective Function Value = 13

30.

<table>
<thead>
<tr>
<th>Extreme Points</th>
<th>Objective Function Value</th>
<th>Surplus Demand</th>
<th>Surplus Total Production</th>
<th>Slack Processing Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A = 250, B = 100)$</td>
<td>800</td>
<td>125</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$(A = 125, B = 225)$</td>
<td>925</td>
<td>—</td>
<td>—</td>
<td>125</td>
</tr>
<tr>
<td>$(A = 125, B = 350)$</td>
<td>1300</td>
<td>—</td>
<td>125</td>
<td>—</td>
</tr>
</tbody>
</table>
31. a.

Optimal Solution: $x_1 = 3, x_2 = 1$, value = 5

b.

(1) $3 + 4(1) = 7$  Slack = 21 - 7 = 14
(2) $2(3) + 1 = 7$  Surplus = 7 - 7 = 0
(3) $3(3) + 1.5 = 10.5$  Slack = 21 - 10.5 = 10.5
(4) $-2(3) + 6(1) = 0$  Surplus = 0 - 0 = 0

c.

Optimal Solution: $x_1 = 6, x_2 = 2$, value = 34
32. a.

\[ x_2 \]
\[ 4 \]
\[ 3 \]
\[ 2 \]
\[ 1 \]
\[ 0 \]

\[ x_1 \]

Feasible Region

(21/4, 9/4)

(4,1)

b. There are two extreme points: \((x_1 = 4, x_2 = 1)\) and \((x_1 = 21/4, x_2 = 9/4)\)

c. The optimal solution is \(x_1 = 4, x_2 = 1\)

33. a. 

\[
\begin{align*}
\text{Min} & \quad 6x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 \\
\text{s.t.} & \quad 2x_1 + 1x_2 - s_1 = 12 \\
& \quad 1x_1 + 1x_2 - s_2 = 10 \\
& \quad 1x_2 + s_3 = 4 \\
& \quad x_1, x_2, s_1, s_2, s_3 \geq 0
\end{align*}
\]

b. The optimal solution is \(x_1 = 6, x_2 = 4\).

c. \(s_1 = 4, s_2 = 0, s_3 = 0\).

34. a. Let \(T\) = number of training programs on teaming

\[ P = \text{number of training programs on problem solving} \]

\[
\begin{align*}
\text{Max} \quad & \quad 10,000T + 8,000P \\
\text{s.t.} \quad & \quad T \geq 8 \quad \text{Minimum Teaming} \\
& \quad P \geq 10 \quad \text{Minimum Problem Solving} \\
& \quad T + P \geq 25 \quad \text{Minimum Total} \\
& \quad 3T + 2P \leq 84 \quad \text{Days Available} \\
& \quad T, P \geq 0
\end{align*}
\]
b. There are four extreme points: (15,10); (21.33,10); (8,30); (8,17)

d. The minimum cost solution is $T = 8, P = 17$
Total cost = $216,000$

35.

<table>
<thead>
<tr>
<th></th>
<th>Regular</th>
<th>Zesty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild</td>
<td>80%</td>
<td>60%</td>
</tr>
<tr>
<td>Extra Sharp</td>
<td>20%</td>
<td>40%</td>
</tr>
</tbody>
</table>

Let $R =$ number of containers of Regular  
$Z =$ number of containers of Zesty

Each container holds 12/16 or 0.75 pounds of cheese

Pounds of mild cheese used  
$= 0.80 \times 0.75 \times R + 0.60 \times 0.75 \times Z$
$= 0.60 \times R + 0.45 \times Z$

Pounds of extra sharp cheese used  
$= 0.20 \times 0.75 \times R + 0.40 \times 0.75 \times Z$
$= 0.15 \times R + 0.30 \times Z$
Cost of Cheese = Cost of mild + Cost of extra sharp
= 1.20 (0.60 R + 0.45 Z) + 1.40 (0.15 R + 0.30 Z)
= 0.72 R + 0.54 Z + 0.21 R + 0.42 Z
= 0.93 R + 0.96 Z

Packaging Cost = 0.20 R + 0.20 Z

Total Cost = (0.93 R + 0.96 Z) + (0.20 R + 0.20 Z)
= 1.13 R + 1.16 Z

Revenue = 1.95 R + 2.20 Z

Profit Contribution = Revenue - Total Cost
= (1.95 R + 2.20 Z) - (1.13 R + 1.16 Z)
= 0.82 R + 1.04 Z

Max 0.82 R + 1.04 Z
s.t.
0.60 R + 0.45 Z ≤ 8100 Mild
0.15 R + 0.30 Z ≤ 3000 Extra Sharp
R, Z ≥ 0

Optimal Solution: R = 9600, Z = 5200, profit = 0.82(9600) + 1.04(5200) = $13,280

36. a. Let S = yards of the standard grade material per frame
P = yards of the professional grade material per frame

Min 7.50S + 9.00P
s.t.
0.10S + 0.30P ≥ 6 carbon fiber (at least 20% of 30 yards)
0.06S + 0.12P ≤ 3 kevlar (no more than 10% of 30 yards)
S + P = 30 total (30 yards)
S, P ≥ 0
b. 

![Graph showing Extreme Points and Feasible Region]

**c.**

<table>
<thead>
<tr>
<th>Extreme Point</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15, 15)</td>
<td>7.50(15) + 9.00(15) = 247.50</td>
</tr>
<tr>
<td>(10, 20)</td>
<td>7.50(10) + 9.00(20) = 255.00</td>
</tr>
</tbody>
</table>

The optimal solution is $S = 15, P = 15$

d. Optimal solution does not change: $S = 15$ and $P = 15$. However, the value of the optimal solution is reduced to $7.50(15) + 8(15) = \$232.50$.

e. At $7.40$ per yard, the optimal solution is $S = 10, P = 20$. The value of the optimal solution is reduced to $7.50(10) + 7.40(20) = \$223.00$. A lower price for the professional grade will not change the $S = 10, P = 20$ solution because of the requirement for the maximum percentage of kevlar (10%).

37. a. Let $S =$ number of units purchased in the stock fund  
   $M =$ number of units purchased in the money market fund

   Min $8S + 3M$

   s.t.

   $50S + 100M \leq 1,200,000$ Funds available
   $5S + 4M \geq 60,000$ Annual income
   $M \geq 3,000$ Minimum units in money market
   $S, M \geq 0$
Optimal Solution: $S = 4000, M = 10000$, value = 62000

b. Annual income = $5(4000) + 4(10000) = 60,000$

c. Invest everything in the stock fund.

38. Let $P_1$ = gallons of product 1
    $P_2$ = gallons of product 2

Min $P_1 + P_2$

s.t.

$P_1 \geq 30$  Product 1 minimum

$P_2 \geq 20$  Product 2 minimum

$P_1 + 2P_2 \geq 80$  Raw material

$P_1, P_2 \geq 0$
Optimal Solution: $P_1 = 30, P_2 = 25$  Cost = $55$

39. a. Let $R$ = number of gallons of regular gasoline produced
    $P$ = number of gallons of premium gasoline produced

Max $0.30R + 0.50P$

s.t.

$0.30R + 0.60P \leq 18,000$  Grade A crude oil available

$1R + 1P \leq 50,000$  Production capacity

$1P \leq 20,000$  Demand for premium

$R, P \geq 0$
b. Optimal Solution:
40,000 gallons of regular gasoline
10,000 gallons of premium gasoline
Total profit contribution = $17,000

c. Constraint | Value of Slack Variable | Interpretation
--- | --- | ---
1 | 0 | All available grade A crude oil is used
2 | 0 | Total production capacity is used
3 | 10,000 | Premium gasoline production is 10,000 gallons less than the maximum demand
d. Grade A crude oil and production capacity are the binding constraints.
40. Satisfies Constraint #2

Satisfies Constraint #1

Infeasibility

41. Unbounded
42. a. 

![Diagram of a graph with a linear objective function and constraints. The optimal solution is marked with coordinates (30/16, 30/16) and a value of 60/16.]

b. New optimal solution is $x_1 = 0, x_2 = 3$, value = 6.

c. Slope of constraint is $-3/5$

Slope of objective function when $c_1 = 1$ is $-1/c_2$

Set slopes equal: $-1/c_2 = -3/5$

$-5 = -3c_2$

$c_2 = 5/3$

Objective function needed: $\max x_1 + \frac{5}{3}x_2$

43. a. 

![Diagram of a graph with a linear objective function and constraints. The feasible region is shaded and the optimal solution is marked with coordinates $x_1 = 3, x_2 = 0$ and a value of 3.]

b. Feasible region is unbounded.
c. Optimal Solution: $x_1 = 3, x_2 = 0, z = 3$.

d. An unbounded feasible region does not imply the problem is unbounded. This will only be the case when it is unbounded in the direction of improvement for the objective function.

44. Let $N =$ number of sq. ft. for national brands
    $G =$ number of sq. ft. for generic brands

Problem Constraints:

$$
\begin{align*}
N + G & \leq 200 \quad \text{Space available} \\
N & \geq 120 \quad \text{National brands} \\
G & \geq 20 \quad \text{Generic}
\end{align*}
$$

<table>
<thead>
<tr>
<th>Extreme Point</th>
<th>$N$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>80</td>
</tr>
</tbody>
</table>

a. Optimal solution is extreme point 2; 180 sq. ft. for the national brand and 20 sq. ft. for the generic brand.

b. Alternative optimal solutions. Any point on the line segment joining extreme point 2 and extreme point 3 is optimal.

c. Optimal solution is extreme point 3; 120 sq. ft. for the national brand and 80 sq. ft. for the generic brand.
Alternative optimal solutions exist at extreme points \((A = 125, B = 225)\) and \((A = 250, B = 100)\).

\[
\text{Cost} = 3(125) + 3(225) = 1050
\]

or

\[
\text{Cost} = 3(250) + 3(100) = 1050
\]

The solution \((A = 250, B = 100)\) uses all available processing time. However, the solution \((A = 125, B = 225)\) uses only \(2(125) + 1(225) = 475\) hours.

Thus, \((A = 125, B = 225)\) provides \(600 - 475 = 125\) hours of slack processing time which may be used for other products.
Possible Actions:

i. Reduce total production to \( A = 125, B = 350 \) on 475 gallons.

ii. Make solution \( A = 125, B = 375 \) which would require \( 2(125) + 1(375) = 625 \) hours of processing time. This would involve 25 hours of overtime or extra processing time.

iii. Reduce minimum \( A \) production to 100, making \( A = 100, B = 400 \) the desired solution.
47. a. 

The graph shows the feasible region with extreme points labeled 1 through 5. 

b. Yes. New optimal solution is \( F = 18.75, S = 25 \). Value of the new optimal solution is 
   
   \[ 40(18.75) + 60(25) = 2250. \]

c. An optimal solution occurs at extreme point 3, extreme point 4, and any point on the line segment joining these two points. This is the special case of alternative optimal solutions. For the manager attempting to implement the solution this means that the manager can select the specific solution that is most appropriate.

48. a. 

The graph shows the constraints with feasible regions labeled. 

There are no points satisfying both sets of constraints; thus there will be no feasible solution.
b. 

<table>
<thead>
<tr>
<th>Materials</th>
<th>Minimum Tons Required for $F = 30$, $S = 15$</th>
<th>Tons Available</th>
<th>Additional Tons Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material 1</td>
<td>$2/5(30) + 1/2(15) = 19.5$</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Material 2</td>
<td>$0(30) + 1/5(15) = 3$</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Material 3</td>
<td>$3/5(30) + 3/10(15) = 22.5$</td>
<td>21</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Thus RMC will need 1.5 additional tons of material 3.

49. a. Let $P =$ number of full-time equivalent pharmacists  
$T =$ number of full-time equivalent physicians

The model and the optimal solution obtained using The Management Scientist is shown below:

\[
\begin{align*}
\text{MIN } & 40P + 10T \\
\text{S.T.} & \\
& 1P + 1T > 250 \\
& 2P - 1T > 0 \\
& 1P > 90 \\
\end{align*}
\]

OPTIMAL SOLUTION

Objective Function Value = 5200.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>90.000</td>
<td>0.000</td>
</tr>
<tr>
<td>T</td>
<td>160.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>-10.000</td>
</tr>
<tr>
<td>2</td>
<td>20.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>-30.000</td>
</tr>
</tbody>
</table>

The optimal solution requires 90 full-time equivalent pharmacists and 160 full-time equivalent technicians. The total cost is $5200 per hour.

b. 

<table>
<thead>
<tr>
<th>Current Levels</th>
<th>Attrition</th>
<th>Optimal Values</th>
<th>New Hires Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pharmacists</td>
<td>85</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>Technicians</td>
<td>175</td>
<td>30</td>
<td>160</td>
</tr>
</tbody>
</table>

The payroll cost using the current levels of 85 pharmacists and 175 technicians is $40(85) + 10(175) = \$5150$ per hour.

The payroll cost using the optimal solution in part (a) is $5200 per hour.

Thus, the payroll cost will go up by $50.
Let \( M \) = number of Mount Everest Parkas

\( R \) = number of Rocky Mountain Parkas

Max \( 100M + 150R \)

s.t.

\[
\begin{align*}
30M + 20R & \leq 7200 \text{ Cutting time} \\
45M + 15R & \leq 7200 \text{ Sewing time} \\
0.8M - 0.2R & \geq 0 \text{ % requirement}
\end{align*}
\]

Note: Students often have difficulty formulating constraints such as the % requirement constraint. We encourage our students to proceed in a systematic step-by-step fashion when formulating these types of constraints. For example:

- \( M \) must be at least 20\% of total production
  
  \( M \geq 0.2 \) (total production)

- \( M \geq 0.2 \) (\( M + R \))

- \( M \geq 0.2M + 0.2R \)

- \( 0.8M - 0.2R \geq 0 \)

The optimal solution is \( M = 65.45 \) and \( R = 261.82 \); the value of this solution is \( z = 100(65.45) + 150(261.82) = 45,818 \). If we think of this situation as an on-going continuous production process, the fractional values simply represent partially completed products. If this is not the case, we can approximate the optimal solution by rounding down; this yields the solution \( M = 65 \) and \( R = 261 \) with a corresponding profit of $45,650.
51. Let \( C \) = number sent to current customers  
\( N \) = number sent to new customers  

Note:  
Number of current customers that test drive = .25 \( C \)  
Number of new customers that test drive = .20 \( N \)  
Number sold = \( .12 \times (.25 \times C) + .20 \times (.20 \times N) \)  
\[ = .03 \times C + .04 \times N \]  

Max \( .03 \times C + .04 \times N \)  

s.t.  
\[ .25 \times C \geq 30,000 \text{ Current Min} \]  
\[ .20 \times N \geq 10,000 \text{ New Min} \]  
\[ .25 \times C - .40 \times N \geq 0 \text{ Current vs. New} \]  
\[ 4 \times C + 6 \times N \leq 1,200,000 \text{ Budget} \]  
\[ C, N \geq 0 \]  

Optimal Solution  
\( C = 225,000, N = 50,000 \)  
Value = 8,750
52. Let \( S \) = number of standard size rackets
\( O \) = number of oversize size rackets

\[
\begin{align*}
\text{Max} & \quad 10S + 15O \\
\text{s.t.} & \quad 0.8S - 0.2O \geq 0 \quad \% \text{ standard} \\
& \quad 10S + 12O \leq 4800 \quad \text{Time} \\
& \quad 0.125S + 0.4O \leq 80 \quad \text{Alloy} \\
& \quad S, O \geq 0
\end{align*}
\]

53. a. Let \( R \) = time allocated to regular customer service
\( N \) = time allocated to new customer service

\[
\begin{align*}
\text{Max} & \quad 1.2R + N \\
\text{s.t.} & \quad R + N \leq 80 \\
& \quad 25R + 8N \geq 800 \\
& \quad -0.6R + N \geq 0 \\
& \quad R, N \geq 0
\end{align*}
\]

b. OPTIMAL SOLUTION

Objective Function Value = 90.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>50.000</td>
<td>0.000</td>
</tr>
<tr>
<td>N</td>
<td>30.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>1.125</td>
</tr>
<tr>
<td>2</td>
<td>690.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>-0.125</td>
</tr>
</tbody>
</table>
Optimal solution:  \( R = 50, N = 30, \text{ value } = 90 \)

HTS should allocate 50 hours to service for regular customers and 30 hours to calling on new customers.

54. a. Let  
\[
M_1 = \text{number of hours spent on the M-100 machine} \\
M_2 = \text{number of hours spent on the M-200 machine}
\]

Total Cost  
\[
6(40)M_1 + 6(50)M_2 + 50M_1 + 75M_2 = 290M_1 + 375M_2
\]

Total Revenue  
\[
25(18)M_1 + 40(18)M_2 = 450M_1 + 720M_2
\]

Profit Contribution  
\[
(450 - 290)M_1 + (720 - 375)M_2 = 160M_1 + 345M_2
\]

Max  
\[
160M_1 + 345M_2
\]

s.t.  
\[
M_1 \leq 15 \quad \text{M-100 maximum} \\
M_2 \leq 10 \quad \text{M-200 maximum} \\
M_1 \geq 5 \quad \text{M-100 minimum} \\
M_2 \geq 5 \quad \text{M-200 minimum} \\
40M_1 + 50M_2 \leq 1000 \quad \text{Raw material available}
\]

\( M_1, M_2 \geq 0 \)

b.

**OPTIMAL SOLUTION**

Objective Function Value = 5450.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>12.500</td>
<td>0.000</td>
</tr>
<tr>
<td>M2</td>
<td>10.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.500</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>145.000</td>
</tr>
<tr>
<td>3</td>
<td>7.500</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>5.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>4.000</td>
</tr>
</tbody>
</table>

The optimal decision is to schedule 12.5 hours on the M-100 and 10 hours on the M-200.
Chapter 3
Linear Programming: Sensitivity Analysis and Interpretation of Solution

Learning Objectives

1. Be able to conduct graphical sensitivity analysis for two variable linear programming problems.

2. Be able to compute and interpret the range of optimality for objective function coefficients.

3. Be able to compute and interpret the dual price for a constraint.

4. Learn how to formulate, solve, and interpret the solution for linear programs with more than two decision variables.

5. Understand the following terms:

   - sensitivity analysis
   - range of optimality
   - dual price
   - reduced cost
   - range of feasibility
   - 100 percent rule
   - sunk cost
   - relevant cost
Solutions:

1. Note: Feasible region is shown as part of the solution to problem 21 in Chapter 2.

Optimal Solution: \( F = 25, S = 20 \)

Binding Constraints: material 1 and material 3

Let Line A = material 1 = \( \frac{2}{5} F + \frac{1}{2} S = 20 \)
Line B = material 3 = \( \frac{3}{5} F + \frac{3}{10} S = 21 \)

The slope of Line A = \(-\frac{4}{5}\)
The slope of Line B = \(-2\)

Current solution is optimal for

\[ -2 \leq \frac{C_F}{30} \leq -\frac{4}{5} \]

or

\[ 24 \leq C_F \leq 60 \]

Current solution is optimal for

\[ -2 \leq -\frac{40}{C_S} \leq -\frac{4}{5} \]

or

\[ 20 \leq C_S \leq 50 \]

2.

Application of the graphical solution procedure to the problem with the enlarged feasible region shows that the extreme point with \( F = 100/3 \) and \( S = 40/3 \) now provides the optimal solution. The new value for
the objective function is \(40(100/3) + 30(40/3) = 1733.33\), providing an increase in profit of \(\$1733.33 - 1600 = \$133.33\). Thus the increased profit occurs at a rate of \(\$133.33/3 = \$44.44\) per ton of material 3 added. Thus the dual price for the material 3 constraint is \$44.44.

3. a. 

Optimal Value = 27

b. Slope of Line B = -1
Slope of Line A = -1/3

Let \(C_1\) = objective function coefficient of \(x_1\)
\(C_2\) = objective function coefficient of \(x_2\)

\[-1 \leq -C_1/3 \leq -1/3\]

\[1 \geq C_1/3 \quad C_1/3 \geq 1/3\]

\[C_1 \leq 3 \quad C_1 \geq 1\]

Range: \(1 \leq C_1 \leq 3\)

c. \(-1 \leq -2/C_2 \leq -1/3\)

\[1 \geq 2/C_2 \quad 2/C_2 \geq 1/3\]

\[C_2 \geq 2 \quad C_2 \leq 6\]

Range: \(2 \leq C_2 \leq 6\)

d. Since this change leaves \(C_1\) in its range of optimality, the same solution \((x_1 = 3, x_2 = 7)\) is optimal.
e. This change moves $C_2$ outside its range of optimality. The new optimal solution is shown below.

![Feasible Region Diagram]

Alternative optimal solutions exist. Extreme points 2 and 3 and all points on the line segment between them are optimal.

4. By making a small increase in the right-hand side of constraint one and resolving we find a dual price of 1.5 for the constraint. Thus the objective function will increase at the rate of 1.5 per unit increase in the right-hand side.

Since constraint two is not binding, its dual price is zero.

5. a.

![Feasible Region Diagram]

Optimal Solution: $x_1 = 1, x_2 = 3$, Value = 4
b. Slope of Line B = -2  
Slope of Line A = -1/2  

Let $C_1$ = objective function coefficient of $x_1$
$C_2$ = objective function coefficient of $x_2$

\[-2 \leq -C_1/1 \leq -1/2\]
\[2 \geq C_1 \quad C_1 \geq 1/2\]

Range: $1/2 \leq C_1 \leq 2$

c. \[-2 \leq -1/C_2 \leq -1/2\]
\[2 \geq 1/C_2 \quad 1/C_2 \geq 1/2\]
\[C_2 \geq 1/2 \quad 2 \leq C_2\]

Range: $1/2 \leq C_2 \leq 2$

d. Since this change leaves $C_1$ in its range of optimality, the same solution is optimal.

e. This change moves $C_2$ outside of its range of optimality. The new optimal solution is found at extreme point 1; $x_1 = 0, x_2 = 5$.

6.  

Constraint 1: Dual price = -0.333  
Constraint 2: Dual price = -0.333  
Constraint 3: Dual price = 0  

Since this is a minimization problem, the negative dual prices for constraints one and two indicate that by increasing the right-hand side of these constraints by one unit, the value of the objective function will increase by 0.333. The dual price for constraint three indicates that increasing the right hand side a small amount will not affect the value of the optimal solution.
7. a. 

b. Slope of Line B = -3/2  
Slope of Line A = -3/7  

Let \( C_1 \) = objective function coefficient of \( x_1 \)  
\( C_2 \) = objective function coefficient of \( x_2 \)  

\[
-3/2 \leq -\frac{C_1}{7} \leq -3/7  
3/2 \geq \frac{C_1}{7}  
C_1/7 \geq 3/7  
C_1 \leq 21/2  
C_1 \geq 3
\]

Range: \( 3 \leq C_1 \leq 10.5 \)

c. \[-3/2 \leq -\frac{5}{C_2} \leq -3/7 \]

\[
3/2 \geq \frac{5}{C_2}  
5/C_2 \geq 3/7  
C_2 \geq 10/3  
C_2 \leq 35/3
\]

Range: \( 10/3 \leq C_2 \leq 35/3 \)

d. This change moves \( C_1 \) outside its range of optimality. The new optimal solution is found at extreme point 6. It is \( x_1 = 0, x_2 = 10 \). The value is 70.
e. Since this change leaves $C_2$ in its range of optimality, the same solution, $x_1 = 7$ and $x_2 = 7$, with a value of $5(7) + 10(7) = 105$, is optimal.

8. a. 

![Graph showing feasible region and optimal solution](image.png)

b. Constraint 2: Dual price = 0

Constraint 3: Dual price = 0.0769

9. From the solution to Problem 3, we see that the optimal solution will not change as long as the slope of the objective function stays in the following interval:

\[-1 \leq -\frac{C_1}{C_2} \leq -\frac{1}{3}\]

a. The slope of the new objective function is

\[-\frac{C_1}{C_2} = -\frac{3}{4}\]

Since this is in the above interval, these simultaneous changes do not cause a change in the optimal solution.

b. The slope of the new objective function is

\[-\frac{C_1}{C_2} = -\frac{3}{2}.\]

This is outside the above interval; therefore, the optimal solution will change. Extreme point 3 is now optimal; the optimal solution is $x_1 = 6$, $x_2 = 4$, and value = 26.
10. From the Solution to problem 7, we see that the optimal solution will not change as long as the slope of the objective function stays in the following interval:

\[-\frac{3}{2} \leq -\frac{C_1}{C_2} \leq -\frac{3}{7}\]

a. The slope of the new objective function is

\[-\frac{C_1}{C_2} = -\frac{4}{10} = -0.40\]

Since -0.40 > -3/7, we conclude that the optimal solution will change. Extreme point 6 is now optimal. The new optimal solution is \(x_1 = 0, x_2 = 10\). The value of the new optimal solution is 100.

b. The slope of the new objective function is

\[-\frac{C_1}{C_2} = -\frac{4}{8} = -0.50\]

Since \(-0.50 \leq -\frac{3}{2} \leq -\frac{3}{7}\), these simultaneous changes do not cause a change in the optimal solution; it remains \(x_1 = 7, x_2 = 7\).

11. a. Regular Glove = 500
   Catcher’s Mitt = 150
   Value = 3700

b. The finishing and packaging and shipping constraints are binding.

c. Cutting and Sewing = 0
   Finishing = 3
   Packaging and Shipping = 28

   Additional finishing time is worth $3 per unit and additional packaging and shipping time is worth $28 per unit.

d. In the packaging and shipping department. Each additional hour is worth $28.

12. a. 

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular Glove</td>
<td>4 to 12</td>
</tr>
<tr>
<td>Catcher’s Mitt</td>
<td>3.33 to 10</td>
</tr>
</tbody>
</table>

b. As long as the profit contribution for the regular glove is between $4.00 and $12.00, the current solution is optimal.

As long as the profit contribution for the catcher’s mitt stays between $3.33 and $10.00, the current solution is optimal.

The optimal solution is not sensitive to small changes in the profit contributions for the gloves.
c. The dual prices for the resources are applicable over the following ranges:

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Range of Feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting and Sewing</td>
<td>725 to No Upper Limit</td>
</tr>
<tr>
<td>Finishing</td>
<td>133.33 to 400</td>
</tr>
<tr>
<td>Packaging</td>
<td>75 to 135</td>
</tr>
</tbody>
</table>

d. Amount of increase = (28)(20) = $560

13. a. \( U = 800 \)
\( H = 1200 \)
Estimated Annual Return = $8400

b. Constraints 1 and 2. All funds available are being utilized and the maximum permissible risk is being incurred.

c. Constraints Dual Prices

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funds Avail.</td>
<td>0.09</td>
</tr>
<tr>
<td>Risk Max</td>
<td>1.33</td>
</tr>
<tr>
<td>U.S. Oil Max</td>
<td>0</td>
</tr>
</tbody>
</table>

d. No, the optimal solution does not call for investing the maximum amount in U.S. Oil.

14. a. By more than $7.00 per share.

b. By more than $3.50 per share.

c. None. This is only a reduction of 100 shares and the allowable decrease is 200. Management may want to address.

15. a. Optimal solution calls for the production of 560 jars of Western Foods Salsa and 240 jars of Mexico City Salsa; profit is $860.

b. Variables Range of Optimality

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Western Foods Salsa</td>
<td>0.893 to 1.250</td>
</tr>
<tr>
<td>Mexico City Salsa</td>
<td>1.000 to 1.400</td>
</tr>
</tbody>
</table>

c. Constraints Dual Price Interpretation

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Dual Price</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>One more ounce of whole tomatoes will increase profits by $0.125</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>Additional ounces of tomato sauce will not improve profits; slack of 160 ounces.</td>
</tr>
<tr>
<td>3</td>
<td>0.187</td>
<td>One more ounce of tomato paste will increase profits by $0.187</td>
</tr>
</tbody>
</table>

d. Constraints Range of Feasibility

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Range of Feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4320 to 5600</td>
</tr>
<tr>
<td>2</td>
<td>1920 to No Upper Limit</td>
</tr>
<tr>
<td>3</td>
<td>1280 to 1640</td>
</tr>
</tbody>
</table>
16. a. $S = 4000$
   $M = 10,000$
   Total risk = $62,000$

b. 

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>3.75 to No Upper Limit</td>
</tr>
<tr>
<td>$M$</td>
<td>No Upper Limit to 6.4</td>
</tr>
</tbody>
</table>

c. $5(4000) + 4(10,000) = 60,000$

d. $60,000 / 1,200,000 = 0.05$ or 5%

e. 0.057 risk units

f. $0.057(100) = 5.7\%$

17. a. No change in optimal solution; there is no upper limit for the range of optimality for the objective coefficient for $S$.

b. No change in the optimal solution; the objective coefficient for $M$ can increase to 6.4.

c. There is no upper limit on the allowable increase for $C_S$; thus the percentage increase is 0%.

For $C_M$, we obtain $0.3/3.4 = 0.088$. The accumulated percentage change is 8.8%. Thus, the 100% rule is satisfied and the optimal solution will not change.

18. a. $E = 80$, $S = 120$, $D = 0$
   Profit = $16,440$

b. Fan motors and cooling coils

c. Labor hours; 320 hours available.

d. Objective function coefficient range of optimality

   No lower limit to 159.

Since $150$ is in this range, the optimal solution would not change.

19. a. Range of optimality

   $E$ 47.5 to 75
   $S$ 87 to 126
   $D$ No lower limit to 159.

b. 

<table>
<thead>
<tr>
<th>Model</th>
<th>Profit</th>
<th>Change</th>
<th>Allowable Increase/Decrease</th>
<th>[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$63$</td>
<td>Increase $6$</td>
<td>$75 - 63 = 12$</td>
<td>$6/12 = 0.50$</td>
</tr>
<tr>
<td>$S$</td>
<td>$95$</td>
<td>Decrease $2$</td>
<td>$95 - 87 = 8$</td>
<td>$2/8 = 0.25$</td>
</tr>
<tr>
<td>$D$</td>
<td>$135$</td>
<td>Increase $4$</td>
<td>$159 - 135 = 24$</td>
<td>$4/24 = 0.17$</td>
</tr>
</tbody>
</table>

0.92
Since changes are 92% of allowable changes, the optimal solution of $E = 80$, $S = 120$, $D = 0$ will not change.

However, the change in total profit will be:

$$
\begin{align*}
E & \quad 80 \text{ unit } @ \ + \ 6 = \ 480 \\
S & \quad 120 \text{ unit } @ \ - \ 2 = \ -240 \\
\therefore \quad \text{Profit} & = \ 16,440 + 240 = 16,680.
\end{align*}
$$

c. Range of feasibility

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>2080</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>

d. Yes, fan motors = 200 + 100 = 300 is outside the range of feasibility.

The dual price will change.

20. a. Manufacture 100 cases of model A
Manufacture 60 cases of model B
Purchase 90 cases of model B
Total Cost = $2170

b. Demand for model A
Demand for model B
Assembly time
c.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Dual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12.25</td>
</tr>
<tr>
<td>2</td>
<td>-9.0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>.375</td>
</tr>
</tbody>
</table>

If demand for model A increases by 1 unit, total cost will increase by $12.25
If demand for model B increases by 1 unit, total cost will increase by $9.00
If an additional minute of assembly time is available, total cost will decrease by $.375
d. The assembly time constraint. Each additional minute of assembly time will decrease costs by $.375. Note that this will be true up to a value of 1133.33 hours.

Some students may say that the demand constraint for model A should be selected because decreasing the demand by one unit will decrease cost by $12.25. But, carrying this argument to the extreme would argue for a demand of 0.

21. a.

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Ranges of Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM</td>
<td>No lower limit to 11.75</td>
</tr>
<tr>
<td>BM</td>
<td>3.667 to 9</td>
</tr>
<tr>
<td>AP</td>
<td>12.25 to No Upper Limit</td>
</tr>
<tr>
<td>BP</td>
<td>6 to 11.333</td>
</tr>
</tbody>
</table>
Provided a single change of an objective function coefficient is within its above range, the optimal solution AM = 100, BM = 60, AP = 0, and BP = 90 will not change.

b. This change is within the range of optimality. The optimal solution remains AM = 100, BM = 60, AP = 0, and BP = 90. The $11.20 - $10.00 = $1.20 per unit cost increase will increase the total cost to $2170 = $1.20(100) = $2290.

c. Variable | Cost | Change | Allowable Increase/Decrease | Percentage Change
---|---|---|---|---
AM | 10 | Increase 1.20 | 11.75 - 10 = 1.75 | (1.20/1.75)100 = 68.57
BM | 6 | Decrease 1 | 6.0 - 3.667 = 2.333 | (1/2.333)100 = 42.86

111.43% exceeds 100%; therefore, we must resolve the problem.

Resolving the problem provides the new optimal solution: AM = 0, BM = 135, AP = 100, and BP = 15; the total cost is $22,100.

22. a. The optimal solution calls for the production of 100 suits and 150 sport coats. Forty hours of cutting overtime should be scheduled, and no hours of sewing overtime should be scheduled. The total profit is $40,900.

b. The objective coefficient range for suits shows an upper limit of $225. Thus, the optimal solution will not change. But, the value of the optimal solution will increase by ($210-$190)100 = $2000. Thus, the total profit becomes $42,990.

c. The slack for the material coefficient is 0. Because this is a binding constraint, Tucker should consider ordering additional material. The dual price of $34.50 is the maximum extra cost per yard that should be paid. Because the additional handling cost is only $8 per yard, Tucker should order additional material. Note that the dual price of $34.50 is valid up to 1333.33 - 1200 = 133.33 additional yards.

d. The dual price of -$35 for the minimum suit requirement constraint tells us that lowering the minimum requirement by 25 suits will improve profit by $35(25) = $875.

23. a. Let S1 = SuperSaver rentals allocated to room type I  
   S2 = SuperSaver rentals allocated to room type II  
   D1 = Deluxe rentals allocated to room type I  
   D2 = Deluxe rentals allocated to room type II  
   B1 = Business rentals allocated to room type II

The linear programming formulation and solution is given.

MAX 30S1 + 20S2 + 35D1 + 30D2 + 40B2

S.T.
1) 1S1 + 1S2 < 130
2) 1D1 + 1D2 < 60
3) 1B2 < 50
4) 1S1 + 1D1 < 100
5) 1S2 + 1D2 + 1B2 < 120

OPTIMAL SOLUTION
Objective Function Value = 7000.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>100.000</td>
<td>0.000</td>
</tr>
<tr>
<td>S2</td>
<td>10.000</td>
<td>0.000</td>
</tr>
<tr>
<td>D1</td>
<td>0.000</td>
<td>5.000</td>
</tr>
<tr>
<td>D2</td>
<td>60.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B2</td>
<td>50.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>10.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>20.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>30.000</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>20.000</td>
</tr>
</tbody>
</table>

OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>25.000</td>
<td>30.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>S2</td>
<td>0.000</td>
<td>20.000</td>
<td>25.000</td>
</tr>
<tr>
<td>D1</td>
<td>No Lower Limit</td>
<td>35.000</td>
<td>40.000</td>
</tr>
<tr>
<td>D2</td>
<td>25.000</td>
<td>30.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>B2</td>
<td>20.000</td>
<td>40.000</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>

RIGHT HAND SIDE RANGES

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110.000</td>
<td>130.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>2</td>
<td>40.000</td>
<td>60.000</td>
<td>70.000</td>
</tr>
<tr>
<td>3</td>
<td>30.000</td>
<td>50.000</td>
<td>60.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>100.000</td>
<td>120.000</td>
</tr>
<tr>
<td>5</td>
<td>110.000</td>
<td>120.000</td>
<td>140.000</td>
</tr>
</tbody>
</table>

20 SuperSaver rentals will have to be turned away if demands materialize as forecast.

b. RoundTree should accept 110 SuperSaver reservations, 60 Deluxe reservations and 50 Business reservations.

c. Yes, the effect of a person upgrading is an increase in demand for Deluxe accommodations from 60 to 61. From constraint 2, we see that such an increase in demand will increase profit by $10. The added cost of the breakfast is only $5.

d. Convert to a Type I room. From the dual price to constraint 4 we see that this will increase profit by $30.

e. Yes. We would need the forecast of demand for each rental class on the next night. Using the demand forecasts, we would modify the right-hand sides of the first three constraints and resolve.
24. a. Let \( H \) = amount allocated to home loans
    \( P \) = amount allocated to personal loans
    \( A \) = amount allocated to automobile loans

Max \( 0.07H + 0.12P + 0.09A \)

s.t.
\[
\begin{align*}
H + P + A &= 1,000,000 \quad \text{Amount of New Funds} \\
0.6H - 0.4P - 0.4A &\geq 0 \quad \text{Minimum Home Loans} \\
P - 0.6A &\leq 0 \quad \text{Personal Loan Requirement}
\end{align*}
\]

b. \( H = $400,000 \quad P = $225,000 \quad A = $375,000 \)
Total annual return = $88,750
Annual percentage return = 8.875%

c. The range of optimality for \( H \) is No Lower Limit to 0.101. Since 0.09 is within the range of optimality, the solution obtained in part (b) will not change.

d. The dual price for constraint 1 is 0.089. The range of feasibility for constraint 1 is 0 to No Upper Limit. Therefore, increasing the amount of new funds available by $10,000 will increase the total annual return by 0.089 (10,000) = $890.

e. The second constraint now becomes
\[ -0.61H - 0.39P - 0.39A \geq 0 \]

The new optimal solution is
\( H = $390,000 \quad P = $228,750 \quad A = $381,250 \)
Total annual return = $89,062.50, an increase of $312.50
Annual percentage return = 8.906%, an increase of approximately 0.031%.

25. a. Let \( P_1 \) = units of product 1
    \( P_2 \) = units of product 2
    \( P_3 \) = units of product 3

Max \( 30P_1 + 50P_2 + 20P_3 \)

s.t.
\[
\begin{align*}
0.5P_1 + 2P_2 + 0.75P_3 &\leq 40 \quad \text{Machine 1} \\
P_1 + P_2 + 0.5P_3 &\leq 40 \quad \text{Machine 2} \\
2P_1 + 5P_2 + 2P_3 &\leq 100 \quad \text{Labor} \\
0.5P_1 - 0.5P_2 - 0.5P_3 &\leq 0 \quad \text{Max } P_1 \\
-0.2P_1 - 0.2P_2 + 0.8P_3 &\geq 0 \quad \text{Min } P_3 \\
P_1, P_2, P_3 &\geq 0
\end{align*}
\]

A portion of the optimal solution obtained using The Management Scientist is shown.

Objective Function Value = 1250.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>25.000</td>
<td>0.000</td>
</tr>
<tr>
<td>P2</td>
<td>0.000</td>
<td>7.500</td>
</tr>
<tr>
<td>P3</td>
<td>25.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
### Constraint Analysis

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.750</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>2.500</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>12.500</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>10.000</td>
</tr>
<tr>
<td>5</td>
<td>15.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Right Hand Side Ranges

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.250</td>
<td>40.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>2</td>
<td>37.500</td>
<td>40.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>100.000</td>
<td>106.667</td>
</tr>
<tr>
<td>4</td>
<td>-25.000</td>
<td>0.000</td>
<td>5.000</td>
</tr>
<tr>
<td>5</td>
<td>No Lower Limit</td>
<td>0.000</td>
<td>15.000</td>
</tr>
</tbody>
</table>

b. Machine Hours Schedule:
   - Machine 1: 31.25 Hours
   - Machine 2: 37.50 Hours

c. $12.50

d. Increase labor hours to 120; the new optimal product mix is

\[
P_1 = 24 \\
P_2 = 8 \\
P_3 = 16 \\
\text{Profit} = $1440
\]

26. a. Let 
   - \( L \) = number of hours assigned to Lisa 
   - \( D \) = number of hours assigned to David 
   - \( S \) = amount allocated to Sarah

Max \( 30L + 25D + 18S \)

s.t.

\[
\begin{align*}
L + D + S &= 100 & \text{Total Time} \\
0.6L - 0.4D &\geq 0 & \text{Lisa 40% requirement} \\
-0.15L - 0.15D + 0.85S &\geq 0 & \text{Minimum Sarah} \\
-0.25L - 0.25D + S &\leq 0 & \text{Maximum Sarah} \\
L &\leq 50 & \text{Maximum Lisa}
\end{align*}
\]

b. \( L = 48 \) hours \( D = 72 \) hours \( S = 30 \) hours
   Total Cost = $3780

c. The dual price for constraint 5 is 0. Therefore, additional hours for Lisa will not change the solution.

d. The dual price for constraint 3 is 0. Because there is No Lower Limit on the range of feasibility, the optimal solution will not change. Resolving the problem without this constraint will also show that the solution obtained in (b) does not change. Constraint 3, therefore, is really a redundant constraint.
27 a. Let
\[ C_1 = \text{units of component 1 manufactured} \]
\[ C_2 = \text{units of component 2 manufactured} \]
\[ C_3 = \text{units of component 3 manufactured} \]

Max \[ 8C_1 + 6C_2 + 9C_3 \]
s.t. \[ 6C_1 + 4C_2 + 4C_3 \leq 7200 \]
\[ 4C_1 + 5C_2 + 2C_3 \leq 6600 \]
\[ C_3 \leq 200 \]
\[ C_1 \leq 1000 \]
\[ C_2 \leq 1000 \]
\[ C_1 \geq 600 \]
\[ C_2 \geq 0 \]
\[ C_3 \geq 0 \]

The optimal solution is
\[ C_1 = 600 \]
\[ C_2 = 700 \]
\[ C_3 = 200 \]

b. Variable Range of Optimality
<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>No Lower Limit to 9.0</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>5.33 to 9.0</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>6.00 to No Lower Limit</td>
</tr>
</tbody>
</table>

Individual changes in the profit coefficients within these ranges will not cause a change in the optimal number of components to produce.

Constraint Range of Feasibility
<table>
<thead>
<tr>
<th>Constraint</th>
<th>Range of Feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4400 to 7440</td>
</tr>
<tr>
<td>2</td>
<td>6300 to No Upper Limit</td>
</tr>
<tr>
<td>3</td>
<td>100 to 900</td>
</tr>
<tr>
<td>4</td>
<td>600 to No Upper Limit</td>
</tr>
<tr>
<td>5</td>
<td>700 to No Upper Limit</td>
</tr>
<tr>
<td>6</td>
<td>514.29 to 1000</td>
</tr>
</tbody>
</table>

These are the ranges over which the dual prices for the associated constraints are applicable.

d. Nothing, since there are 300 minutes of slack time on the grinder at the optimal solution.
e. No, since at that price it would not be profitable to produce any of component 3.

28. Let \( A = \text{number of shares of stock A} \)
\[ B = \text{number of shares of stock B} \]
\[ C = \text{number of shares of stock C} \]
\[ D = \text{number of shares of stock D} \]
a. To get data on a per share basis multiply price by rate of return or risk measure value.

\[
\begin{align*}
\text{Min} & \quad 10A + 3.5B + 4C + 3.2D \\
\text{s.t.} & \quad 100A + 50B + 80C + 40D = 200,000 \\
& \quad 12A + 4B + 4.8C + 4D \geq 18,000 & (9\% \text{ of } 200,00) \\
& \quad 100A \leq 100,000 \\
& \quad 50B \leq 100,000 \\
& \quad 80C \leq 100,000 \\
& \quad 40D \leq 100,000 \\
& \quad A, B, C, D \geq 0
\end{align*}
\]

Solution: \( A = 333.3, B = 0, C = 833.3, D = 2500 \)

Risk: 14,666.7
Return: 18,000 (9\%) from constraint 2

b. \[
\begin{align*}
\text{Max} & \quad 12A + 4B + 4.8C + 4D \\
\text{s.t.} & \quad 100A + 50B + 80C + 40D = 200,000 \\
& \quad 100A \leq 100,000 \\
& \quad 50B \leq 100,000 \\
& \quad 80C \leq 100,000 \\
& \quad 40D \leq 100,000 \\
& \quad A, B, C, D \geq 0
\end{align*}
\]

Solution: \( A = 1000, B = 0, C = 0, D = 2500 \)

Risk: 10.4 + 3.5B + 4C + 3.2D = 18,000
Return: 22,000 (11\%)

c. The return in part (b) is $4,000 or 2\% greater, but the risk index has increased by 3,333.

Obtaining a reasonable return with a lower risk is a preferred strategy in many financial firms. The more speculative, higher return investments are not always preferred because of their associated higher risk.

29. a. Let:
\[
\begin{align*}
O1 &= \text{percentage of Oak cabinets assigned to cabinetmaker 1} \\
O2 &= \text{percentage of Oak cabinets assigned to cabinetmaker 2} \\
O3 &= \text{percentage of Oak cabinets assigned to cabinetmaker 3} \\
C1 &= \text{percentage of Cherry cabinets assigned to cabinetmaker 1} \\
C2 &= \text{percentage of Cherry cabinets assigned to cabinetmaker 2} \\
C3 &= \text{percentage of Cherry cabinets assigned to cabinetmaker 3}
\end{align*}
\]

\[
\begin{align*}
\text{Min} & \quad 1800O1 + 1764O2 + 1650O3 + 2160C1 + 2016C2 + 1925C3 \\
\text{s.t.} & \quad 50O1 + 60C1 \leq 40 \quad \text{Hours avail. 1} \\
& \quad 42O2 + 48C2 \leq 30 \quad \text{Hours avail. 2} \\
& \quad 30O3 + 35C3 \leq 35 \quad \text{Hours avail. 3} \\
& \quad O1 + O2 + O3 = 1 \quad \text{Oak} \\
& \quad C1 + C2 + C3 = 1 \quad \text{Cherry} \\
& \quad O1, O2, O3, C1, C2, C3 \geq 0
\end{align*}
\]
Note: objective function coefficients are obtained by multiplying the hours required to complete all the oak or cherry cabinets times the corresponding cost per hour. For example, 1800 for $O_1$ is the product of 50 and 36, 1764 for $O_2$ is the product of 42 and 42 and so on.

b.  
<table>
<thead>
<tr>
<th></th>
<th>Cabinetmaker 1</th>
<th>Cabinetmaker 2</th>
<th>Cabinetmaker 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oak</td>
<td>$O_1 = 0.271$</td>
<td>$O_2 = 0.000$</td>
<td>$O_3 = 0.729$</td>
</tr>
<tr>
<td>Cherry</td>
<td>$C_1 = 0.000$</td>
<td>$C_2 = 0.625$</td>
<td>$C_3 = 0.375$</td>
</tr>
</tbody>
</table>

Total Cost = $3672.50

c.  
No, since cabinetmaker 1 has a slack of 26.458 hours. Alternatively, since the dual price for constraint 1 is 0, increasing the right hand side of constraint 1 will not change the value of the optimal solution.

d.  
The dual price for constraint 2 is 1.750. The upper limit on the range of feasibility is 41.143. Therefore, each additional hour of time for cabinetmaker 2 will reduce total cost by $1.75 per hour, up to a maximum of 41.143 hours.

e.  
The new objective function coefficients for $O_2$ and $C_2$ are 42(38) = 1596 and 48(38) = 1824, respectively. The optimal solution does not change but the total cost decreases to $3552.50.

30.  
a.  
Let $M_1 =$ units of component 1 manufactured  
$M_2 =$ units of component 2 manufactured  
$M_3 =$ units of component 3 manufactured  
$P_1 =$ units of component 1 purchased  
$P_2 =$ units of component 2 purchased  
$P_3 =$ units of component 3 purchased

Min $4.50M_1 + 5.00M_2 + 2.75M_3 + 6.50P_1 + 8.80P_2 + 7.00P_3$

s.t.  
$2M_1 + 3M_2 + 4M_3 \leq 21,600 \text{ Production}$

$1M_1 + 1.5M_2 + 3M_3 \leq 15,000 \text{ Assembly}$

$1.5M_1 + 2M_2 + 5M_3 \leq 18,000 \text{ Testing/Packaging}$

$M_1 = 6,000 \text{ Component 1}$

$M_2 = 4,000 \text{ Component 2}$

$M_3 = 3,500 \text{ Component 3}$

$M_1, M_2, M_3, P_1, P_2, P_3 \geq 0$

b.  

<table>
<thead>
<tr>
<th>Source</th>
<th>Component 1</th>
<th>Component 2</th>
<th>Component 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacture</td>
<td>2000</td>
<td>4000</td>
<td>1400</td>
</tr>
<tr>
<td>Purchase</td>
<td>4000</td>
<td>0</td>
<td>2100</td>
</tr>
</tbody>
</table>

Total Cost: $73,550

c.  
Since the slack is 0 in the production and the testing & packaging departments, these department are limiting Benson's manufacturing quantities.
Sensitivity Analysis and Interpretation

Dual prices information:
Production $0.906/minute x 60 minutes = $54.36 per hour
Testing/Packaging $0.125/minute x 60 minutes = $ 7.50 per hour

d. The dual price is -$7.969. This tells us that the value of the optimal solution will worsen (the cost will increase) by $7.969 for an additional unit of component 2. Note that although component 2 has a purchase cost per unit of $8.80, it would only cost Benson $7.969 to obtain an additional unit of component 2.

31. Let $RS$ = number of regular flex shafts made in San Diego
   $RT$ = number of regular flex shafts made in Tampa
   $SS$ = number of stiff flex shafts made in San Diego
   $ST$ = number of shift flex shafts made in Tampa

Min $5.25 RS + 4.95 RT + 5.40 SS + 5.70 ST$

s.t.

$RS + SS \leq 120,000$

$RT + ST \leq 180,000$

$RS + RT = 200,000$

$SS + ST = 75,000$

$RS, RT, SS, ST \geq 0$

OPTIMAL SOLUTION

Objective Function Value = 1401000.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
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<td>0.600</td>
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</table>

Constraint Slack/Surplus Dual Prices

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25000.000</td>
<td>0.000</td>
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<tr>
<td>4</td>
<td>0.000</td>
<td>-5.40</td>
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OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
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<th>Upper Limit</th>
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</thead>
<tbody>
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<tr>
<td>ST</td>
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<td>5.250</td>
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RIGHT HAND SIDE RANGES

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<th>Upper Limit</th>
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<td>200000.000</td>
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<td>4</td>
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<td>75000.000</td>
<td>100000.000</td>
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</table>

32. a. Let $G = \text{amount invested in growth stock fund}$
   $S = \text{amount invested in income stock fund}$
Sensitivity Analysis and Interpretation

\( M = \) amount invested in money market fund

\[
\begin{align*}
\text{Max} & \quad 0.20G + 0.10S + 0.06M \\
\text{s.t.} & \quad 0.10G + 0.05S + 0.01M \leq (0.05)(300,000) \quad \text{Hartmann's max risk} \\
& \quad G \geq (0.10)(300,000) \quad \text{Growth fund min.} \\
& \quad S \geq (0.10)(300,000) \quad \text{Income fund min.} \\
& \quad M \geq (0.20)(300,000) \quad \text{Money market min.} \\
& \quad G + S + M \leq 300,000 \quad \text{Funds available} \\
G, S, M \geq 0
\end{align*}
\]

b. The solution to Hartmann's portfolio mix problem is given.

Objective Function Value = 36000.000

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<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
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<tr>
<td>S</td>
<td>30000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>M</td>
<td>150000.000</td>
<td>0.000</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
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<td>0.044</td>
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OBJECTIVE COEFFICIENT RANGES

<table>
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<tr>
<th>Variable</th>
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<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
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<td>0.200</td>
<td>0.600</td>
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<tr>
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</tr>
<tr>
<td>M</td>
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<td>0.060</td>
<td>0.200</td>
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RIGHT HAND SIDE RANGES

<table>
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<tr>
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<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
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<td>15000.000</td>
<td>23100.000</td>
</tr>
<tr>
<td>2</td>
<td>No Lower Limit</td>
<td>30000.000</td>
<td>120000.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>30000.000</td>
<td>192000.016</td>
</tr>
<tr>
<td>4</td>
<td>No Lower Limit</td>
<td>60000.000</td>
<td>150000.000</td>
</tr>
<tr>
<td>5</td>
<td>219000.000</td>
<td>300000.000</td>
<td>1110000.500</td>
</tr>
</tbody>
</table>

c. These are given by the ranges of optimality on the objective function coefficients. The portfolio above will be optimal as long as the yields remain in the following intervals:

- Growth stock \( 0.15 \leq c_1 \leq 0.60 \)
- Income stock No Lower Limit \( < c_2 \leq 0.122 \)
- Money Market \( 0.02 \leq c_3 \leq 0.20 \)

d. The dual price for the first constraint provides this information. A change in the risk index from 0.05 to 0.06 would increase the constraint RHS by 3000 (from 15,000 to 18,000). This is within the range of
feasibility, so the dual price of 1.556 is applicable. The value of the optimal solution would increase by 
(3000)(1.556) = 4668.

Hartmann's yield with a risk index of 0.05 is 

\[ \frac{36,000}{300,000} = 0.12 \]

His yield with a risk index of 0.06 would be 

\[ \frac{40,668}{300,000} = 0.1356 \]

e. This change is outside the range of optimality so we must resolve the problem. The solution is shown 
below.

LINEAR PROGRAMMING PROBLEM

MAX .1G + .1S + .06M

S.T.

1) .1G + .05S + .01M < 15000
2) G > 30000
3) S > 30000
4) M > 60000
5) G + S + M < 300000

OPTIMAL SOLUTION

Objective Function Value = 27600.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>48000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>S</td>
<td>192000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>M</td>
<td>60000.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Slack</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>18000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>162000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>-0.040</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.100</td>
</tr>
</tbody>
</table>

OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.100</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>S</td>
<td>0.078</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>M</td>
<td>No Lower Limit</td>
<td>0.060</td>
<td>0.100</td>
</tr>
</tbody>
</table>

RIGHT HAND SIDE RANGES

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14100.000</td>
<td>15000.000</td>
<td>23100.000</td>
</tr>
</tbody>
</table>
f. The client’s risk index and the amount of funds available.

g. With the new yield estimates, Pfeiffer would solve a new linear program to find the optimal portfolio mix for each client. Then by summing across all 50 clients he would determine the total amount that should be placed in a growth fund, an income fund, and a money market fund. Pfeiffer then would make the necessary switches to have the correct total amount in each account. There would be no actual switching of funds for individual clients.

33. a. Relevant cost since LaJolla Beverage Products can purchase wine and fruit juice on an as-needed basis.

b. Let $W =$ gallons of white wine $R =$ gallons of rose wine $F =$ gallons of fruit juice

Max $1.5W + 1R + 2F$

s.t.

- $0.5W - 0.5R - 0.5F \geq 0$ % white
- $-0.2W + 0.8R - 0.2F \geq 0$ % rose minimum
- $-0.3W + 0.7R - 0.3F \leq 0$ % rose maximum
- $-0.2W - 0.2R + 0.8F = 0$ % fruit juice
- $W \leq 10000$ Available white
- $R \leq 8000$ Available rose
- $W, R, F \geq 0$

Optimal Solution: $W = 10,000, R = 6000, F = 4000$

profit contribution = $29,000.$

c. Since the cost of the wine is a relevant cost, the dual price of $2.90 is the maximum premium (over the normal price of $1.00) that LaJolla Beverage Products should be willing to pay to obtain one additional gallon of white wine. In other words, at a price of $3.90 = 2.90 + 1.00$, the additional cost is exactly equal to the additional revenue.

d. No; only 6000 gallons of the rose are currently being used.

e. Requiring 50% plus one gallon of white wine would reduce profit by $2.40. Note to instructor: Although this explanation is technically correct, it does not provide an explanation that is especially useful in the context of the problem. Alternatively, we find it useful to explore the question of what would happen if the white wine requirement were changed to at least 51%. Note that in this case, the first constraint would change to $0.49W - 0.51R - 0.51F \geq 0$. This shows the student that the coefficients on the left-hand side are changing; note that this is beyond the scope of sensitivity analysis discussed in this chapter. Resolving the problem with this revised constraint will show the effect on profit of a 1% change.

f. Allowing the amount of fruit juice to exceed 20% by one gallon will increase profit by $1.00.

34. a. Let $L =$ minutes devoted to local news $N =$ minutes devoted to national news $W =$ minutes devoted to weather $S =$ minutes devoted to sports

Min $300L + 200N + 100W + 100S$

s.t.
Sensitivity Analysis and Interpretation

\[ \begin{align*}
L + N + W + S &= 20 \quad \text{Time available} \\
L &\geq 3 \quad \text{15\% local} \\
L + N &\geq 10 \quad \text{50\% requirement} \\
W - S &\leq 0 \quad \text{Weather - sports} \\
-L - N + S &\leq 0 \quad \text{Sports requirement} \\
W &\geq 4 \quad \text{20\% weather} \\
L, N, W, S &\geq 0
\end{align*} \]

Optimal Solution: \( L = 3, N = 7, W = 5, S = 5 \)
Total cost = $3,300

b. Each additional minute of broadcast time increases cost by $100; conversely, each minute reduced will decrease cost by $100. These interpretations are valid for increase up to 10 minutes and decreases up to 2 minutes from the current level of 20 minutes.

c. If local coverage is increased by 1 minute, total cost will increase by $100.

d. If the time devoted to local and national news is increased by 1 minute, total cost will increase by $100.

e. Increasing the sports by one minute will have no effect for this constraint since the dual price is 0.

35. a. Let
   \( B \) = number of copies done by Benson Printing
   \( J \) = number of copies done by Johnson Printing
   \( L \) = number of copies done by Lakeside Litho

   \[ \begin{align*}
   \text{min} & \quad 2.45B + 2.5J + 2.75L \\
   \text{s.t.} & \quad B \leq 30,000 \quad \text{Benson} \\
               & \quad J \leq 50,000 \quad \text{Johnson} \\
               & \quad L \leq 50,000 \quad \text{Lakeside} \\
               & \quad 0.9B + 0.99J + 0.995L = 75,000 \quad \# \text{useful reports} \\
               & \quad B - 0.1J \geq 0 \quad \text{Benson - Johnson \%} \\
               & \quad L \geq 30,000 \quad \text{Minimum Lakeside} \\
               & \quad B, J, L \geq 0
\end{align*} \]

Optimal Solution: \( B = 4,181, J = 41,806, L = 30,000 \)

b. Suppose that Benson printing has a defective rate of 2\% instead of 10\%. The new optimal solution would increase the copies assigned to Benson printing to 30,000. In this case, the additional copies assigned to Benson Printing would reduce on a one-for-one basis the number assigned to Johnson Printing.

c. If the Lakeside Litho requirement is reduced by 1 unit, total cost will decrease by $0.2210.
Chapter 4
Linear Programming Applications

Learning Objectives

1. Learn about applications of linear programming that have been encountered in practice.
2. Develop an appreciation for the diversity of problems that can be modeled as linear programs.
3. Obtain practice and experience in formulating realistic linear programming models.
4. Understand linear programming applications such as:
   
<table>
<thead>
<tr>
<th>Media selection</th>
<th>Production scheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio selection</td>
<td>Work force assignments</td>
</tr>
<tr>
<td>Financial mix strategy</td>
<td>Blending problems</td>
</tr>
<tr>
<td>Data envelopment analysis</td>
<td>Revenue management</td>
</tr>
</tbody>
</table>
Solutions:

1. a. Let $T =$ number of television spot advertisements  
   $R =$ number of radio advertisements  
   $N =$ number of newspaper advertisements  

Max $100,000T + 18,000R + 40,000N$  

s.t. 

- $2,000T + 300R + 600N \leq 18,200$  
  Budget  
- $T \leq 10$  
  Max TV  
- $R \leq 20$  
  Max Radio  
- $N \leq 10$  
  Max News  
- $-0.5T + 0.5R - 0.5N \leq 0$  
  Max 50% Radio  
- $0.9T - 0.1R - 0.1N \geq 0$  
  Min 10% TV  

$T, R, N, \geq 0$

Solution:  

- $T = 4$  
  $\$8,000  
- $R = 14$  
  $4,200$  
- $N = 10$  
  $6,000$  

$\$18,200  
Audience = 1,052,000.

This information can be obtained from *The Management Scientist* as follows.

OPTIMAL SOLUTION

Objective Function Value = 1052000.000

<table>
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<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
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<tr>
<td>$R$</td>
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<tr>
<td>$N$</td>
<td>10.000</td>
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</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
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</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>5217.391</td>
</tr>
<tr>
<td>6</td>
<td>1.200</td>
<td>0.000</td>
</tr>
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</table>
OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>-18000.000</td>
<td>100000.000</td>
<td>120000.000</td>
</tr>
<tr>
<td>R</td>
<td>15000.000</td>
<td>18000.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>N</td>
<td>28173.913</td>
<td>40000.000</td>
<td>No Upper Limit</td>
</tr>
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</table>

RIGHT HAND SIDE RANGES

<table>
<thead>
<tr>
<th>Constraint</th>
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<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14750.000</td>
<td>18200.000</td>
<td>31999.996</td>
</tr>
<tr>
<td>2</td>
<td>4.000</td>
<td>10.000</td>
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</tr>
<tr>
<td>3</td>
<td>14.000</td>
<td>20.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>10.000</td>
<td>12.339</td>
</tr>
<tr>
<td>5</td>
<td>-8.050</td>
<td>0.000</td>
<td>2.936</td>
</tr>
<tr>
<td>6</td>
<td>No Lower Limit</td>
<td>0.000</td>
<td>1.200</td>
</tr>
</tbody>
</table>

b. The dual price for the budget constraint is 51.30. Thus, a $100 increase in budget should provide an increase in audience coverage of approximately 5,130. The right-hand-side range for the budget constraint will show this interpretation is correct.

2. a. Let $x_1 =$ units of product 1 produced  
     $x_2 =$ units of product 2 produced

Max $30x_1 + 15x_2$

s.t. $x_1 + 0.35x_2 \leq 100$ Dept. A
     $0.30x_1 + 0.20x_2 \leq 36$ Dept. B
     $0.20x_1 + 0.50x_2 \leq 50$ Dept. C

$x_1, x_2 \geq 0$

Solution: $x_1 = 77.89, x_2 = 63.16$ Profit = 3284.21

b. The dual price for Dept. A is $15.79, for Dept. B it is $47.37, and for Dept. C it is $0.00. Therefore we would attempt to schedule overtime in Departments A and B. Assuming the current labor available is a sunk cost, we should be willing to pay up to $15.79 per hour in Department A and up to $47.37 in Department B.

c. Let $x_A =$ hours of overtime in Dept. A  
   $x_B =$ hours of overtime in Dept. B  
   $x_C =$ hours of overtime in Dept. C
Max \[ 30x_1 + 15x_2 - 18A - 22.5B - 12C \]
\[
\text{s.t.}\]
\[ x_1 + 0.35x_2 - A \leq 100 \]
\[ 0.30x_1 + 0.20x_2 - B \leq 36 \]
\[ 0.20x_1 + 0.50x_2 - C \leq 50 \]
\[ A \leq 10 \]
\[ B \leq 6 \]
\[ C \leq 8 \]
\[ x_1, x_2, x_A, x_B, x_C \geq 0 \]

\[ x_1 = 87.21 \]
\[ x_2 = 65.12 \]
Profit = $3341.34

<table>
<thead>
<tr>
<th>Overtime</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dept. A</td>
<td>10 hrs.</td>
</tr>
<tr>
<td>Dept. B</td>
<td>3.186 hrs</td>
</tr>
<tr>
<td>Dept. C</td>
<td>0 hours</td>
</tr>
</tbody>
</table>

Increase in Profit from overtime = $3341.34 - 3284.21 = $57.13

3. \[ x_1 = \$ \text{automobile loans} \]
\[ x_2 = \$ \text{furniture loans} \]
\[ x_3 = \$ \text{other secured loans} \]
\[ x_4 = \$ \text{signature loans} \]
\[ x_5 = \$ \text{"risk free" securities} \]

Max \[ 0.08x_1 + 0.10x_2 + 0.11x_3 + 0.12x_4 + 0.09x_5 \]
\[
\text{s.t.}\]
\[ x_5 \leq 600,000 \] [1]
\[ x_4 \leq 0.10(x_1 + x_2 + x_3 + x_4) \]
\[ x_2 + x_3 \leq x_1 \] [2]
\[ x_1 + x_2 + x_3 \leq 0 \] [3]
\[ x_3 + x_4 \leq x_5 \] [4]
\[ x_1 + x_2 + x_3 + x_4 + x_5 = 2,000,000 \] [5]
\[ x_1, x_2, x_3, x_4, x_5 \geq 0 \]

Solution:
Automobile Loans \((x_1)\) = $630,000
Furniture Loans \((x_2)\) = $170,000
Other Secured Loans \((x_3)\) = $460,000
Signature Loans \((x_4)\) = $140,000
Risk Free Loans \((x_5)\) = $600,000

Annual Return $188,800 (9.44%)
4.  

a. \[ x_1 = \text{pounds of bean 1} \]
\[ x_2 = \text{pounds of bean 2} \]
\[ x_3 = \text{pounds of bean 3} \]

Max \[ 0.50x_1 + 0.70x_2 + 0.45x_3 \]

s.t.

\[ 75x_1 + 85x_2 + 60x_3 \geq 75 \]
\[ x_1 + x_2 + x_3 \]

or

\[ 10x_2 - 15x_3 \geq 0 \]

Aroma

\[ 86x_1 + 88x_2 + 75x_3 \geq 80 \]

Taste

\[ 6x_1 + 8x_2 - 5x_3 \geq 0 \]

\[ x_1, x_2, x_3 \geq 0 \]

Optimal Solution: \[ x_1 = 500, x_2 = 300, x_3 = 200 \] Cost: $550

b. Cost per pound = $550/1000 = $0.55

c. Surplus for aroma: \[ s_1 = 0; \] thus aroma rating = 75

Surplus for taste: \[ s_2 = 4400; \] thus taste rating = \( 80 + 4400/1000 \) lbs. = 84.4

d. Dual price = -$0.60. Extra coffee can be produced at a cost of $0.60 per pound.

5. Let \[ x_1 = \text{amount of ingredient A} \]
\[ x_2 = \text{amount of ingredient B} \]
\[ x_3 = \text{amount of ingredient C} \]

Min \[ 0.10x_1 + 0.03x_2 + 0.09x_3 \]

s.t.

\[ x_1 + x_2 + x_3 \geq 10 \] [1]
\[ x_1 + x_2 + x_3 \leq 15 \] [2]
\[ x_2 \geq x_1 \] [3]

or

\[ x_1 - x_2 \geq 0 \] [3]

\[ x_3 \geq 0 \] [3]

or

\[ -1/2x_1 + x_3 \geq 0 \] [4]

\[ x_1, x_2, x_3 \geq 0 \]

Solution: \[ x_1 = 4, x_2 = 4, x_3 = 2 \] Cost = $0.70 per gallon.
6. Let \( x_1 \) = units of product 1  
\( x_2 \) = units of product 2  
\( b_1 \) = labor-hours Dept. A  
\( b_2 \) = labor-hours Dept. B

Max \( 25x_1 + 20x_2 + 0b_1 + 0b_2 \)  
\( \text{s.t.} \)
\( 6x_1 + 8x_2 - b_1 = 0 \)  
\( 12x_1 + 10x_2 - b_2 = 0 \)  
\( b_1 + b_2 \leq 900 \)  
\( x_1, x_2, b_1, b_2 \geq 0 \)

Solution: \( x_1 = 50, x_2 = 0, b_1 = 300, b_2 = 600 \) Profit: $1,250

7. a. Let \( F \) = total funds required to meet the six years of payments  
\( G_1 \) = units of government security 1  
\( G_2 \) = units of government security 2  
\( S_i \) = investment in savings at the beginning of year \( i \)

Note: All decision variables are expressed in thousands of dollars

MIN \( F \)  
\( \text{s.t.} \)
1) \( F - 1.055G_1 - 1.000G_2 - S_1 = 190 \)  
2) \( 0.0675G_1 + 0.05125G_2 + 1.04S_1 - S_2 = 215 \)  
3) \( 0.0675G_1 + 0.05125G_2 + 1.04S_2 - S_3 = 240 \)  
4) \( 1.0675G_1 + 0.05125G_2 + 1.04S_3 - S_4 = 285 \)  
5) \( 1.05125G_2 + 1.04S_4 - S_5 = 315 \)  
6) \( 1.04S_5 - S_6 = 460 \)

OPTIMAL SOLUTION

Objective Function Value = \( \boxed{1484.96655} \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1484.96655</td>
<td>0.00000</td>
</tr>
<tr>
<td>G1</td>
<td>232.39356</td>
<td>0.00000</td>
</tr>
<tr>
<td>G2</td>
<td>720.38782</td>
<td>0.00000</td>
</tr>
<tr>
<td>S1</td>
<td>329.40353</td>
<td>0.00000</td>
</tr>
<tr>
<td>S2</td>
<td>180.18611</td>
<td>0.00000</td>
</tr>
<tr>
<td>S3</td>
<td>0.00000</td>
<td>0.02077</td>
</tr>
<tr>
<td>S4</td>
<td>0.00000</td>
<td>0.01942</td>
</tr>
<tr>
<td>S5</td>
<td>442.30769</td>
<td>0.00000</td>
</tr>
<tr>
<td>S6</td>
<td>0.00000</td>
<td>0.78551</td>
</tr>
</tbody>
</table>
The current investment required is $1,484,967. This calls for investing $232,394 in government security 1 and $720,388 in government security 2. The amounts, placed in savings are $329,404, $180,186 and $442,308 for years 1, 2 and 5 respectively. No funds are placed in savings for years 3, 4 and 6.

b. The dual price for constraint 6 indicates that each $1 reduction in the payment required at the beginning of year 6 will reduce the amount of money Hoxworth must pay the trustee by $0.78551. The lower limit on the right-hand-side range is zero so a $60,000 reduction in the payment at the beginning of year 6 will save Hoxworth $60,000 (0.78551) = $47,131.

c. The dual price for constraint 1 shows that every dollar of reduction in the initial payment is worth $1.00 to Hoxworth. So Hoxworth should be willing to pay anything less than $40,000.

d. To reformulate this problem, one additional variable needs to be added, the right-hand sides for the original constraints need to be shifted ahead by one, and the right-hand side of the first constraint needs to be set equal to zero. The value of the optimal solution with this formulation is $1,417,739. Hoxworth will save $67,228 by having the payments moved to the end of each year.

The revised formulation is shown below:

MIN $F$

S.T.

1) $F - 1.055G1 - 1.000G2 - S1 = 0$
2) $.0675G1 + .05125G2 + 1.04S1 - S2 = 190$
3) $.0675G1 + .05125G2 + 1.04S2 - S3 = 215$
4) $1.0675G1 + .05125G2 + 1.04S3 - S4 = 240$
5) $1.05125G2 + 1.04S4 - S5 = 285$
6) $1.04S5 - S6 = 315$
7) $1.04S6 - S7 = 460$

8. Let $x_1$ = the number of officers scheduled to begin at 8:00 a.m.
   $x_2$ = the number of officers scheduled to begin at noon
   $x_3$ = the number of officers scheduled to begin at 4:00 p.m.
   $x_4$ = the number of officers scheduled to begin at 8:00 p.m.
   $x_5$ = the number of officers scheduled to begin at midnight
   $x_6$ = the number of officers scheduled to begin at 4:00 a.m.
The objective function to minimize the number of officers required is as follows:

\[
\text{Min } x_1 + x_2 + x_3 + x_4 + x_5 + x_6
\]

The constraints require the total number of officers of duty each of the six four-hour periods to be at least equal to the minimum officer requirements. The constraints for the six four-hour periods are as follows:

<table>
<thead>
<tr>
<th>Time of Day</th>
<th>(x_1) + (x_2) + (x_3) + (x_4) + (x_5) + (x_6) (\geq) 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - noon</td>
<td>(x_1) + (x_6) (\geq) 5</td>
</tr>
<tr>
<td>noon to 4:00 p.m.</td>
<td>(x_1) + (x_2) (\geq) 6</td>
</tr>
<tr>
<td>4:00 p.m. - 8:00 p.m.</td>
<td>(x_2) + (x_3) (\geq) 10</td>
</tr>
<tr>
<td>8:00 p.m. - midnight</td>
<td>(x_3) + (x_4) (\geq) 7</td>
</tr>
<tr>
<td>midnight - 4:00 a.m.</td>
<td>(x_4) + (x_5) (\geq) 4</td>
</tr>
<tr>
<td>4:00 a.m. - 8:00 a.m.</td>
<td>(x_5) + (x_6) (\geq) 6</td>
</tr>
</tbody>
</table>

Schedule 19 officers as follows:

- \(x_1 = 3\) begin at 8:00 a.m.
- \(x_2 = 3\) begin at noon
- \(x_3 = 7\) begin at 4:00 p.m.
- \(x_4 = 0\) begin at 8:00 p.m.
- \(x_5 = 4\) begin at midnight
- \(x_6 = 2\) begin at 4:00 a.m.

9. a. Let each decision variable, A, P, M, H and G, represent the fraction or proportion of the total investment placed in each investment alternative.

\[
\text{Max } .073A + .103P + .064M + .075H + .045G
\]

s.t.

\[
A + P + M + H + G = 1
\]

\[
.5A + .5P - .5M - .5H \leq 0
\]

\[
-.5A - .5P + .5M + .5H \leq 0
\]

\[
-.25M - .25H + G \geq 0
\]

\[
-.6A + .4P \leq 0
\]

\[
A, P, M, H, G \geq 0
\]

Solution: Objective function = 0.079 with

- Atlantic Oil = 0.178
- Pacific Oil = 0.267
- Midwest Oil = 0.000
- Huber Steel = 0.444
- Government Bonds = 0.111
b. For a total investment of $100,000, we show

<table>
<thead>
<tr>
<th>Investment</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic Oil</td>
<td>$17,800</td>
</tr>
<tr>
<td>Pacific Oil</td>
<td>$26,700</td>
</tr>
<tr>
<td>Midwest Oil</td>
<td>$0.00</td>
</tr>
<tr>
<td>Huber Steel</td>
<td>$44,400</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>$11,100</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$100,000</strong></td>
</tr>
</tbody>
</table>

c. Total earnings = $100,000 (.079) = $7,900

d. Marginal rate of return is .079

10. a. Let \( S \) = the proportion of funds invested in stocks
    \( B \) = the proportion of funds invested in bonds
    \( M \) = the proportion of funds invested in mutual funds
    \( C \) = the proportion of funds invested in cash

The linear program and optimal solution obtained using The Management Scientist is as follows:

\[
\text{MAX} \quad 0.1S + 0.03B + 0.04M + 0.01C
\]

\[
\text{S.T.}
\]

1) \( 1S + 1B + 1M + 1C = 1 \)
2) \( 0.8S + 0.2B + 0.3M < 0.4 \)
3) \( 1S < 0.75 \)
4) \( -1B + 1M > 0 \)
5) \( 1C > 0.1 \)
6) \( 1C < 0.3 \)

**OPTIMAL SOLUTION**

Objective Function Value = 0.054

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.409</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>0.145</td>
<td>0.000</td>
</tr>
<tr>
<td>M</td>
<td>0.145</td>
<td>0.000</td>
</tr>
<tr>
<td>C</td>
<td>0.300</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.118</td>
</tr>
<tr>
<td>3</td>
<td>0.341</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.200</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.005</td>
</tr>
</tbody>
</table>

**OBJECTIVE COEFFICIENT RANGES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.090</td>
<td>0.100</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>B</td>
<td>0.028</td>
<td>0.030</td>
<td>0.036</td>
</tr>
<tr>
<td>M</td>
<td>No Lower Limit</td>
<td>0.040</td>
<td>0.042</td>
</tr>
<tr>
<td>C</td>
<td>0.005</td>
<td>0.010</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>

**RIGHT HAND SIDE RANGES**
LP Applications

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.800</td>
<td>1.000</td>
<td>1.900</td>
</tr>
<tr>
<td>2</td>
<td>0.175</td>
<td>0.400</td>
<td>0.560</td>
</tr>
<tr>
<td>3</td>
<td>0.409</td>
<td>0.750</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>4</td>
<td>-0.267</td>
<td>0.000</td>
<td>0.320</td>
</tr>
<tr>
<td>5</td>
<td>No Lower Limit</td>
<td>0.100</td>
<td>0.300</td>
</tr>
<tr>
<td>6</td>
<td>0.100</td>
<td>0.300</td>
<td>0.500</td>
</tr>
</tbody>
</table>

The optimal allocation among the four investment alternatives is

- Stocks 40.9%
- Bonds 14.5%
- Mutual Funds 14.5%
- Cash 30.0%

The annual return associated with the optimal portfolio is 5.4%

The total risk = 0.409(0.8) + 0.145(0.2) + 0.145(0.3) + 0.300(0.0) = 0.4

b. Changing the right-hand-side value for constraint 2 to 0.18 and resolving using The Management Scientist we obtain the following optimal solution:

- Stocks 0.0%
- Bonds 36.0%
- Mutual Funds 36.0%
- Cash 28.0%

The annual return associated with the optimal portfolio is 2.52%

The total risk = 0.0(0.8) + 0.36(0.2) + 0.36(0.3) + 0.28(0.0) = 0.18

c. Changing the right-hand-side value for constraint 2 to 0.7 and resolving using The Management Scientist we obtain the following optimal solution:

The optimal allocation among the four investment alternatives is

- Stocks 75.0%
- Bonds 0.0%
- Mutual Funds 15.0%
- Cash 10.0%

The annual return associated with the optimal portfolio is 8.2%

The total risk = 0.75(0.8) + 0.0(0.2) + 0.15(0.3) + 0.10(0.0) = 0.65

d. Note that a maximum risk of 0.7 was specified for this aggressive investor, but that the risk index for the portfolio is only 0.65. Thus, this investor is willing to take more risk than the solution shown above provides. There are only two ways the investor can become even more aggressive: increase the proportion invested in stocks to more than 75% or reduce the cash requirement of at least 10% so that additional cash could be put into stocks. For the data given here, the investor should ask the investment advisor to relax either or both of these constraints.
e. Defining the decision variables as proportions means the investment advisor can use the linear programming model for any investor, regardless of the amount of the investment. All the investor advisor needs to do is to establish the maximum total risk for the investor and resolve the problem using the new value for maximum total risk.

11. Let $x_{ij}$ = units of component $i$ purchased from supplier $j$

$$\text{Min} \quad \frac{12x_{11} + 13x_{12} + 14x_{13} + 10x_{21} + 11x_{22} + 10x_{23}}{2}$$

s.t.

$$x_{11} + x_{12} = 1000$$

$$x_{11} + x_{21} + x_{22} = 800$$

$$x_{11} + x_{21} = 600$$

$$x_{12} + x_{22} \leq 1000$$

$$x_{13} + x_{23} \leq 800$$

$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0$$

Solution:

<table>
<thead>
<tr>
<th>Supplier</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component 1</td>
<td>600</td>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>Component 2</td>
<td>0</td>
<td>0</td>
<td>800</td>
</tr>
</tbody>
</table>

Purchase Cost = $20,400$

12. Let $B_i$ = pounds of shrimp bought in week $i$, $i = 1,2,3,4$

$S_i$ = pounds of shrimp sold in week $i$, $i = 1,2,3,4$

$I_i$ = pounds of shrimp held in storage (inventory) in week $i$

Total purchase cost = $6.00B_1 + 6.20B_2 + 6.65B_3 + 5.55B_4$

Total sales revenue = $6.00S_1 + 6.20S_2 + 6.65S_3 + 5.55S_4$

Total storage cost = $0.15I_1 + 0.15I_2 + 0.15I_3 + 0.15I_4$

Total profit contribution = (total sales revenue) - (total purchase cost) - (total storage cost)

Objective: maximize total profit contribution subject to balance equations for each week, storage capacity for each week, and ending inventory requirement for week 4.

$$\text{Max} \quad 6.00S_1 + 6.20S_2 + 6.65S_3 + 5.55S_4 - 6.00B_1 - 6.20B_2 - 6.65B_3 - 5.55B_4 - 0.15I_1 - 0.15I_2 - 0.15I_3 - 0.15I_4$$

s.t.

$$20,000 + B_1 - S_1 = I_1 \quad \text{Balance eq. - week 1}$$

$$I_1 + B_2 - S_2 = I_2 \quad \text{Balance eq. - week 2}$$

$$I_2 + B_3 - S_3 = I_3 \quad \text{Balance eq. - week 3}$$

$$I_3 + B_4 - S_4 = I_4 \quad \text{Balance eq. - week 4}$$

$I_1 \leq 100,000 \quad \text{Storage cap. - week 1}$

$I_2 \leq 100,000 \quad \text{Storage cap. - week 2}$

$I_3 \leq 100,000 \quad \text{Storage cap. - week 3}$
\[ I_4 \leq 100,000 \quad \text{Storage cap. - week 4} \]
\[ I_4 \geq 25,000 \quad \text{Req'd inv. - week 4} \]
all variables \( \geq 0 \)

Note that the first four constraints can be written as follows:

\[ I_1 - B_1 + S_1 = 20,000 \]
\[ I_1 - I_2 + B_2 - S_2 = 0 \]
\[ I_2 - I_3 + B_3 - S_3 = 0 \]
\[ I_3 - I_4 + B_4 - S_4 = 0 \]

The optimal solution obtained using *The Management Scientist* follows:

<table>
<thead>
<tr>
<th>Week (i)</th>
<th>( B_i )</th>
<th>( S_i )</th>
<th>( I_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80,000</td>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>100,000</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>25,000</td>
<td>0</td>
<td>25,000</td>
</tr>
</tbody>
</table>

Total profit contribution = $12,500

Note however, ASC started week 1 with 20,000 pounds of shrimp and ended week 4 with 25,000 pounds of shrimp. During the 4-week period, ASC has taken profits to reinvest and build inventory by 5000 pounds in anticipation of future higher prices. The amount of profit reinvested in inventory is \((5.55 + 0.15)(5000) = 28,500\). Thus, total profit for the 4-week period including reinvested profit is $12,500 + $28,500 = $41,000.

13. Let

- \( BR = \) pounds of Brazilian beans purchased to produce Regular
- \( BD = \) pounds of Brazilian beans purchased to produce DeCaf
- \( CR = \) pounds of Colombian beans purchased to produce Regular
- \( CD = \) pounds of Colombian beans purchased to produce DeCaf

<table>
<thead>
<tr>
<th>Type of Bean</th>
<th>Cost per pound ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazilian</td>
<td>1.10(0.47) = 0.517</td>
</tr>
<tr>
<td>Colombian</td>
<td>1.10(0.62) = 0.682</td>
</tr>
</tbody>
</table>

Total revenue = 3.60(BR + CR) + 4.40(BD + CD)

Total cost of beans = 0.517(BR + BD) + 0.682(CR + CD)

Total production cost = 0.80(BR + CR) + 1.05(BD + CD)

Total packaging cost = 0.25(BR + CR) + 0.25(BD + CD)

Total contribution to profit = (total revenue) - (total cost of beans) - (total production cost)

\[ \therefore \text{Total contribution to profit} = 2.033BR + 2.583BD + 1.868CR + 2.418CD \]

Regular % constraint

\[ BR = 0.75(BR + CR) \]
\[ 0.25BR - 0.75CR = 0 \]

DeCaf % constraint
BD = 0.40(BD + CD)
0.60BD - 0.40CD = 0

Pounds of Regular: BR + CR = 1000
Pounds of DeCaf: BD + CD = 500

The complete linear program is

Max 2.033BR + 2.583BD + 1.868CR + 2.418CD
s.t.
0.25BR - 0.75CR = 0
0.60BD - 0.40CD = 0
BR + CR = 1000
BD + CD = 500
BR, BD, CR, CD ≥ 0

Using *The Management Scientist*, the optimal solution is BR = 750, BD = 200, CR = 250, and CD = 300.

The value of the optimal solution is $3233.75

14. a. Let $x_i$ = number of Classic 2l boats produced in Quarter $i$; $i = 1,2,3,4$
$s_i$ = ending inventory of Classic 2l boats in Quarter $i$; $i = 1,2,3,4$

Min 10,000$x_1$ + 11,000$x_2$ + 12,100$x_3$ + 13,310$x_4$ + 250$s_1$ + 250$s_2$ + 300$s_3$ + 300$s_4$

s.t.
$x_1 - s_1 = 1900$ Quarter 1 demand
$s_1 + x_2 - s_2 = 4000$ Quarter 2 demand
$s_2 + x_3 - s_3 = 3000$ Quarter 3 demand
$s_3 + x_4 - s_4 = 1500$ Quarter 4 demand
$s_4 ≥ 500$ Ending Inventory
$x_1 ≤ 4000$ Quarter 1 capacity
$x_2 ≤ 3000$ Quarter 2 capacity
$x_3 ≤ 2000$ Quarter 3 capacity
$x_4 ≤ 4000$ Quarter 4 capacity

b.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Production</th>
<th>Ending Inventory</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000</td>
<td>2100</td>
<td>40,525,000</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>1100</td>
<td>33,275,000</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>100</td>
<td>24,230,000</td>
</tr>
<tr>
<td>4</td>
<td>1900</td>
<td>500</td>
<td>25,439,000</td>
</tr>
</tbody>
</table>

$123,469,000$

14. c. The dual prices tell us how much it would cost if demand were to increase by one additional unit. For example, in Quarter 2 the dual price is -12,760; thus, demand for one more boat in Quarter 2 will increase costs by $12,760.
d. The dual price of 0 for Quarter 4 tells us we have excess capacity in Quarter 4. The positive dual prices in Quarters 1-3 tell us how much increasing the production capacity will improve the objective function. For example, the dual price of $2510 for Quarter 1 tells us that if capacity is increased by 1 unit for this quarter, costs will go down $2510.

15. Let $x_{11} =$ gallons of crude 1 used to produce regular
   $x_{12} =$ gallons of crude 1 used to produce high-octane
   $x_{21} =$ gallons of crude 2 used to produce regular
   $x_{22} =$ gallons of crude 2 used to produce high-octane

Min $0.10x_{11} + 0.10x_{12} + 0.15x_{21} + 0.15x_{22}$

s.t.

Each gallon of regular must have at least 40% A.

$$x_{11} + x_{21} = \text{amount of regular produced}$$

$$0.4(x_{11} + x_{21}) = \text{amount of A required for regular}$$

$$0.2x_{11} + 0.50x_{21} = \text{amount of A in } (x_{11} + x_{21}) \text{ gallons of regular gas}$$

$$\therefore 0.2x_{11} + 0.50x_{21} \geq 0.4x_{11} + 0.40x_{21} \quad [1]$$

$$\therefore -0.2x_{11} + 0.10x_{21} \geq 0$$

Each gallon of high octane can have at most 50% B.

$$x_{12} + x_{22} = \text{amount high-octane}$$

$$0.5(x_{12} + x_{22}) = \text{amount of B required for high octane}$$

$$0.60x_{12} + 0.30x_{22} = \text{amount of B in } (x_{12} + x_{22}) \text{ gallons of high octane.}$$

$$\therefore 0.60x_{12} + 0.30x_{22} \leq 0.5x_{12} + 0.5x_{22}$$

$$\therefore 0.1x_{12} - 0.2x_{22} \leq 0 \quad [2]$$

$$x_{11} + x_{21} \geq 800,000 \quad [3]$$

$$x_{12} + x_{22} \geq 500,000 \quad [4]$$

$$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$$

Optimal Solution: $x_{11} = 266,667, x_{12} = 333,333, x_{21} = 533,333, x_{22} = 166,667$

Cost = $165,000

16. Let $x_i =$ number of 10-inch rolls of paper processed by cutting alternative $i; \ i = 1,2,...,7$

Min $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

s.t.

$$6x_1 + 2x_3 + x_5 + x_6 + 4x_7 \geq 1000 \quad \text{1 1/2" production}$$

$$4x_2 + x_4 + 3x_5 + 2x_6 \geq 2000 \quad \text{2 1/2" production}$$

$$2x_3 + 2x_4 + x_6 + x_7 \geq 4000 \quad \text{3 1/2" production}$$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$
\[
\begin{align*}
x_1 &= 0 \\
x_2 &= 125 \\
x_3 &= 500 & & \text{2125 Rolls} \\
x_4 &= 1500 \\
x_5 &= 0 & & \text{Production:} \\
x_6 &= 0 & & 1\ 1/2"\ 1000 \\
x_7 &= 0 & & 2\ 1/2"\ 2000 \\
& & & 3\ 1/2"\ 4000 \\
\end{align*}
\]

Waste: Cut alternative #4 (1/2" per roll) 
\therefore\ 750\ inches.

b. Only the objective function needs to be changed. An objective function minimizing waste production and the new optimal solution are given.

\[
\begin{align*}
\text{Min }\ & x_1 + 0x_2 + 0x_3 + 0.5x_4 + x_5 + 0x_6 + 0.5x_7 \\
\end{align*}
\]

\[
\begin{align*}
x_1 &= 0 \\
x_2 &= 500 \\
x_3 &= 2000 & & \text{2500 Rolls} \\
x_4 &= 0 \\
x_5 &= 0 & & \text{Production:} \\
x_6 &= 0 & & 1\ 1/2"\ 4000 \\
x_7 &= 0 & & 2\ 2/1"\ 2000 \\
& & & 3\ 1/2"\ 4000 \\
\end{align*}
\]

Waste is 0; however, we have over-produced the 1\ 1/2" size by 3000 units. Perhaps these can be inventoried for future use.

c. Minimizing waste may cause you to over-produce. In this case, we used 375 more rolls to generate a 3000 surplus of the 1\ 1/2" product. Alternative b might be preferred on the basis that the 3000 surplus could be held in inventory for later demand. However, in some trim problems, excess production cannot be used and must be scrapped. If this were the case, the 3000 unit 1\ 1/2" size would result in 4500 inches of waste, and thus alternative a would be the preferred solution.

17. a. Let \[
\begin{align*}
\text{FM} &= \text{number of frames manufactured} \\
\text{FP} &= \text{number of frames purchased} \\
\text{SM} &= \text{number of supports manufactured} \\
\text{SP} &= \text{number of supports purchased} \\
\text{TM} &= \text{number of straps manufactured} \\
\text{TP} &= \text{number of straps purchased} \\
\end{align*}
\]
Min 38FM + 51FP + 11.5SM + 15SP + 6.5TM + 7.5TP
s.t.
3.5FM + 1.3SM + 0.8TM ≤ 21,000
2.2FM + 1.7SM ≤ 25,200
3.1FM + 2.6SM + 1.7TM ≤ 40,800
FM + FP ≥ 5,000
SM + SP ≥ 10,000
TM + TP ≥ 5,000
FM, FP, SM, SP, TM, TP ≥ 0.

Solution:

<table>
<thead>
<tr>
<th>Manufacture</th>
<th>Purchase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frames</td>
<td>5000</td>
</tr>
<tr>
<td>Supports</td>
<td>2692</td>
</tr>
<tr>
<td>Straps</td>
<td>0</td>
</tr>
</tbody>
</table>

b. Total Cost = $368,076.91

c. Subtract values of slack variables from minutes available to determine minutes used. Divide by 60 to determine hours of production time used.

d. Nothing, there are already more hours available than are being used.

e. Yes. The current purchase price is $51.00 and the reduced cost of 3.577 indicates that for a purchase price below $47.423 the solution may improve. Resolving with the coefficient of FP = 45 shows that 2714 frames should be purchased.

The optimal solution is as follows:

OPTIMAL SOLUTION

Objective Function Value = 361500.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM</td>
<td>2285.714</td>
<td>0.000</td>
</tr>
<tr>
<td>FP</td>
<td>2714.286</td>
<td>0.000</td>
</tr>
<tr>
<td>SM</td>
<td>10000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>SP</td>
<td>0.000</td>
<td>0.900</td>
</tr>
<tr>
<td>TM</td>
<td>0.000</td>
<td>0.600</td>
</tr>
<tr>
<td>TP</td>
<td>5000.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
18. a. Let $x_1 =$ number of Super Tankers purchased  
$x_2 =$ number of Regular Line Tankers purchased  
$x_3 =$ number of Econo-Tankers purchased

Min $550x_1 + 425x_2 + 350x_3$

s.t.

$6700x_1 + 55000x_2 + 4600x_3 \leq 600,000$  
Budget

$15(5000)x_1 + 20(2500)x_2 + 25(1000)x_3 \geq 550,000$

or

$75000x_1 + 50000x_2 + 25000x_3 \geq 550,000$  
Meet Demand

$x_1 + x_2 + x_3 \leq 15$  
Max. Total Vehicles

$x_3 \geq 3$  
Min. Econo-Tankers

$x_1 \leq 1/2(x_1 + x_2 + x_3)$

or

$1/2x_1 - 1/2x_2 - 1/2x_3 \leq 0$  
No more than 50% Super Tankers

$x_1, x_2, x_3 \geq 0$

Solution: 5 Super Tankers, 2 Regular Tankers, 3 Econo-Tankers

Total Cost: $583,000

Monthly Operating Cost: $4,650

b. The last two constraints in the formulation above must be deleted and the problem resolved.

The optimal solution calls for 7 1/3 Super Tankers at an annual operating cost of $4033. However, 
since a partial Super Tanker can't be purchased we must round up to find a feasible solution of 8 
Super Tankers with a monthly operating cost of $4,400.

Actually this is an integer programming problem, since partial tankers can't be purchased. We were 
fortunate in part (a) that the optimal solution turned out integer.

The true optimal integer solution to part (b) is $x_1 = 6$ and $x_2 = 2$ with a monthly operating cost of 
$4150. This is 6 Super Tankers and 2 Regular Line Tankers.
19. a. Let 
\[ x_{11} = \text{amount of men's model in month 1} \]
\[ x_{21} = \text{amount of women's model in month 1} \]
\[ x_{12} = \text{amount of men's model in month 2} \]
\[ x_{22} = \text{amount of women's model in month 2} \]
\[ s_{11} = \text{inventory of men's model at end of month 1} \]
\[ s_{21} = \text{inventory of women's model at end of month 1} \]
\[ s_{12} = \text{inventory of men's model at end of month 2} \]
\[ s_{22} = \text{inventory of women's model at end of month} \]

The model formulation for part (a) is given.

\[
\text{Min} \quad 120x_{11} + 90x_{21} + 120x_{12} + 90x_{22} + 2.4s_{11} + 1.8s_{21} + 2.4s_{12} + 1.8s_{22} \\
\text{s.t.} \quad 20 + x_{11} - s_{11} = 150 \quad \text{Satisfy Demand} \quad [1] \\
\text{or} \quad x_{11} - s_{11} = 130 \\
\text{or} \quad 30 + x_{21} - s_{21} = 125 \quad \text{Satisfy Demand} \quad [2] \\
\text{or} \quad x_{21} - s_{21} = 95 \\
\quad s_{11} + x_{12} - s_{12} = 200 \quad \text{Satisfy Demand} \quad [3] \\
\quad s_{21} + x_{22} - s_{22} = 150 \quad \text{Satisfy Demand} \quad [4] \\
\quad s_{12} \geq 25 \quad \text{Ending Inventory} \quad [5] \\
\quad s_{22} \geq 25 \quad \text{Ending Inventory} \quad [6] \\
\]

Labor Hours: Men’s = 2.0 + 1.5 = 3.5  
Women’s = 1.6 + 1.0 = 2.6

\[
3.5x_{11} + 2.6x_{21} \geq 900 \quad \text{Labor Smoothing for} \quad [7] \\
3.5x_{11} + 2.6x_{21} \leq 1100 \quad \text{Month 1} \quad [8] \\
3.5x_{11} + 2.6x_{21} - 3.5x_{12} - 2.6x_{22} \leq 100 \quad \text{Labor Smoothing for} \quad [9] \\
-3.5x_{11} + 2.6x_{21} + 3.5x_{12} + 2.6x_{22} \leq 100 \quad \text{Month 2} \quad [10] \\
\]

\[ x_{11}, x_{12}, x_{21}, x_{22}, s_{11}, s_{12}, s_{21}, s_{22} \geq 0 \]

The optimal solution is to produce 193 of the men's model in month 1, 162 of the men's model in month 2, 95 units of the women's model in month 1, and 175 of the women's model in month 2. Total Cost = $67,156

<table>
<thead>
<tr>
<th>Inventory Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month 1</td>
</tr>
</tbody>
</table>

4 - 18
Month 2 25 Men's 25 Women's

<table>
<thead>
<tr>
<th>Labor Levels</th>
<th>Previous month</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000.00 hours</td>
<td>922.25</td>
<td>1022.25</td>
</tr>
</tbody>
</table>

b. To accommodate this new policy the right-hand sides of constraints [7] to [10] must be changed to 950, 1050, 50, and 50 respectively. The revised optimal solution is given.

\[ x_{11} = 201 \]
\[ x_{21} = 95 \]
\[ x_{12} = 154 \]
\[ x_{22} = 175 \]  Total Cost = $67,175

We produce more men's models in the first month and carry a larger men's model inventory; the added cost however is only $19. This seems to be a small expense to have less drastic labor force fluctuations. The new labor levels are 1000, 950, and 994.5 hours each month. Since the added cost is only $19, management might want to experiment with the labor force smoothing restrictions to enforce even less fluctuations. You may want to experiment yourself to see what happens.

20. Let \( x_m \) = number of units produced in month \( m \)
\( I_m \) = increase in the total production level in month \( m \)
\( D_m \) = decrease in the total production level in month \( m \)
\( s_m \) = inventory level at the end of month \( m \)

where
\( m = 1 \) refers to March
\( m = 2 \) refers to April
\( m = 3 \) refers to May

Min \[ 1.25 I_1 + 1.25 I_2 + 1.25 I_3 + 1.00 D_1 + 1.00 D_2 + 1.00 D_3 \]
s.t.

Change in production level in March
\[ x_1 - 10,000 = I_1 - D_1 \]
or
\[ x_1 - I_1 + D_1 = 10,000 \]

Change in production level in April
\[ x_2 - x_1 = I_2 - D_2 \]
or
\[ x_2 - x_1 - I_2 + D_2 = 0 \]

Change in production level in May
\[ x_3 - x_2 = I_3 \cdot D_3 \]

or

\[ x_3 - x_2 - I_3 + D_3 = 0 \]

Demand in March

\[ 2500 + x_1 - s_1 = 12,000 \]

or

\[ x_1 - s_1 = 9,500 \]

Demand in April

\[ s_1 + x_2 - s_2 = 8,000 \]

Demand in May

\[ s_2 + x_3 = 15,000 \]

Inventory capacity in March

\[ s_1 \leq 3,000 \]

Inventory capacity in April

\[ s_2 \leq 3,000 \]

Optimal Solution:

Total cost of monthly production increases and decreases = $2,500

\[
\begin{align*}
x_1 &= 10,250 & I_1 &= 250 & D_1 &= 0 \\
x_2 &= 10,250 & I_2 &= 0 & D_2 &= 0 \\
x_3 &= 12,000 & I_3 &= 1750 & D_3 &= 0 \\
s_1 &= 750 & \\
s_2 &= 3000 &
\end{align*}
\]

21. Decision variables: Regular

<table>
<thead>
<tr>
<th>Model</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>B1R</td>
<td>B2R</td>
</tr>
<tr>
<td>Floor</td>
<td>F1R</td>
<td>F2R</td>
</tr>
</tbody>
</table>

Decision variables: Overtime

<table>
<thead>
<tr>
<th>Model</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>B1O</td>
<td>B2O</td>
</tr>
<tr>
<td>Floor</td>
<td>F1O</td>
<td>F2O</td>
</tr>
</tbody>
</table>

Labor costs per unit

<table>
<thead>
<tr>
<th>Model</th>
<th>Regular</th>
<th>Overtime</th>
</tr>
</thead>
</table>
IB = Month 1 ending inventory for bookshelf units
IF = Month 1 ending inventory for floor model

Objective function

Min \[ 15.40 \cdot B1R + 15.40 \cdot B2R + 22 \cdot F1R + 22 \cdot F2R + 23.10 \cdot B1O + 23.10 \cdot B2O + 33 \cdot F1O + 33 \cdot F2O + 10 \cdot B1R + 10 \cdot B2R + 12 \cdot F1R + 12 \cdot F2R + 10 \cdot B1O + 10 \cdot B2O + 12 \cdot F1O + 12 \cdot F2O + 5 \cdot IB + 5 \cdot IF \]

or

Min \[ 25.40 \cdot B1R + 25.40 \cdot B2R + 34 \cdot F1R + 34 \cdot F2R + 33.10 \cdot B1O + 33.10 \cdot B2O + 45 \cdot F1O + 45 \cdot F2O + 5 \cdot IB + 5 \cdot IF \]

s.t.

\[ 0.7 \cdot B1R + 1 \cdot F1R \leq 2400 \quad \text{Regular time: month 1} \]
\[ 0.7 \cdot B2R + 1 \cdot F2R \leq 2400 \quad \text{Regular time: month 2} \]
\[ 0.7 \cdot B1O + 1 \cdot F1O \leq 1000 \quad \text{Overtime: month 1} \]
\[ 0.7 \cdot B2O + 1 \cdot F2O \leq 1000 \quad \text{Overtime: month 2} \]
\[ B1R + B1O - IB = 2100 \quad \text{Bookshelf: month 1} \]
\[ IB + B2R + B2O = 1200 \quad \text{Bookshelf: month 2} \]
\[ F1R + F1O - IF = 1500 \quad \text{Floor: month 1} \]
\[ IF + F2R + F2O = 2600 \quad \text{Floor: month 2} \]

OPTIMAL SOLUTION

Objective Function Value = 241130.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1R</td>
<td>2100.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B2R</td>
<td>1200.000</td>
<td>0.000</td>
</tr>
<tr>
<td>F1R</td>
<td>930.000</td>
<td>0.000</td>
</tr>
<tr>
<td>F2R</td>
<td>1560.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B1O</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B2O</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>F1O</td>
<td>610.000</td>
<td>0.000</td>
</tr>
<tr>
<td>F2O</td>
<td>1000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>IB</td>
<td>0.000</td>
<td>1.500</td>
</tr>
<tr>
<td>IF</td>
<td>40.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>11.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
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4. 0.000  5.000
5. 0.000 -33.100
6. 0.000 -36.600
7. 0.000 -45.000
8. 0.000 -50.000

OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
<tr>
<th>Variable</th>
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</tr>
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<td>34.000</td>
<td>50.000</td>
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<tr>
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<td>33.100</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>F1O</td>
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<td>45.000</td>
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<tr>
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RIGHT HAND SIDE RANGES

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</tr>
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<td>3010.000</td>
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<td>2400.000</td>
<td>2440.000</td>
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</tr>
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<td>1040.000</td>
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<td>2657.143</td>
</tr>
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<td>1142.857</td>
<td>1200.000</td>
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<td>1500.000</td>
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<tr>
<td>8</td>
<td>2560.000</td>
<td>2600.000</td>
<td>2990.000</td>
</tr>
</tbody>
</table>

22. Let
   SM1 = No. of small on machine M1
   SM2 = No. of small on machine M2
   SM3 = No. of small on machine M3
   LM1 = No. of large on machine M1
   LM2 = No. of large on machine M2
   LM3 = No. of large on machine M3
   MM2 = No. of meal on machine M2
   MM3 = No. of meal on machine M3

Output from *The Management Scientist* showing the formulation and solution follows. Note that constraints 1-3 guarantee that next week's schedule will be met and constraints 4-6 enforce machine capacities.

LINEAR PROGRAMMING PROBLEM

MIN 20SM1+24SM2+32SM3+15LM1+28LM2+35LM3+18MM2+36MM3

S.T.

1) 1SM1+1SM2+1SM3>80000
2) +1LM1+1LM2+1LM3>80000
3)  +1MM2+1MM3>65000
4)  0.03333SM1+0.04LM1<2100
5)  +0.02222SM2+0.025LM2+0.03333MM2<2100
6)  +0.01667SM3+0.01923LM3+0.02273MM3<2400

OPTIMAL SOLUTION

Objective Function Value = 5515886.5866

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM1</td>
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<td>4.66500</td>
</tr>
<tr>
<td>SM2</td>
<td>0.00000</td>
<td>4.00000</td>
</tr>
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<td>SM3</td>
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</tr>
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</tr>
<tr>
<td>LM2</td>
<td>0.00000</td>
<td>6.50135</td>
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<tr>
<td>LM3</td>
<td>27500.0000</td>
<td>0.00000</td>
</tr>
<tr>
<td>MM2</td>
<td>63006.30063</td>
<td>0.00000</td>
</tr>
<tr>
<td>MM3</td>
<td>1993.69937</td>
<td>0.00000</td>
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</table>

Constraint Slack/Surplus Dual Prices

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
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<tr>
<td>5</td>
<td>0.00000</td>
<td>540.05401</td>
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<td>6</td>
<td>492.25821</td>
<td>0.00000</td>
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OBJECTIVE COEFFICIENT RANGES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
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</tr>
<tr>
<td>SM2</td>
<td>20.00000</td>
<td>24.00000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>SM3</td>
<td>0.00000</td>
<td>32.00000</td>
<td>36.00000</td>
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<tr>
<td>LM1</td>
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<tr>
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</tr>
<tr>
<td>LM3</td>
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<td>35.00000</td>
<td>41.50135</td>
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<tr>
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</tr>
<tr>
<td>MM3</td>
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<td>36.00000</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>

RIGHT HAND SIDE RANGES

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
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<td>3</td>
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<td>65000.00000</td>
<td>86656.76257</td>
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<tr>
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<td>2166.45000</td>
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<tr>
<td>6</td>
<td>1907.74179</td>
<td>2400.00000</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>

Note that 5,515,887 square inches of waste are generated. Machine 3 has 492 minutes of idle capacity.

23. Let \( F = \) number of windows manufactured in February
    \( M = \) number of windows manufactured in March
A = number of windows manufactured in April
\( I_m \) = increase in production level necessary during month \( m \)
\( D_m \) = decrease in production level necessary during month \( m \)
\( s_m \) = ending inventory in month \( m \)

\[
\text{Min } 1I_1 + 1I_2 + 1I_3 + 0.65D_1 + 0.65D_2 + 0.65D_3
\]
s.t.
\[
9000 + F - s_1 = 15,000 \quad \text{February Demand}
\]
or
\[
(1) \quad F - s_1 = 6000
\]
(2) \( s_1 + M - s_2 = 16,500 \quad \text{March Demand} \)
(3) \( s_2 + A - s_3 = 20,000 \quad \text{April Demand} \)
\[
F - 15,000 = I_1 - D_1 \quad \text{Change in February Production}
\]
or
\[
(4) \quad F - I_1 + D_1 = 15,000
\]
\[
M - F = I_2 - D_2 \quad \text{Change in March Production}
\]
or
\[
(5) \quad M - F - I_2 + D_2 = 0
\]
\[
A - M = I_3 - D_3 \quad \text{Change in April Production}
\]
or
\[
(6) \quad A - M - I_3 + D_3 = 0
\]
(7) \( F \leq 14,000 \quad \text{February Production Capacity} \)
(8) \( M \leq 14,000 \quad \text{March Production Capacity} \)
(9) \( A \leq 18,000 \quad \text{April Production Capacity} \)
(10) \( s_1 \leq 6,000 \quad \text{February Storage Capacity} \)
(11) \( s_2 \leq 6,000 \quad \text{March Storage Capacity} \)
(12) \( s_3 \leq 6,000 \quad \text{April Storage Capacity} \)

Optimal Solution: Cost = $6,450

<table>
<thead>
<tr>
<th></th>
<th>February</th>
<th>March</th>
<th>April</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Level</td>
<td>12,000</td>
<td>14,000</td>
<td>16,500</td>
</tr>
<tr>
<td>Increase in Production</td>
<td>0</td>
<td>2,000</td>
<td>2,500</td>
</tr>
<tr>
<td>Decrease in Production</td>
<td>3,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ending Inventory</td>
<td>6,000</td>
<td>3,500</td>
<td>0</td>
</tr>
</tbody>
</table>
Let  
\( x_1 = \) proportion of investment A undertaken  
\( x_2 = \) proportion of investment B undertaken  
\( s_1 = \) funds placed in savings for period 1  
\( s_2 = \) funds placed in savings for period 2  
\( s_3 = \) funds placed in savings for period 3  
\( s_4 = \) funds placed in savings for period 4  
\( L_1 = \) funds received from loan in period 1  
\( L_2 = \) funds received from loan in period 2  
\( L_3 = \) funds received from loan in period 3  
\( L_4 = \) funds received from loan in period 4

Objective Function:

In order to maximize the cash value at the end of the four periods, we must consider the value of investment A, the value of investment B, savings income from period 4, and loan expenses for period 4.

\[
\text{Max } 3200x_1 + 2500x_2 + 1.1s_4 - 1.18L_4
\]

Constraints require the use of funds to equal the source of funds for each period.

**Period 1:**
\[
1000x_1 + 800x_2 + s_1 = 1500 + L_1
\]

or
\[
1000x_1 + 800x_2 + s_1 - L_1 = 1500
\]

**Period 2:**
\[
800x_1 + 500x_2 + s_2 + 1.18L_1 = 400 + 1.1s_1 + L_2
\]

or
\[
800x_1 + 500x_2 - 1.1s_1 + s_2 + 1.18L_1 - L_2 = 400
\]

**Period 3**
\[
200x_1 + 300x_2 + s_3 + 1.18L_2 = 500 + 1.1s_2 + L_3
\]

or
\[
200x_1 + 300x_2 - 1.1s_2 + s_3 + 1.18L_2 - L_3 = 500
\]

**Period 4**
\[
s_4 + 1.18L_3 = 100 + 200x_1 + 300x_2 + 1.1s_3 + L_4
\]

or
\[
-200x_1 - 300x_2 - 1.1s_3 + s_4 + 1.18L_3 - L_4 = 100
\]

Limits on Loan Funds Available
\[
L_1 \leq 200
\]
\[
L_2 \leq 200 \\
L_3 \leq 200 \\
L_4 \leq 200
\]

Proportion of Investment Undertaken

\[
\begin{align*}
    x_1 & \leq 1 \\
    x_2 & \leq 1
\end{align*}
\]

Optimal Solution: \$4340.40

Investment A \( x_1 = 0.458 \) or 45.8%
Investment B \( x_2 = 1.0 \) or 100.0%

Savings/Loan Schedule:

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings</td>
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<td>—</td>
<td>—</td>
<td>341.04</td>
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<tr>
<td>Loan</td>
<td>—</td>
<td>200.00</td>
<td>127.58</td>
<td>—</td>
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</tbody>
</table>

25. Let \( x_1 \) = number of part-time employees beginning at 11:00 a.m.
\( x_2 \) = number of part-time employees beginning at 12:00 p.m.
\( x_3 \) = number of part-time employees beginning at 1:00 p.m.
\( x_4 \) = number of part-time employees beginning at 2:00 p.m.
\( x_5 \) = number of part-time employees beginning at 3:00 p.m.
\( x_6 \) = number of part-time employees beginning at 4:00 p.m.
\( x_7 \) = number of part-time employees beginning at 5:00 p.m.
\( x_8 \) = number of part-time employees beginning at 6:00 p.m.

Each part-time employee assigned to a four-hour shift will be paid \$7.60 (4 hours) = \$30.40.

\[
\begin{align*}
    \text{Min } & 30.4x_1 + 30.4x_2 + 30.4x_3 + 30.4x_4 + 30.4x_5 + 30.4x_6 + 30.4x_7 + 30.4x_8 \\
    \text{s.t. } & \\
    x_1 & \geq 8 \quad \text{11:00 a.m.} \\
    x_1 + x_2 & \geq 8 \quad \text{12:00 p.m.} \\
    x_1 + x_2 + x_3 & \geq 7 \quad \text{1:00 p.m.} \\
    x_1 + x_2 + x_3 + x_4 & \geq 1 \quad \text{2:00 p.m.} \\
    x_2 + x_3 + x_4 + x_5 & \geq 2 \quad \text{3:00 p.m.} \\
    x_3 + x_4 + x_5 + x_6 & \geq 1 \quad \text{4:00 p.m.} \\
    x_4 + x_5 + x_6 + x_7 & \geq 5 \quad \text{5:00 p.m.} \\
    x_5 + x_6 + x_7 + x_8 & \geq 10 \quad \text{6:00 p.m.} \\
    x_6 + x_7 + x_8 & \geq 10 \quad \text{7:00 p.m.} \\
    x_7 + x_8 & \geq 6 \quad \text{8:00 p.m.} \\
    x_8 & \geq 6 \quad \text{9:00 p.m.}
\end{align*}
\]
\[ x_j \geq 0 \quad j = 1,2,...,8 \]

Full-time employees reduce the number of part-time employees needed.

A portion of *The Management Scientist* solution to the model follows.

**OPTIMAL SOLUTION**

**Objective Function Value = 608.000**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>X2</td>
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<tr>
<td>X3</td>
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<tr>
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<td>X5</td>
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<td>X6</td>
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<td>X7</td>
<td>4.000</td>
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<tr>
<td>X8</td>
<td>6.000</td>
<td>0.000</td>
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</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
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<td>0.000</td>
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<tr>
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<td>0.000</td>
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<tr>
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</tr>
<tr>
<td>11</td>
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<td>0.000</td>
</tr>
</tbody>
</table>

The optimal schedule calls for
8 starting at 11:00 a.m.
2 starting at 3:00 p.m.
4 starting at 5:00 p.m.
6 starting at 6:00 p.m.

b. Total daily salary cost = $608

There are 7 surplus employees scheduled from 2:00 - 3:00 p.m. and 4 from 8:00 - 9:00 p.m. suggesting the desirability of rotating employees off sooner.

c. Considering 3-hour shifts

Let \( x \) denote 4-hour shifts and \( y \) denote 3-hour shifts where

\[ y_1 = \text{number of part-time employees beginning at 11:00 a.m.} \]
\[ y_2 = \text{number of part-time employees beginning at 12:00 p.m.} \]
Each part-time employee assigned to a three-hour shift will be paid $7.60 (3 hours) = $22.80

New objective function:

\[
\min \sum_{j=1}^{8} 30.40x_j + \sum_{i=1}^{9} 22.80y_i
\]

Each constraint must be modified with the addition of the \(y_i\) variables. For instance, the first constraint becomes

\[
x_1 + y_1 \geq 8
\]

and so on. Each \(y_i\) appears in three constraints because each refers to a three hour shift. The optimal solution is shown below.

\[
\begin{align*}
x_8 &= 6 \quad &y_1 &= 8 \\
y_3 &= 1 \quad &y_5 &= 1 \\
y_7 &= 4
\end{align*}
\]

Optimal schedule for part-time employees:

<table>
<thead>
<tr>
<th>4-Hour Shifts</th>
<th>3-Hour Shifts</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_8 = 6)</td>
<td>(y_1 = 8)</td>
</tr>
<tr>
<td>(y_3 = 1)</td>
<td>(y_5 = 1)</td>
</tr>
<tr>
<td>(y_7 = 4)</td>
<td>(y_7 = 4)</td>
</tr>
</tbody>
</table>

Total cost reduced to $501.60. Still have 20 part-time shifts, but 14 are 3-hour shifts. The surplus has been reduced by a total of 14 hours.

26. a.

Min \(E\)

s.t.

\[
\begin{align*}
wg + &wu + wc + ws &= 1 \\
48.14wg + &34.62wu + 36.72wc + 33.16ws &\geq 48.14 \\
43.10wg + &27.11wu + 45.98wc + 56.46ws &\geq 43.10 \\
253wg + &148wu + 175wc + 160ws &\geq 253 \\
41wg + &27wu + 23wc + 84ws &\geq 41 \\
-285.2E + &285.2wg + 162.3wu + 275.7wc + 210.4ws &\leq 0
\end{align*}
\]
\[-123.80E + 1123.80wg + 128.70wu + 348.50wc + 154.10ws \leq 0\]
\[-106.72E + 106.72wg + 64.21wu + 104.10wc + 104.04ws \leq 0\]

\[wg, wu, wc, ws \geq 0\]

b. Since \(wg = 1.0\), the solution does not indicate General Hospital is relatively inefficient.

c. The composite hospital is General Hospital. For any hospital that is not relatively inefficient, the composite hospital will be that hospital because the model is unable to find a weighted average of the other hospitals that is better.

27. a.

\[\text{Min } E\]

\[wa + wb + wc + wd + we + wf + wg = 1\]

\[
\begin{align*}
55.31wa + 37.64wb + 32.91wc + 33.53wd + 32.48we + 48.78wf + 58.41wg & \geq 33.53 \\
49.52wa + 55.63wb + 25.77wc + 41.99wd + 55.30we + 81.92wf + 119.70wg & \geq 41.99 \\
281wa + 156wb + 141wc + 160wd + 157we + 285wf + 111wg & \geq 160 \\
47wa + 3wb + 26wc + 21wd + 82we + 92wf + 89wg & \geq 21 \\
-250E + 310wa + 278.5wb + 165.6wc + 250wd + 206.4we + 384wf + 530.1wg & \leq 0 \\
-316E + 134.6wa + 114.3wb + 131.3wc + 316wd + 151.2we + 217wf + 770.8wg & \leq 0 \\
-94.4E + 116wa + 106.8wb + 65.52wc + 94.4wd + 102.1we + 153.7wf + 215wg & \leq 0
\end{align*}
\]

\[wa, wb, wc, wd, we, wf, wg \geq 0\]

b. \(E = 0.924\)

\[wa = 0.074\]

\[wc = 0.436\]

\[we = 0.489\]

All other weights are zero.

c. \(D\) is relatively inefficient.

Composite requires 92.4 of \(D\)'s resources.

d. 34.37 patient days (65 or older)

41.99 patient days (under 65)

e. Hospitals A, C, and E.

28. a. Make the following changes to the model in problem 27.

New Right-Hand Side Values for

<table>
<thead>
<tr>
<th>Constraint</th>
<th>New Right-Hand Side Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>32.48</td>
</tr>
<tr>
<td>3</td>
<td>55.30</td>
</tr>
<tr>
<td>4</td>
<td>157</td>
</tr>
<tr>
<td>5</td>
<td>82</td>
</tr>
</tbody>
</table>

New Coefficients for \(E\) in

<table>
<thead>
<tr>
<th>Constraint</th>
<th>New Coefficients for (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-206.4</td>
</tr>
<tr>
<td>7</td>
<td>-151.2</td>
</tr>
<tr>
<td>8</td>
<td>-102.1</td>
</tr>
</tbody>
</table>
b. $E = 1; \text{ we} = 1; \text{ all other weights} = 0$

c. No; $E = 1$ indicates that all the resources used by Hospital $E$ are required to produce the outputs of Hospital $E$.

d. Hospital $E$ is the only hospital in the composite. If a hospital is not relatively inefficient, the hospital will make up the composite hospital with weight equal to 1.

29. a. 

Min $E$

s.t.

\[
\begin{align*}
wb + wc + wj + wn + ws &= 1 \\
3800wb + 4600wc + 4400wj + 6500wn + 6000ws &\geq 4600 \\
25wb + 32wc + 35wj + 30wn + 28ws &\geq 32 \\
8wb + 8.5wc + 8wj + 10wn + 9ws &\geq 8.5 \\
-110E + 96wb + 110wc + 100wj + 125wn + 120ws &\leq 0 \\
-22E + 16wb + 22wc + 18wj + 25wn + 24ws &\leq 0 \\
-1400E + 850wb + 1400wc + 1200wj + 1500wn + 1600ws &\leq 0 \\
\end{align*}
\]

wb, wc, wj, wn, ws $\geq 0$

b.

OPTIMAL SOLUTION

Objective Function Value = 0.960

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.960</td>
<td>0.000</td>
</tr>
<tr>
<td>WB</td>
<td>0.175</td>
<td>0.000</td>
</tr>
<tr>
<td>WC</td>
<td>0.000</td>
<td>0.040</td>
</tr>
<tr>
<td>WJ</td>
<td>0.575</td>
<td>0.000</td>
</tr>
<tr>
<td>WN</td>
<td>0.250</td>
<td>0.000</td>
</tr>
<tr>
<td>WS</td>
<td>0.000</td>
<td>0.085</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.200</td>
</tr>
<tr>
<td>2</td>
<td>220.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>-0.004</td>
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<tr>
<td>4</td>
<td>0.000</td>
<td>-0.123</td>
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<tr>
<td>5</td>
<td>0.000</td>
<td>0.009</td>
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<tr>
<td>6</td>
<td>1.710</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>129.614</td>
<td>0.000</td>
</tr>
</tbody>
</table>

c. Yes; $E = 0.960$ indicates a composite restaurant can produce Clarksville's output with 96% of Clarksville's available resources.

d. More Output (Constraint 2 Surplus) $\$220$ more profit per week. Less Input
Hours of Operation \(110E = 105.6\) hours  
FTE Staff \(22E - 1.71\) (Constraint 6 Slack) = 19.41  
Supply Expense \(1400E - 129.614\) (Constraint 7 Slack) = $1214.39

The composite restaurant uses 4.4 hours less operation time, 2.6 less employees and $185.61 less supplies expense when compared to the Clarksville restaurant.

e. \(w_b = 0.175, w_j = 0.575,\) and \(w_n = 0.250\). Consider the Bardstown, Jeffersonville, and New Albany restaurants.

30. a. If the larger plane is based in Pittsburgh, the total revenue increases to $107,849. If the larger plane is based in Newark, the total revenue increases to $108,542. Thus, it would be better to locate the larger plane in Newark.

Note: The optimal solution to the original Leisure Air problem resulted in a total revenue of $103,103. The difference between the total revenue for the original problem and the problem that has a larger plane based in Newark is $108,542 - $103,103 = $5,439. In order to make the decision to change to a larger plane based in Newark, management must determine if the $5,439 increase in revenue is sufficient to cover the cost associated with changing to the larger plane.

b. Using a larger plane based in Newark, the optimal allocations are:

<table>
<thead>
<tr>
<th>PCQ</th>
<th>PMQ</th>
<th>POQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>23</td>
<td>43</td>
</tr>
<tr>
<td>PCY</td>
<td>PMY</td>
<td>POY</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>NCQ</td>
<td>NMQ</td>
<td>NOQ</td>
</tr>
<tr>
<td>26</td>
<td>56</td>
<td>39</td>
</tr>
<tr>
<td>NCY</td>
<td>NMY</td>
<td>NOY</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>CMQ</td>
<td>CMY</td>
<td>COY</td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>COQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The differences between the new allocations above and the allocations for the original Leisure Air problem involve the five ODIFs that are boldfaced in the solution shown above.

c. Using a larger plane based in Pittsburgh and a larger plane based in Newark, the optimal allocations are:

<table>
<thead>
<tr>
<th>PCQ</th>
<th>PMQ</th>
<th>POQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>44</td>
<td>45</td>
</tr>
<tr>
<td>PCY</td>
<td>PMY</td>
<td>POY</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>NCQ</td>
<td>NMQ</td>
<td>NOQ</td>
</tr>
<tr>
<td>26</td>
<td>56</td>
<td>39</td>
</tr>
<tr>
<td>NCY</td>
<td>NMY</td>
<td>NOY</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>CMQ</td>
<td>CMY</td>
<td>COY</td>
</tr>
<tr>
<td>37</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>COQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The differences between the new allocations above and the allocations for the original Leisure Air problem involve the four ODIFs that are boldfaced in the solution shown above. The total revenue associated with the new optimal solution is $115,073, which is a difference of $115,073 - $103,103 = $11,970.

d. In part (b), the ODIF that has the largest bid price is COY, with a bid price of $443. The bid price tells us that if one more Y class seat were available from Charlotte to Myrtle Beach that revenue would increase by $443. In other words, if all 10 seats allocated to this ODIF had been sold, accepting another reservation will provide additional revenue of $443.
31. a. The calculation of the number of seats still available on each flight leg is shown below:

<table>
<thead>
<tr>
<th>ODIF</th>
<th>ODIF Code</th>
<th>Original Allocation</th>
<th>Seats Sold</th>
<th>Seats Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PCQ</td>
<td>33</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>PMQ</td>
<td>44</td>
<td>44</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>POQ</td>
<td>22</td>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>PCY</td>
<td>16</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>PMY</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>POY</td>
<td>11</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>NCQ</td>
<td>26</td>
<td>20</td>
<td>6</td>
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<tr>
<td>8</td>
<td>NMQ</td>
<td>36</td>
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<td>3</td>
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<tr>
<td>9</td>
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<td>39</td>
<td>37</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>NCY</td>
<td>15</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>NMY</td>
<td>7</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>NOY</td>
<td>9</td>
<td>8</td>
<td>1</td>
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<tr>
<td>13</td>
<td>CMQ</td>
<td>31</td>
<td>27</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>CMY</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>COQ</td>
<td>41</td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td>16</td>
<td>COY</td>
<td>10</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Flight Leg 1: $8 + 0 + 4 + 4 + 1 + 2 = 19$
Flight Leg 2: $6 + 3 + 2 + 4 + 2 + 1 = 18$
Flight Leg 3: $0 + 1 + 3 + 2 + 4 + 2 = 12$
Flight Leg 4: $4 + 2 + 2 + 1 + 6 + 3 = 18$

Note: See the demand constraints for the ODIFs that make up each flight leg.

b. The calculation of the remaining demand for each ODIF is shown below:

<table>
<thead>
<tr>
<th>ODIF</th>
<th>ODIF Code</th>
<th>Original Allocation</th>
<th>Seats Sold</th>
<th>Seats Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PCQ</td>
<td>33</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>PMQ</td>
<td>44</td>
<td>44</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>POQ</td>
<td>45</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>PCY</td>
<td>16</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>PMY</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>POY</td>
<td>11</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>NCQ</td>
<td>26</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>NMQ</td>
<td>56</td>
<td>33</td>
<td>23</td>
</tr>
<tr>
<td>9</td>
<td>NOQ</td>
<td>39</td>
<td>37</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>NCY</td>
<td>15</td>
<td>11</td>
<td>4</td>
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<tr>
<td>11</td>
<td>NMY</td>
<td>7</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>NOY</td>
<td>9</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>CMQ</td>
<td>64</td>
<td>27</td>
<td>37</td>
</tr>
<tr>
<td>14</td>
<td>CMY</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>COQ</td>
<td>46</td>
<td>35</td>
<td>11</td>
</tr>
<tr>
<td>16</td>
<td>COY</td>
<td>10</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>
c. The LP model and solution are shown below:

\[
\text{MAX} \\
178PCQ+268PMQ+228POQ+380PCY+456PMY+560POY+199NCQ+249NMQ+349NOQ+385NCY+444NMY+580NOY+179CMQ+380CMY+224COQ+582COY \\
\text{S.T.} \\
1) 1PCQ+1PMQ+1POQ+1PCY+1PMY+1POY<19 \\
2) 1NCQ+1NMQ+1NOQ+1NCY+1NMY+1NOY<18 \\
3) 1PMQ+1PMY+1NMQ+1NMY+1CMQ+1CMY<12 \\
4) 1POQ+1POY+1NOQ+1NOY+1COQ+1COY<18 \\
5) 1PCQ<8 \\
6) 1PMQ<1 \\
7) 1POQ<27 \\
8) 1PCY<4 \\
9) 1PMY<1 \\
10) 1POY<2 \\
11) 1NCQ<6 \\
12) 1NMQ<23 \\
13) 1NOQ<2 \\
14) 1NCY<4 \\
15) 1NMY<2 \\
16) 1NOY<1 \\
17) 1CMQ<37 \\
18) 1CMY<2 \\
19) 1COQ<11 \\
20) 1COY<3 \\
\]

\text{OPTIMAL SOLUTION} \\
Objective Function Value = 15730.000 \\

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCQ</td>
<td>8.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PMQ</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>POQ</td>
<td>3.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PCY</td>
<td>4.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PMY</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>POY</td>
<td>2.000</td>
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</tr>
<tr>
<td>NCQ</td>
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<tr>
<td>NMY</td>
<td>2.000</td>
<td>0.000</td>
</tr>
<tr>
<td>NOY</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CMQ</td>
<td>3.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CMY</td>
<td>2.000</td>
<td>0.000</td>
</tr>
<tr>
<td>COQ</td>
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<td>0.000</td>
</tr>
<tr>
<td>COY</td>
<td>3.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: The values shown above provide the allocations for the remaining seats available. The bid prices for each ODIF are provide by the deal prices in the following output.
<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>4.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>70.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
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<td>0.000</td>
<td>224.000</td>
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<td>0.000</td>
<td>174.000</td>
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<td>0.000</td>
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<td>0.000</td>
</tr>
<tr>
<td>20</td>
<td>0.000</td>
<td>358.000</td>
</tr>
</tbody>
</table>

32. a. Let CT = number of convention two-night rooms
   CF = number of convention Friday only rooms
   CS = number of convention Saturday only rooms
   RT = number of regular two-night rooms
   RF = number of regular Friday only rooms
   RS = number of regular Saturday only room

   b./c. The formulation and output obtained using *The Management Scientist* is shown below.

**LINEAR PROGRAMMING PROBLEM**

**MAX** 225CT + 123CF + 130CS + 295RT + 146RF + 152RS

**S.T.**

1) 1CT < 40
2) 1CF < 20
3) 1CS < 15
4) 1RT < 20
5) 1RF < 30
6) 1RS < 25
7) 1CT + 1CF > 48
8) 1CT + 1CS > 48
9) 1CT + 1CF + 1RT + 1RF < 96
10) 1CT + 1CS + 1RT + 1RS < 96
**OPTIMAL SOLUTION**

Objective Function Value = 25314.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>36.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CF</td>
<td>12.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CS</td>
<td>15.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RT</td>
<td>20.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RF</td>
<td>28.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RS</td>
<td>25.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>8.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>28.000</td>
</tr>
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<td>4</td>
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<td>47.000</td>
</tr>
<tr>
<td>5</td>
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<td>0.000</td>
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<tr>
<td>6</td>
<td>0.000</td>
<td>50.000</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>-23.000</td>
</tr>
<tr>
<td>8</td>
<td>3.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>146.000</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>102.000</td>
</tr>
</tbody>
</table>

**OBJECTIVE COEFFICIENT RANGES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>123.000</td>
<td>225.000</td>
<td>253.000</td>
</tr>
<tr>
<td>CF</td>
<td>95.000</td>
<td>123.000</td>
<td>146.000</td>
</tr>
<tr>
<td>CS</td>
<td>102.000</td>
<td>130.000 No Upper Limit</td>
<td></td>
</tr>
<tr>
<td>RT</td>
<td>248.000</td>
<td>295.000 No Upper Limit</td>
<td></td>
</tr>
<tr>
<td>RF</td>
<td>123.000</td>
<td>146.000</td>
<td>193.000</td>
</tr>
<tr>
<td>RS</td>
<td>102.000</td>
<td>152.000 No Upper Limit</td>
<td></td>
</tr>
</tbody>
</table>

**RIGHT HAND SIDE RANGES**

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.000</td>
<td>40.000 No Upper Limit</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.000</td>
<td>20.000 No Upper Limit</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11.000</td>
<td>15.000 23.000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>18.000</td>
<td>20.000 23.000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>28.000</td>
<td>30.000 No Upper Limit</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>21.000</td>
<td>25.000 28.000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>46.000</td>
<td>48.000 56.000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>No Lower Limit</td>
<td>48.000 51.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>68.000</td>
<td>96.000 98.000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>93.000</td>
<td>96.000 100.000</td>
<td></td>
</tr>
</tbody>
</table>
d. The dual price for constraint 10 shows an added profit of $50 if this additional reservation is accepted.
Chapter 5
Linear Programming: The Simplex Method

Learning Objectives

1. Learn how to find basic and basic feasible solutions to systems of linear equations when the number of variables is greater than the number of equations.

2. Learn how to use the simplex method for solving linear programming problems.

3. Obtain an understanding of why and how the simplex calculations are made.

4. Understand how to use slack, surplus, and artificial variables to set up tableau form to get started with the simplex method for all types of constraints.

5. Understand the following terms:
   - simplex method
   - net evaluation row
   - basic solution
   - basis
   - basic feasible solution
   - iteration
   - tableau form
   - pivot element
   - simplex tableau
   - artificial variable

6. Know how to recognize the following special situations when using the simplex method to solve linear programs.
   - infeasibility
   - unboundedness
   - alternative optimal solutions
   - degeneracy
Solutions:

1. a. With \( x_1 = 0 \), we have

\[
\begin{align*}
  x_2 & = 6 \quad (1) \\
  4x_2 + x_3 & = 12 \quad (2)
\end{align*}
\]

From (1), we have \( x_2 = 6 \). Substituting for \( x_2 \) in (2) yields

\[
\begin{align*}
  4(6) + x_3 & = 12 \\
  x_3 & = 12 - 24 = -12
\end{align*}
\]

Basic Solution: \( x_1 = 0, x_2 = 6, x_3 = -12 \)

b. With \( x_2 = 0 \), we have

\[
\begin{align*}
  3x_1 + x_3 & = 6 \quad (3) \\
  2x_1 + x_3 & = 12 \quad (4)
\end{align*}
\]

From (3), we find \( x_1 = 2 \). Substituting for \( x_1 \) in (4) yields

\[
\begin{align*}
  2(2) + x_3 & = 12 \\
  x_3 & = 12 - 4 = 8
\end{align*}
\]

Basic Solution: \( x_1 = 2, x_2 = 0, x_3 = 8 \)

c. With \( x_3 = 0 \), we have

\[
\begin{align*}
  3x_1 + x_2 & = 6 \quad (5) \\
  2x_1 + 4x_2 & = 12 \quad (6)
\end{align*}
\]

Multiplying (6) by \( 3/2 \) and Subtracting form (5) yields

\[
\begin{align*}
  3x_1 + x_2 & = 6 \\
  -3x_1 - 6x_2 & = -18
\end{align*}
\]

\[
\begin{align*}
  -5x_2 & = -12 \\
  x_2 & = 12/5
\end{align*}
\]

Substituting \( x_2 = 12/5 \) into (5) yields

\[
\begin{align*}
  3x_1 + 12/5 & = 6 \\
  3x_1 & = 18/5 \\
  x_1 & = 6/5
\end{align*}
\]

Basic Solution: \( x_1 = 6/5, x_2 = 12/5, x_3 = 0 \)

d. The basic solutions found in (b) and (c) are basic feasible solutions. The one in (a) is not because \( x_3 = -12 \).
2. a. Standard Form:

Max \[ x_1 + 2x_2 \]

s.t.

\[ \begin{align*}
    x_1 + 5x_2 + s_1 &= 10 \\
    2x_1 + 6x_2 + s_2 &= 16 \\
    x_1, x_2, s_1, s_2 &\geq 0
\end{align*} \]

b. We have \( n = 4 \) and \( m = 2 \) in standard form. So \( n - m = 4 - 2 = 2 \) variables must be set equal to zero in each basic solution.

c. There are 6 combinations of the two variables that may be set equal to zero and hence 6 possible basic solutions.

\( x_1 = 0, x_2 = 0 \)

\[ \begin{align*}
    s_1 &= 10 \\
    s_2 &= 16 \\
    \text{This is a basic feasible solution.}
\end{align*} \]

\( x_1 = 0, s_1 = 0 \)

\[ \begin{align*}
    5x_2 &= 10 \quad (1) \\
    6x_2 + s_2 &= 16 \quad (2)
\end{align*} \]

From (1) we have \( x_2 = 2 \). And substituting for \( x_2 \) in (2) yields

\[ \begin{align*}
    6(2) + s_2 &= 16 \\
    s_2 &= 16 - 12 = 4 \\
    \text{This is a basic feasible solution.}
\end{align*} \]

\( x_1 = 0, s_2 = 0 \)

\[ \begin{align*}
    5x_2 + s_1 &= 10 \quad (3) \\
    6x_2 &= 16 \quad (4)
\end{align*} \]

From (4), we have \( x_2 = 8/3 \). Substituting for \( x_2 \) in (3) yields

\[ \begin{align*}
    5(8/3) + s_1 &= 10 \\
    s_1 &= 10 - 40/3 = -10/3 \\
    \text{This is not a basic feasible solution.}
\end{align*} \]

\( x_2 = 0, s_1 = 0 \)

\[ \begin{align*}
    x_1 &= 10 \quad (5) \\
    2x_1 + s_2 &= 16 \quad (6)
\end{align*} \]

From (5) we have \( x_1 = 10 \). And substituting for \( x_1 \) in (6) yields

\[ \begin{align*}
    2(10) + s_2 &= 16 \\
    s_2 &= 16 - 20 = -4 \\
    \text{This is not a basic feasible solution.}
\end{align*} \]
\[ x_2 = 0, s_2 = 0 \]

\[
\begin{align*}
x_1 &+ s_1 = 10 \quad (7) \\
2x_1 &= 16 \quad (8)
\end{align*}
\]

From (8) we find \( x_1 = 8 \). And substituting for \( x_1 \) in (7) yields

\[
\begin{align*}
8 &+ s_1 = 10 \\
s_1 &= 2
\end{align*}
\]

This is a basic feasible solution

\[ s_1 = 0, s_2 = 0 \]

\[
\begin{align*}
x_1 &+ 5x_2 = 10 \quad (9) \\
2x_1 &+ 6x_2 = 16 \quad (10)
\end{align*}
\]

From (9) we have \( x_1 = 10 - 5x_2 \). Substituting for \( x_1 \) in (10) yields

\[
\begin{align*}
2(10 - 5x_2) &+ 6x_2 = 16 \\
20 - 10x_2 &+ 6x_2 = 16 \\
-4x_2 &= 16 - 20 \\
4x_2 &= -4 \\
x_2 &= 1
\end{align*}
\]

Then, \( x_1 = 10 - 5(1) = 5 \)

This is a basic feasible solution.

d. The optimal solution is the basic feasible solution with the largest value of the objective function. There are 4 basic feasible solutions from part (c) to evaluate in the objective function.

\[ \begin{align*}
x_1 &= 0, x_2 = 0, s_1 = 10, s_2 = 16 \\
\text{Value} &= 1(0) + 2(0) = 0 \\
x_1 &= 0, x_2 = 2, s_1 = 0, s_2 = 4 \\
\text{Value} &= 1(0) + 2(2) = 4 \\
x_1 &= 8, x_2 = 0, s_1 = 2, s_2 = 0 \\
\text{Value} &= 1(8) + 2(0) = 8 \\
x_1 &= 5, x_2 = 1, s_1 = 0, s_2 = 0 \\
\text{Value} &= 1(5) + 2(1) = 7
\end{align*} \]

The optimal solution is \( x_1 = 8, x_2 = 0 \) with value = 8.
Simplex Method

3. a. \[ \text{Max } 5x_1 + 9x_2 + 0s_1 + 0s_2 + 0s_3 \]
   \[ \text{s.t. } \begin{align*}
   \frac{1}{3}x_1 + 1x_2 + 1s_1 &= 8 \\
   1x_1 + 1x_2 - 1s_2 &= 10 \\
   \frac{1}{4}x_1 + \frac{3}{2}x_2 - 1s_3 &= 6 \\
   x_1, x_2, s_1, s_2, s_3 \geq 0
   \end{align*} \]

b. 2

c. \( x_1 = 4, x_2 = 6, \) and \( s_3 = 4. \)

d. \( x_2 = 4, s_1 = 4, \) and \( s_2 = -6. \)

e. The answer to part c is a basic feasible solution and an extreme point solution. The answer to part d is not a basic feasible solution because \( s_2 \) is negative.

f. The graph below shows that the basic solution for part c is an extreme point and the one for part d is not.

![Graph showing extreme point solution and basic solution](image)

4. a. Standard Form:
   \[ \text{Max } 60x_1 + 90x_2 \]
   \[ \text{s.t. } \begin{align*}
   15x_1 + 45x_2 + s_1 &= 90 \\
   5x_1 + 5x_2 + s_2 &= 20 \\
   x_1, x_2, s_1, s_2 \geq 0
   \end{align*} \]
b. Partial initial simplex tableau:

```
<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>s1</th>
<th>s2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>45</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
```

5. a. Initial Tableau

```
<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s2</td>
<td>0</td>
<td>10</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>z_j</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
```

b. We would introduce \( x_2 \) at the first iteration.

c. Max \( 5x_1 + 9x_2 \)

s.t.

\[
\begin{align*}
10x_1 + 9x_2 & \leq 90 \\
-5x_1 + 3x_2 & \leq 15 \\
x_1, x_2 & \geq 0
\end{align*}
\]

6. a.

```
<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>s2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>s3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>-1/2</td>
<td>0</td>
</tr>
<tr>
<td>z_j</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>5</td>
<td>20</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
```
b. Max \[5x_1 + 20x_2 + 25x_3 + 0s_1 + 0s_2 + 0s_3\]
s.t. \[
2x_1 + 1x_2 + 1x_3 + 1s_1 = 40 \\
2x_2 + 1x_3 + 1s_2 = 30 \\
3x_1 - 1/2x_3 + 1s_3 = 15
\]
\[x_1, x_2, x_3, s_1, s_2, s_3, \geq 0.\]

The original basis consists of \(s_1, s_2,\) and \(s_3.\) It is the origin since the nonbasic variables are \(x_1, x_2,\) and \(x_3\) and are all zero.

d. 0.

e. \(x_3\) enters because it has the largest \(c_j - z_j\) and \(s_2\) will leave because row 2 has the only positive coefficient.

f. 30; objective function value is 30 times 25 or 750.

g. Optimal Solution:
\[
\begin{align*}
x_1 &= 10 & s_1 &= 20 \\
x_2 &= 0 & s_2 &= 0 \\
x_3 &= 30 & s_3 &= 0
\end{align*}
\]
\[z = 800.\]

7.

Sequence of extreme points generated by the simplex method:
(x_1 = 0, x_2 = 0)
(x_1 = 0, x_2 = 6)
(x_1 = 7, x_2 = 3)

8. a. Initial simplex tableau

<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th>x_1</th>
<th>x_2</th>
<th>s_1</th>
<th>s_2</th>
<th>s_3</th>
<th>s_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_1</td>
<td>0</td>
<td>10</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s_2</td>
<td>0</td>
<td>7/10</td>
<td>1/10</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s_3</td>
<td>0</td>
<td>1/2</td>
<td>5/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s_4</td>
<td>0</td>
<td>1</td>
<td>2/3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>s_5</td>
<td>0</td>
<td>1/10</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>z_j</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>10</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Final simplex tableau

<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th>x_1</th>
<th>x_2</th>
<th>s_1</th>
<th>s_2</th>
<th>s_3</th>
<th>s_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_2</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>30/16</td>
<td>0</td>
<td>-21/16</td>
<td>0</td>
</tr>
<tr>
<td>s_2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-15/16</td>
<td>1</td>
<td>5/32</td>
<td>0</td>
</tr>
<tr>
<td>x_1</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>-20/16</td>
<td>0</td>
<td>30/16</td>
<td>0</td>
</tr>
<tr>
<td>s_4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-11/32</td>
<td>0</td>
<td>9/64</td>
<td>1</td>
</tr>
<tr>
<td>s_5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>111/16</td>
<td>0</td>
<td>111/16</td>
<td>0</td>
</tr>
<tr>
<td>z_j</td>
<td>10</td>
<td>9</td>
<td>70/16</td>
<td>0</td>
<td>111/16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>0</td>
<td>0</td>
<td>-70/16</td>
<td>0</td>
<td>-111/16</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

x_1 = 540 standard bags
x_2 = 252 deluxe bags

b. $7668
c. & d.

<table>
<thead>
<tr>
<th>Slack</th>
<th>Production Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_1 = 0</td>
<td>Cutting and dyeing time = 630 hours</td>
</tr>
<tr>
<td>s_2 = 120</td>
<td>Sewing time = 600 - 120 = 480 hours</td>
</tr>
<tr>
<td>s_3 = 0</td>
<td>Finishing time = 708 hours</td>
</tr>
<tr>
<td>s_4 = 18</td>
<td>Inspection and Packaging time = 135 - 18 = 117 hours</td>
</tr>
</tbody>
</table>
9. Note: Refer to Chapter 2, problem 21 for a graph showing the location of the extreme points.

Initial simplex tableau (corresponds to the origin)

<table>
<thead>
<tr>
<th>Basis</th>
<th>( c_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( \frac{b_i}{a_{ij}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0</td>
<td>2/5</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>0</td>
<td>3/5</td>
<td>3/10</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>( z_j )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_j \cdot z_j )</td>
<td>40</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

First iteration: \( x_1 \) enters the basis and \( s_3 \) leaves (new basic feasible solution)

<table>
<thead>
<tr>
<th>Basis</th>
<th>( c_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( \frac{\bar{b}<em>i}{\bar{a}</em>{ij}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0</td>
<td>0</td>
<td>3/10</td>
<td>1</td>
<td>0</td>
<td>-2/3</td>
<td>6</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>40</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>5/3</td>
<td>35</td>
</tr>
<tr>
<td>( z_j )</td>
<td>40</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>200/3</td>
<td>1400</td>
<td>1400</td>
</tr>
<tr>
<td>( c_j \cdot z_j )</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>-200/3</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Next iteration: \( x_2 \) enters the basis and \( s_1 \) leaves (new basic feasible solution)

<table>
<thead>
<tr>
<th>Basis</th>
<th>( c_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>30</td>
<td>0</td>
<td>1</td>
<td>10/3</td>
<td>0</td>
<td>-20/9</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2/3</td>
<td>1</td>
<td>4/9</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>40</td>
<td>1</td>
<td>0</td>
<td>-5/3</td>
<td>0</td>
<td>25/9</td>
</tr>
<tr>
<td>( z_j )</td>
<td>40</td>
<td>30</td>
<td>100/3</td>
<td>0</td>
<td>400/9</td>
<td>1600</td>
</tr>
<tr>
<td>( c_j \cdot z_j )</td>
<td>0</td>
<td>0</td>
<td>-100/3</td>
<td>0</td>
<td>-400/9</td>
<td>—</td>
</tr>
</tbody>
</table>

Optimal Solution:

\( x_1 = 25 \quad x_2 = 20 \)
\( s_1 = 0 \quad s_2 = 1 \quad s_3 = 0. \)
10. Initial simplex tableau:

<table>
<thead>
<tr>
<th>Basis</th>
<th>$B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$\bar{b}<em>i / \bar{a}</em>{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>15</td>
<td>4</td>
<td>12</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2800 / 12 = 233.33</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>15</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6000 / 0 =</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1200 / 8 = 150</td>
</tr>
</tbody>
</table>

$z_j$: 0 0 0 0 0 0 0
$c_j - z_j$: 5 5 24 0 0 0

First iteration: $x_3$ enters, $s_3$ leaves

<table>
<thead>
<tr>
<th>Basis</th>
<th>$B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$\bar{b}<em>i / \bar{a}</em>{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>27/2</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-3/2</td>
<td>1000 / 4 = 250</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>15</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6000 / 8 = 750</td>
</tr>
<tr>
<td>$x_3$</td>
<td>24</td>
<td>1/8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/8</td>
<td>150 / 0 =</td>
</tr>
</tbody>
</table>

$z_j$: 3 0 24 0 0 3
$c_j - z_j$: 2 5 0 0 0 -3

Second iteration: $x_2$ enters, $s_1$ leaves

<table>
<thead>
<tr>
<th>Basis</th>
<th>$B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$\bar{b}<em>i / \bar{a}</em>{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>5</td>
<td>27/8</td>
<td>1</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>-3/8</td>
<td>250</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>1</td>
<td>3</td>
<td>4000</td>
</tr>
<tr>
<td>$x_3$</td>
<td>24</td>
<td>1/8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/8</td>
<td>150</td>
</tr>
</tbody>
</table>

$z_j$: 159/8 5 24 5/4 0 9/8 4850
$c_j - z_j$: -119/8 0 0 -5/4 0 -9/8

Optimal Solution:

$x_2 = 250, \ x_3 = 150, \ s_2 = 4000, \ Value = 4850$
Simplex Method

11.

**Optimal Solution**

\[ x_1 = 4.2, x_2 = 3.6, z = 37.2 \]

**Extreme Points:**

- \((x_1 = 6, x_2 = 0), (x_1 = 4.2, x_2 = 3.6), (x_1 = 15, x_2 = 0)\)

**Simplex Solution Sequence:**

- \((x_1 = 0, x_2 = 0)\)
- \((x_1 = 6, x_2 = 0)\)
- \((x_1 = 4.2, x_2 = 3.6)\)

12.

Let \( x_1 = \) units of product A.
- \( x_2 = \) units of product B.
- \( x_3 = \) units of product C.

Maximize \( \) 20\( x_1 \) + 20\( x_2 \) + 15\( x_3 \)

Subject to:

- \( 7x_1 + 6x_2 + 3x_3 \leq 100 \)
- \( 5x_1 + 4x_2 + 2x_3 \leq 200 \)
- \( x_1, x_2, x_3 \geq 0 \)

**Optimal Solution:** \( x_1 = 0, x_2 = 0, x_3 = 33 \frac{1}{3} \)

**Profit:** \( 500. \)
13. Let \( x_1 \) = number of units of Grade A Plywood produced  
\( x_2 \) = number of units of Grade B Plywood produced  
\( x_3 \) = number of units of Grade X Plywood produced  

Max \( 40x_1 + 30x_2 + 20x_3 \)  

s.t. \( 2x_1 + 5x_2 + 10x_3 \leq 900 \)  
\( 2x_1 + 5x_2 + 3x_3 \leq 400 \)  
\( 4x_1 + 2x_2 - 2x_3 \leq 600 \)  

\( x_1, x_2, x_3 \geq 0 \)  

Optimal Solution:  
\( x_1 = 137.5, x_2 = 25, x_3 = 0 \)  

Profit = 6250.  

14. Let \( x_1 \) = gallons of Heidelberg Sweet produced  
\( x_2 \) = gallons of Heidelberg Regular produced  
\( x_3 \) = gallons of Deutschland Extra Dry produced  

Max \( 1.00x_1 + 1.20x_2 + 2.00x_3 \)  

s.t. \( 1x_1 + 2x_2 \leq 150 \) Grapes Grade A  
\( 1x_1 + x_2 + 2x_3 \leq 150 \) Grapes Grade B  
\( 2x_1 + x_2 \leq 80 \) Sugar  
\( 2x_1 + 3x_2 + x_3 \leq 225 \) Labor-hours  

\( x_1, x_2, x_3, x_4 \geq 0 \)  

a. \( x_1 = 0 \quad s_1 = 50 \)  
\( x_2 = 50 \quad s_2 = 0 \)  
\( x_3 = 75 \quad s_3 = 30 \)  
\( s_4 = 0 \)  

Profit = $210  

b. \( s_1 \) = unused bushels of grapes (Grade A)  
\( s_2 \) = unused bushels of grapes (Grade B)  
\( s_3 \) = unused pounds of sugar  
\( s_4 \) = unused labor-hours  

c. \( s_2 = 0 \) and \( s_4 = 0 \). Therefore the Grade B grapes and the labor-hours are the binding resources. Increasing the amounts of these resources will improve profit.
15. Max $4x_1 + 2x_2 - 3x_3 + 5x_4 + 0s_1 - Ma_1 + 0s_2 - Ma_3$
s.t. $2x_1 - 1x_2 + 1x_3 + 2x_4 - 1s_1 + 1a_1 = 50$
$3x_1 - 1x_3 + 2x_4 + 1s_2 = 80$
$1x_1 + 1x_2 + 1x_4 + 1a_3 = 60$

$x_1, x_2, x_3, x_4, s_1, s_2, a_1, a_3 \geq 0$

16. Max 
- $4x_1 - 5x_2 - 3x_3 + 0s_1 + 0s_2 + 0s_4 - Ma_1 - Ma_2 - Ma_3$

\[ 4x_1 + 2x_3 - 1s_1 + 1a_1 = 20 \]
\[ -1x_1 + 2x_2 + 1s_2 + 1a_2 = 8 \]
\[ 2x_1 + 1x_2 + 1x_3 + 1s_4 = 12 \]

$x_1, x_2, x_3, s_1, s_2, s_4, a_1, a_2, a_3 \geq 0$

17. $x_1 = 1, x_2 = 4, z = 19$

Converting to a max problem and solving using the simplex method, the final simplex tableau is:

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>-3</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1/4</td>
<td>1/8</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1/4</td>
<td>4</td>
</tr>
</tbody>
</table>

| $z_j$ | -3    | -4    | -5    | 3/4   | 5/8   | -19   |
| $c_j - z_j$ | 0    | 0     | 3     | -3/4  | -5/8  |

18. Initial tableau (Note: Min objective converted to Max.)

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$b_i / a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>-84</td>
<td>-4</td>
<td>-30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>240 / 8 = 30</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-M</td>
<td>16</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>480 / 16 = 30</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-M</td>
<td>8</td>
<td>-1</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>160 / 8 = 20</td>
</tr>
</tbody>
</table>

| $z_j$ | -24M  | 0     | -11M  | 0     | M     | M     | -M    | M     | -M    | -640M       |
| $c_j - z_j$ | -84+24M | -4   | -30+11M | 0    | -M    | -M    | 0     | 0     | 0     |             |
Iteration 1: $x_1$ enters, $a_3$ leaves (Drop $a_3$ column)

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-M</td>
<td>0</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-84</td>
<td>1</td>
<td>-1/8</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>-1/8</td>
<td>0</td>
</tr>
<tr>
<td>$z_j$</td>
<td>-84</td>
<td>42/2</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>$c_j - z_j$</td>
<td>0</td>
<td>-29/2</td>
<td>0</td>
<td>-1/2</td>
<td>-1/2</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
</tr>
</tbody>
</table>

Iteration 2: $x_2$ enters, $s_1$ leaves

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>-4</td>
<td>0</td>
<td>1</td>
<td>-1/2</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-M</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>-3/2</td>
<td>-1</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-84</td>
<td>1</td>
<td>0</td>
<td>7/16</td>
<td>1/16</td>
<td>0</td>
<td>-1/16</td>
<td>0</td>
</tr>
<tr>
<td>$z_j$</td>
<td>-84</td>
<td>-4</td>
<td>-139/4</td>
<td>-3M/2</td>
<td>-29/4</td>
<td>+3M/2</td>
<td>M</td>
<td>13/4</td>
</tr>
<tr>
<td>$c_j - z_j$</td>
<td>0</td>
<td>19/4</td>
<td>0</td>
<td>+M/2</td>
<td>29/4</td>
<td>-3M/2</td>
<td>-M</td>
<td>-13/4</td>
</tr>
</tbody>
</table>

Iteration 3: $x_3$ enters, $x_1$ leaves

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>-4</td>
<td>8/7</td>
<td>1</td>
<td>0</td>
<td>4/7</td>
<td>0</td>
<td>3/7</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-M</td>
<td>-8/7</td>
<td>0</td>
<td>0</td>
<td>-11/7</td>
<td>-1</td>
<td>4/7</td>
<td>1</td>
</tr>
<tr>
<td>$x_3$</td>
<td>-30</td>
<td>16/7</td>
<td>0</td>
<td>1</td>
<td>1/7</td>
<td>0</td>
<td>-1/7</td>
<td>0</td>
</tr>
<tr>
<td>$z_j$</td>
<td>-512+8M</td>
<td>-4</td>
<td>-30</td>
<td>-46+11M</td>
<td>M</td>
<td>-42-4M</td>
<td>-M</td>
<td>-13920 - 80M</td>
</tr>
<tr>
<td>$c_j - z_j$</td>
<td>-76-8M</td>
<td>0</td>
<td>0</td>
<td>46-11M</td>
<td>-M</td>
<td>42+4M</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Iteration 4: $s_3$ enters, $a_2$ leaves (Drop $a_2$ column)
### Simplex Method

<table>
<thead>
<tr>
<th>Basis</th>
<th>( c_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>-4</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>0</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>-30</td>
</tr>
<tr>
<td>( z_j )</td>
<td>-68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( c_j \cdot z_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16</td>
</tr>
</tbody>
</table>

Optimal Solution: \( x_2 = 60, x_3 = 60, s_3 = 20 \) Value = 2040

19. Let \( x_1 = \) no. of sailboats rented  
\( x_2 = \) no. of cabin cruisers rented  
\( x_3 = \) no. of luxury yachts rented

The mathematical formulation of this problem is:

\[
\begin{align*}
\text{Max} & \quad 50x_1 + 70x_2 + 100x_3 \\
\text{s.t.} & \quad x_1 \leq 4 \\
& \quad x_2 \leq 8 \\
& \quad x_3 \leq 3 \\
& \quad x_1 + x_2 + x_3 \leq 10 \\
& \quad x_1 + 2x_2 + 3x_3 \leq 18 \\
& \quad x_1, x_2, x_3, \geq 0
\end{align*}
\]

Optimal Solution:
\( x_1 = 4, x_2 = 4, x_3 = 2 \)

Profit = $680.

20. Let \( x_1 = \) number of 20-gallon boxes produced  
\( x_2 = \) number of 30-gallon boxes produced  
\( x_3 = \) number of 33-gallon boxes produced

\[
\begin{align*}
\text{Max} & \quad 0.10x_1 + 0.15x_2 + 0.20x_3 \\
\text{s.t.} & \quad 2x_1 + 3x_2 + 3x_3 \leq 7200 \text{ Cutting} \\
& \quad 2x_1 + 2x_2 + 3x_3 \leq 10800 \text{ Sealing} \\
& \quad 3x_1 + 4x_2 + 5x_3 \leq 14400 \text{ Packaging} \\
& \quad x_1, x_2, x_3, \geq 0
\end{align*}
\]

Optimal Solution
\( x_1 = 0, x_2 = 0, x_3 = 2400 \)

Profit = $480.

21.
Let \( x_1 = \) no. of gallons of Chocolate produced
\( x_2 = \) no. of gallons of Vanilla produced
\( x_3 = \) no. of gallons of Banana produced

Max \( 1.00x_1 + .90x_2 + .95x_3 \)

s.t.
\( .45x_1 + .50x_2 + .40x_3 \leq 200 \text{ Milk} \)
\( .50x_1 + .40x_2 + .20x_3 \leq 150 \text{ Sugar} \)
\( .10x_1 + .15x_2 + .20x_3 \leq 60 \text{ Cream} \)
\( x_1, x_2, x_3, \geq 0 \)

**Optimal Solution**

\( x_1 = 0, x_2 = 300, x_3 = 75 \)

Profit = $341.25. Additional resources: Sugar and Cream.

22.

Let \( x_1 = \) number of cases of Incentive sold by John
\( x_2 = \) number of cases of Temptation sold by John
\( x_3 = \) number of cases of Incentive sold by Brenda
\( x_4 = \) number of cases of Temptation sold by Brenda
\( x_5 = \) number of cases of Incentive sold by Red
\( x_6 = \) number of cases of Temptation sold by Red

Max \( 30x_1 + 25x_2 + 30x_3 + 25x_4 + 30x_5 + 25x_6 \)

s.t.
\( 10x_1 + 15x_2 \leq 4800 \)
\( 15x_3 + 10x_4 \leq 4800 \)
\( 12x_5 + 6x_6 \leq 4800 \)
\( x_1, x_2, x_3, x_4, x_5, x_6, \geq 0 \)

**Optimal Solution:**

\( x_1 = 480 \quad x_4 = 480 \)
\( x_2 = 0 \quad x_5 = 0 \)
\( x_3 = 0 \quad x_6 = 800 \)

Objective Function maximized at 46400.

**Time Allocation:**

<table>
<thead>
<tr>
<th></th>
<th>Incentive</th>
<th>Temptation</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>4800 min.</td>
<td>no time</td>
</tr>
<tr>
<td>Brenda</td>
<td>no time</td>
<td>4800 min.</td>
</tr>
<tr>
<td>Red</td>
<td>no time</td>
<td>4800 min.</td>
</tr>
</tbody>
</table>
23. Final simplex tableau

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-M</td>
<td>-2</td>
<td>0</td>
<td>-1/2</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$z_j$</th>
<th>8 + 2M</th>
<th>8</th>
<th>4 + M/2</th>
<th>+M</th>
<th>-M</th>
<th>40 - 3M</th>
</tr>
</thead>
</table>

| $c_j - z_j$ | -4 - 2M | 0 | -4 - M/2 | -M | 0 |

Infeasible; optimal solution condition is reached with the artificial variable $a_2$ still in the solution.

24. Alternative Optimal Solutions

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-4/3</td>
<td>1</td>
<td>1/6</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-3</td>
<td>1</td>
<td>0</td>
<td>-2/3</td>
<td>0</td>
<td>1/12</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>2/3</td>
<td>0</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$z_j$</th>
<th>-3</th>
<th>-3</th>
<th>0</th>
<th>0</th>
<th>3/4</th>
<th>-24</th>
</tr>
</thead>
</table>

| $c_j - z_j$ | 0       | 0     | 0     | 0     | -3/4  |

indicates alternative optimal solutions exist

$x_1 = 4, x_2 = 4, z = 24$
$x_1 = 8, x_2 = 0, z = 24$

25. Unbounded Solution

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>8/3</td>
<td>0</td>
<td>-1/3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>1</td>
<td>4/3</td>
<td>1</td>
<td>-1/6</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$z_j$</th>
<th>4/3</th>
<th>1</th>
<th>-1/6</th>
<th>0</th>
<th>0</th>
<th>4</th>
</tr>
</thead>
</table>

| $c_j - z_j$ | -1/3   | 0     | 1/6   | 0     | 0     |

Incoming Column

5 - 17
26. Alternative Optimal Solutions

\[
\begin{array}{c|ccccccc}
\text{Basis} & c_B & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\
\hline
x_1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\
\hline
s_2 & 0 & 0 & 0 & -1 & 0 & 1 & -1/2 \\
\hline
s_1 & 0 & 0 & 6 & 0 & 1 & 0 & 1 \\
\hline
z_j & 2 & 4 & 1 & 0 & 0 & 1/4 & 8 \\
c_j - z_j & 0 & -3 & 0 & 0 & 0 & -1/4 & \\
\end{array}
\]

Two possible solutions:
\(x_1 = 4, x_2 = 0, x_3 = 0\) or \(x_1 = 0, x_2 = 0, x_3 = 8\)

27. The final simplex tableau is given by:

\[
\begin{array}{c|cccc}
\text{Basis} & c_B & x_1 & x_2 & s_1 \\
\hline
s_1 & 0 & 1/2 & 0 & 1 \\
\hline
x_2 & 4 & 1 & 1 & 0 \\
\hline
s_3 & 0 & -1/2 & 0 & 0 \\
\hline
z_j & 4 & 4 & 0 & 0 \\
c_j - z_j & -2 & 0 & 0 & 0 \\
\end{array}
\]

This solution is degenerate since the basic variable \(s_3\) is in solution at a zero value.

28. The final simplex tableau is:

\[
\begin{array}{c|ccccccc}
\text{Basis} & c_B & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \\
\hline
a_1 & -M & 1 & -2 & 0 & -1 & 1 & 0 & 1 \\
\hline
x_3 & -5 & -1 & 1 & 1 & 0 & -1 & 0 & 0 \\
\hline
a_3 & -M & -1 & 1 & 0 & 0 & -1 & -1 & 0 & 1 \\
\hline
z_j & +M & +5 & -5 & -5 & +M & +5 & +M & -M \\
c_j - z_j & -M & -5 & -5 & -5 & -M & -M & 0 & 0 \\
\end{array}
\]

Since both artificial variables \(a_1\) and \(a_3\) are contained in this solution, we can conclude that we have an infeasible problem.
29. We must add an artificial variable to the equality constraint to obtain tableau form.

Tableau form:

Max \(120x_1 + 80x_2 + 14x_3 + 0s_1 + 0s_2 - Ma_3\)

s.t.

\[
\begin{align*}
4x_1 + 8x_2 + x_3 + 1s_1 &= 200 \\
2x_2 + x_3 + s_2 &= 300 \\
32x_1 + 4x_2 + 2x_3 + a_3 &= 400
\end{align*}
\]

\(x_1, x_2, x_3, s_1, s_2, a_3 \geq 0\)

Initial Tableau:

<table>
<thead>
<tr>
<th>(b_j / a_{ij})</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1) 0</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(s_2) 0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(a_3) -M</td>
<td>32</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(z_j) -32M</td>
<td>-4M</td>
<td>-2M</td>
<td>0</td>
<td>0</td>
<td>-M</td>
<td>-400M</td>
</tr>
<tr>
<td>(c_j - z_j) 120+32M</td>
<td>80+4M</td>
<td>14+2M</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Iteration 1: \(x_1\) enters, \(a_3\) leaves (drop \(a_3\) column)

<table>
<thead>
<tr>
<th>(b_j / a_{ij})</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(s_1)</th>
<th>(s_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1) 0</td>
<td>0</td>
<td>15/2</td>
<td>3/4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(s_2) 0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(x_1) 120</td>
<td>1</td>
<td>1/8</td>
<td>1/16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(z_j) 120</td>
<td>15</td>
<td>15/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(c_j - z_j) 0 120</td>
<td>65</td>
<td>13/2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Iteration 2: \(x_2\) enters, \(s_1\) leaves

<table>
<thead>
<tr>
<th>(b_j / a_{ij})</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(s_1)</th>
<th>(s_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_2) 120</td>
<td>80</td>
<td>0</td>
<td>1</td>
<td>1/10</td>
<td>2/15</td>
</tr>
<tr>
<td>(s_2) 0</td>
<td>0</td>
<td>0</td>
<td>8/10</td>
<td>-4/15</td>
<td>1</td>
</tr>
<tr>
<td>(x_1) 120</td>
<td>1</td>
<td>0</td>
<td>1/20</td>
<td>-1/60</td>
<td>0</td>
</tr>
<tr>
<td>(z_j) 120</td>
<td>80</td>
<td>14</td>
<td>26/3</td>
<td>0</td>
<td>2800</td>
</tr>
<tr>
<td>(c_j - z_j) 0 120</td>
<td>0</td>
<td>0</td>
<td>-26/3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Optimal solution: \(x_1 = 10\), \(x_2 = 20\), and \(s_2 = 260\), Value = 2800

Note: This problem has alternative optimal solutions; \(x_3\) may be brought in at a value of 200.
30. a. The mathematical formulation of this problem is:

Max \[ 3x_1 + 5x_2 + 4x_3 \]

s.t.

\[ 12x_1 + 10x_2 + 8x_3 \leq 18,000 \quad \text{C \& D} \]
\[ 15x_1 + 15x_2 + 12x_3 \leq 12,000 \quad \text{S} \]
\[ 3x_1 + 4x_2 + 2x_3 \leq 6,000 \quad \text{I and P} \]
\[ x_1 \geq 1,000 \]
\[ x_1, x_2, x_3 \geq 0 \]

There is no feasible solution. Not enough sewing time is available to make 1000 All-Pro footballs.

b. The mathematical formulation of this problem is now

Max \[ 3x_1 + 5x_2 + 4x_3 \]

s.t.

\[ 12x_1 + 10x_2 + 8x_3 \leq 18,000 \quad \text{C \& D} \]
\[ 15x_1 + 15x_2 + 12x_3 \leq 12,000 \quad \text{S} \]
\[ 3x_1 + 4x_2 + 2x_3 \leq 9,000 \quad \text{I \& P} \]
\[ x_1 \geq 1,000 \]
\[ x_1, x_2, x_3 \geq 0 \]

Optimal Solution

\[ x_1 = 1000, x_2 = 0, x_3 = 250 \]

Profit = $4000

There is an alternative optimal solution with \( x_1 = 1000, x_2 = 200, \) and \( x_3 = 0. \)

Note that the additional Inspection and Packaging time is not needed.
Chapter 6
Simplex-Based Sensitivity Analysis and Duality

Learning Objectives
1. Be able to use the final simplex tableau to compute ranges for the coefficients of the objective function.
2. Understand how to use the optimal simplex tableau to identify dual prices.
3. Be able to use the final simplex tableau to compute ranges on the constraint right-hand sides.
4. Understand the concepts of duality and the relationship between the primal and dual linear programming problems.
5. Know the economic interpretation of the dual variables.
6. Be able to convert any maximization or minimization problem into its associated canonical form.
7. Be able to obtain the primal solution from the final simplex tableau of the dual problem.
Solutions:

1. a. Recomputing the $c_j - z_j$ values for the nonbasic variables with $c_1$ as the coefficient of $x_1$ leads to the following inequalities that must be satisfied.

   For $x_2$, we get no inequality since there is a zero in the $x_2$ column for the row $x_1$ is a basic variable in.

   For $s_1$, we get
   \[
   0 + 4 - c_1 \leq 0 \\
   c_1 \geq 4
   \]

   For $s_2$, we get
   \[
   0 - 12 + 2c_1 \leq 0 \\
   2c_1 \leq 12 \\
   c_1 \leq 6
   \]

   Range: $4 \leq c_1 \leq 6$

   b. Since $x_2$ is nonbasic we have
   \[c_2 \leq 8\]

   c. Since $s_1$ is nonbasic we have
   \[c_{s_1} \leq 1\]

2. a. For $s_1$ we get
   \[
   0 - c_2 (8/25) - 50 (-5/25) \leq 0 \\
   c_2 (8/25) \geq 10 \\
   c_2 \geq 31.25
   \]

   For $s_3$ we get
   \[
   0 - c_2 (-3/25) - 50 (5/25) \leq 0 \\
   c_2 (3/25) \leq 10 \\
   c_2 \leq 83.33
   \]

   Range: $31.25 \leq c_2 \leq 83.33$

   b. For $s_1$ we get
   \[
   0 - 40 (8/25) - c_{s_2} (-8/25) - 50 (-5/25) \leq 0 \\
   -64/5 + c_{s_2} (8/25) + 10 \leq 0 \\
   c_{s_2} \leq 25/8 (14/5) = 70/8 = 8.75
   \]
For $s_3$ we get

\[ 0 - 40 \left(-\frac{3}{25}\right) - c_{s_2} \left(\frac{3}{25}\right) - 50 \left(\frac{5}{25}\right) \leq 0 \]

\[ 24/5 - c_{s_2} \left(\frac{3}{25}\right) - 10 \leq 0 \]

\[ c_{s_2} \geq (25/3) \left(-\frac{26}{5}\right) = -130/3 = -43.33 \]

**Range:** \(-43.33 \leq c_{s_2} \leq 8.75\)

c. \(c_{s_3} - 26/5 \leq 0\)

\[ c_{s_3} \leq 26/5 \]

d. No change in optimal solution since \(c_2 = 35\) is within range of optimality. Value of solution decreases to $35 (12) + $50 (30) = $1920.

3. a. It is the \(z_j\) value for \(s_1\). Dual Price = 1.

   b. It is the \(z_j\) value for \(s_2\). Dual Price = 2.

   c. It is the \(z_j\) value for \(s_3\). Dual Price = 0.

d. \(s_3 = 80 + 5(-2) = 70\)

   \(x_3 = 30 + 5(-1) = 25\)

   \(x_1 = 20 + 5(1) = 25\)

   Value = 220 + 5(1) = 225

3. e. \(s_3 = 80 - 10(-2) = 100\)

   \(x_3 = 30 - 10(-1) = 40\)

   \(x_1 = 20 - 10(1) = 10\)

   Value = 220 - 10(1) = 210

4. a.  

   \[ 80 + \Delta b_1 (-2) \geq 0 \rightarrow \Delta b_1 \leq 40 \]

   \[ 30 + \Delta b_1 (-1) \geq 0 \rightarrow \Delta b_1 \leq 30 \]

   \[ 20 + \Delta b_1 (1) \geq 0 \rightarrow \Delta b_1 \geq -20 \]

   \[-20 \leq \Delta b_1 \leq 30 \]

\[ 100 \leq b_1 \leq 150 \]

b.  

\[ 80 + \Delta b_2 (7) \geq 0 \rightarrow \Delta b_2 \geq -80/7 \]

\[ 30 + \Delta b_2 (3) \geq 0 \rightarrow \Delta b_2 \geq -10 \]

\[ 20 + \Delta b_2 (-2) \geq 0 \rightarrow \Delta b_2 \geq 10 \]
-10 ≤ \( \Delta b_2 \) ≤ 10

40 ≤ \( b_2 \) ≤ 60

c. \[
\begin{align*}
80 - \Delta b_3 \ (1) &\geq 0 \quad \rightarrow \quad \Delta b_3 \leq 80 \\
30 - \Delta b_3 \ (0) &\geq 0 \\
20 - \Delta b_3 \ (0) &\geq 0
\end{align*}
\]
\[\Delta b_3 \leq 80\]
\[b_3 \leq 110\]

5 a. \[
\begin{align*}
12 + \Delta b_2 \ (0) &\geq 0 \\
8 + \Delta b_2 \ (1) &\geq 0 \\
30 + \Delta b_2 \ (0) &\geq 0
\end{align*}
\]
Therefore \( \Delta b_2 \geq -8 \)

Range: \( b_2 \geq 12 \)

b. \[
\begin{align*}
12 + \Delta b_3 \ (-3/25) &\geq 0 \quad \rightarrow \quad \Delta b_3 \leq 100 \\
8 + \Delta b_3 \ (3/25) &\geq 0 \quad \rightarrow \quad \Delta b_3 \geq -66 \frac{2}{3} \\
30 + \Delta b_3 \ (5/25) &\geq 0 \quad \rightarrow \quad \Delta b_3 \geq -150
\end{align*}
\]
therefore \(-66 \frac{2}{3} \leq \Delta b_3 \leq 100\)

Range: \( 233 \frac{1}{3} \leq b_3 \leq 400\)

c. The dual price for the warehouse constraint is 26/5 and the 20 unit increase is within the range of feasibility, so the dual price is applicable for the entire increase.

Profit increase = 20 \((26/5) = 104\)

6. a. The final simplex tableau with \( c_1 \) shown as the coefficient of \( x_1 \) is

<table>
<thead>
<tr>
<th>Basis</th>
<th>( c_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_2 )</td>
<td>( 0 )</td>
<td>0</td>
<td>1</td>
<td>30/16</td>
<td>0</td>
<td>-21/16</td>
<td>0</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( 0 )</td>
<td>0</td>
<td>0</td>
<td>-15/16</td>
<td>1</td>
<td>5/32</td>
<td>0</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>( c_1 )</td>
<td>1</td>
<td>0</td>
<td>-20/16</td>
<td>0</td>
<td>30/16</td>
<td>0</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>( 0 )</td>
<td>0</td>
<td>0</td>
<td>-11/32</td>
<td>0</td>
<td>9/64</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\bar{z}_j &\left| c_1 \right| 9 \left(270-20c_1\right)/16 \quad 0 \left(30c_1-189\right)/16 \quad 0 \quad 2268+540c_1 \\
c_j^*-z_j &\left| 0 \left(20c_1-270\right)/16 \quad 0 \left(189-30c_1\right)/16 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \4}{
Simplex-Based Sensitivity Analysis and Duality

(20c_1 - 270) / 16 \leq 0 \quad \rightarrow \quad c_1 \leq 13.5

(189 - 30c_1) / 16 \leq 0 \quad \rightarrow \quad c_1 \geq 6.3

Range: 6.3 \leq c_1 \leq 13.5

b. Following a similar procedure for \( c_2 \) leads to

(200 - 30c_2) / 16 \leq 0 \quad \rightarrow \quad c_2 \geq 6 \frac{2}{3}

(21c_2 - 300) / 16 \leq 0 \quad \rightarrow \quad c_2 \leq 14 \frac{2}{7}

Range : 6 \frac{2}{3} \leq c_2 \leq 14 \frac{2}{7}

c. There would be no change in product mix, but profit will drop to \( 540 (10) + 252 (7) = 7164 \).

d. It would have to drop below \$6 \frac{2}{3} \) or increase above \$14 \frac{2}{7}.

e. We should expect more production of deluxe bags since its profit contribution has increased. The new optimal solution is given by

\[ x_1 = 300, \ x_2 = 420 \]

Optimal Value: \$9300

7. a.

\[
\begin{align*}
252 + \Delta b_1 (30/16) & \geq 0 \quad \rightarrow \quad \Delta b_1 \geq -134.4 \\
120 + \Delta b_1 (-15/16) & \geq 0 \quad \rightarrow \quad \Delta b_1 \leq 128 \\
540 + \Delta b_1 (-20/16) & \geq 0 \quad \rightarrow \quad \Delta b_1 \leq 432 \\
18 + \Delta b_1 (-11/32) & \geq 0 \quad \rightarrow \quad \Delta b_1 \leq 52.36
\end{align*}
\]

therefore \(-134.4 \leq \Delta b_1 \leq 52.36\)

Range: 495.6 \leq b_1 \leq 682.36

b. \( 480 \leq b_2 \)

c. \( 580 \leq b_3 \leq 900 \)

d. \( 117 \leq b_4 \)

e. The cutting and dyeing and finishing since the dual prices and the allowable increases are positive for both.
8. a. 

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>30/16</td>
<td>0</td>
<td>-21/16</td>
<td>0</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-15/16</td>
<td>1</td>
<td>5/32</td>
<td>0</td>
</tr>
<tr>
<td>$x_1$</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>-20/16</td>
<td>0</td>
<td>30/16</td>
<td>0</td>
</tr>
<tr>
<td>$s_4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-11/32</td>
<td>0</td>
<td>9/64</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
z_j = \begin{bmatrix} 10 & 9 & 70/16 & 0 & 111/16 & 0 \\
-21/16 & 5/32 & 0 & 9/64 & 1 & 0 \\
\end{bmatrix} = \begin{bmatrix} 86,868/11 \end{bmatrix} = 7897\frac{1}{11}
\]

\[c_j - z_j = \begin{bmatrix} 0 & 0 & -70/16 & 0 & -111/16 & 0 \\
\end{bmatrix}
\]

b. No, $s_4$ would become nonbasic and $s_1$ would become a basic variable.

9. a. Since this is within the range of feasibility for $b_1$, the increase in profit is given by

\[
\begin{pmatrix} 70 \\ 30 \end{pmatrix} = \begin{pmatrix} 2100 \\ 16 \end{pmatrix}
\]

b. It would not decrease since there is already idle time in this department and 600 - 40 = 560 is still within the range of feasibility for $b_2$.

c. Since 570 is within the range of feasibility for $b_1$, the lost profit would be equal to

\[
\begin{pmatrix} 70 \\ 60 \end{pmatrix} = \begin{pmatrix} 4200 \\ 16 \end{pmatrix}
\]

10. a. The value of the objective function would go up since the first constraint is binding. When there is no idle time, increased efficiency results in increased profits.

b. No. This would just increase the number of idle hours in the sewing department.

11. a. 

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>30</td>
<td>0</td>
<td>1</td>
<td>10/3</td>
<td>0</td>
<td>-20/9</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2/3</td>
<td>1</td>
<td>4/9</td>
</tr>
<tr>
<td>$x_1$</td>
<td>$c_1$</td>
<td>1</td>
<td>0</td>
<td>-5/3</td>
<td>0</td>
<td>25/9</td>
</tr>
</tbody>
</table>

\[
z_j = \begin{bmatrix} c_1 & 30 & 100-(5/3c_1) & 0 & -200/3 + 25/9 c_1 \end{bmatrix} \leq 600 + 25c_1
\]

\[c_j - z_j = \begin{bmatrix} 0 & 0 & 5/3c_1 - 100 & 0 & 200/3 - 25/9 c_1 \end{bmatrix} \leq 0.
\]

Hence

\[\frac{5}{3}c_1 - 100 \leq 0 \quad \text{and} \quad 200/3 - \frac{25}{9}c_1 \leq 0.
\]
Using the first inequality we obtain  
\[ \frac{5}{3}c_1 \leq 100 \quad \text{or} \quad c_1 \leq 60. \]

Using the second inequality we obtain  
\[ \frac{25}{9}c_1 \geq \frac{200}{3} \quad \Rightarrow \quad c_1 \geq \frac{9}{25}(200/3) \]

\[ c_1 \geq 24. \]

Thus the range of optimality for \( c_1 \) is given by  
\[ 24 \leq c_1 \leq 60. \]

A similar approach for \( c_2 \) leads to  
\[ \frac{200 - 10c_2}{3} \leq 0 \quad \Rightarrow \quad c_2 \geq \frac{20}{3} \]

\[ \frac{20c_2 - 1000}{9} \leq 0 \quad \Rightarrow \quad c_2 \leq \frac{50}{3} \]

**Range:** \( 20 \leq c_2 \leq 50 \)

b. Current solution is still optimal. However, the total profit has been reduced to $30 \( 25 \) + $30 \( 20 \) = $1350.

c. From the \( z_j \) entry in the \( s_1 \) column we see that the dual price for the material 1 constraint is $33.33. It is the increase in profit that would result from having one additional ton of material one.

d. Material 3 is the most valuable and RMC should be willing to pay up to $44.44 per ton for it.

12. a.  
\[
\begin{align*}
20 + \Delta b_1 \ (10/3) & \geq 0 \quad \Rightarrow \quad \Delta b_1 \geq -6 \\
1 + \Delta b_1 \ (-2/3) & \geq 0 \quad \Rightarrow \quad \Delta b_1 \leq 3/2 \\
25 + \Delta b_1 \ (-5/3) & \geq 0 \quad \Rightarrow \quad \Delta b_1 \leq 15
\end{align*}
\]

therefore \( -6 \leq \Delta b_1 \leq 1 \frac{1}{2} \)

**Range:** \( 14 \leq b_1 \leq 21 \frac{1}{2} \)

b.  
\[
\begin{align*}
20 + \Delta b_2 \ (0) & \geq 0 \quad \Rightarrow \quad \text{no restriction} \\
1 + \Delta b_2 \ (1) & \geq 0 \quad \Rightarrow \quad \Delta b_2 \geq -1 \\
25 + \Delta b_2 \ (0) & \geq 0 \quad \Rightarrow \quad \text{no restriction}
\end{align*}
\]

**Range:** \( b_2 \geq 4 \)

c.  
\[
\begin{align*}
20 + \Delta b_3 \ (-20/9) & \geq 0 \quad \Rightarrow \quad \Delta b_3 \leq 9 \\
1 + \Delta b_3 \ (4/9) & \geq 0 \quad \Rightarrow \quad \Delta b_3 \geq -9/4 \\
25 + \Delta b_3 \ (25/9) & \geq 0 \quad \Rightarrow \quad \Delta b_3 \geq -9
\end{align*}
\]
therefore \(-2^{1/4} \leq \Delta b_3 \leq 9\)

**Range: 18^{3/4} \leq b_3 \leq 30**

d. Dual price: 400/9

Valid for 18^{3/4} \leq b_3 \leq 30

13. a. The final simplex tableau is given by

<table>
<thead>
<tr>
<th>Basis</th>
<th>(c_B)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(s_2)</th>
<th>(s_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_2)</td>
<td>0</td>
<td>5/2</td>
<td>7/6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>(x_3)</td>
<td>5</td>
<td>3/2</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(x_4)</td>
<td>3</td>
<td>0</td>
<td>2/3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/3</td>
</tr>
<tr>
<td>(z_j)</td>
<td>15/2</td>
<td>9/2</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>(c_j - z_j)</td>
<td>-9/2</td>
<td>-7/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

b. Range: 2 \leq c_3

c. Since 1 is not contained in the range of optimality, a new basis will become optimal.

The new optimal solution and its value is

\(x_1 = 10\)

\(x_4 = 25/3\)

\(s_2 = 40/3\) (Surplus associated with constraint 2)

d. Since \(x_2\) is a nonbasic variable we simply require

\(c_2 - 9/2 \leq 0.\)

Range: \(c_2 \leq 4^{1/2}\)

e. Since 4 is contained in the range, a three unit increase in \(c_2\) would have no effect on the optimal solution or on the value of that solution.

14. a. 400/3 \leq b_1 \leq 800

b. 275 \leq b_2

c. 275/2 \leq b_3 \leq 625
15. The final simplex tableau is given:

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>15</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>30</td>
<td>0</td>
<td>1</td>
<td>1/4</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3/4</td>
<td>-3/4</td>
<td>-1/2</td>
</tr>
<tr>
<td>$z_f$</td>
<td>15</td>
<td>30</td>
<td>45/2</td>
<td>15/2</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>$c_j - z_f$</td>
<td>0</td>
<td>0</td>
<td>-5/2</td>
<td>-15/2</td>
<td>-15</td>
<td>0</td>
</tr>
</tbody>
</table>

a. $x_1 = 4$, $x_2 = 1/2$  Optimal value: 75
b. 75
c. Constraints one and two.
d. There are $1 1/2$ units of slack in constraint three.
e. Dual prices: $15/2$, 15, 0

Increasing the right-hand side of constraint two would have the greatest positive effect on the objective function.
f. $12.5 \leq c_1$
$20 \leq c_2 \leq 60$
$c_3 \leq 22.5$

The optimal values for the decision variables will not change as long as the objective function coefficients stay in these intervals.
g. For $b_1$
$4 + \Delta b_1 (1) \geq 0 \rightarrow \Delta b_1 \geq -4$
$1/2 + \Delta b_1 (-1/4) \geq 0 \rightarrow \Delta b_1 \leq 2$
$3/2 + \Delta b_1 (-3/4) \geq 0 \rightarrow \Delta b_1 \leq 2$

therefore $-4 \leq \Delta b_1 \leq 2$

Range: $0 \leq b_1 \leq 6$

For $b_2$
$4 + \Delta b_2 (0) \geq 0 \rightarrow $ no restriction
$1/2 + \Delta b_2 (1/2) \geq 0 \rightarrow \Delta b_2 \geq -1$
$3/2 + \Delta b_2 (-1/2) \geq 0 \rightarrow \Delta b_2 \leq 3$

therefore $-1 \leq \Delta b_2 \leq 3$

Range: $2 \leq b_2 \leq 6$
For $b_3$

\[ \frac{4}{1} + \Delta b_3 (0) \geq 0 \rightarrow \text{no restriction} \]
\[ \frac{1}{2} + \Delta b_3 (0) \geq 0 \rightarrow \text{no restriction} \]
\[ \frac{3}{2} + \Delta b_3 (1) \geq 0 \rightarrow \Delta b_3 \geq -\frac{3}{2} \]

therefore \(-\frac{3}{2} \leq \Delta b_3\)

**Range:** \(4 \frac{1}{2} \leq b_3\)

The dual prices accurately predict the rate of change of the objective function with respect to an increase in the right-hand side as long as the right-hand side remains within its range of feasibility.

16. a. After converting to a maximization problem by multiplying the objective function by (-1) and solving we obtain the optimal simplex tableau shown.

<table>
<thead>
<tr>
<th>Basis</th>
<th>$c_B$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/60</td>
<td>1/6</td>
<td>1</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-8</td>
<td>1</td>
<td>0</td>
<td>-1/75</td>
<td>-1/3</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-3</td>
<td>0</td>
<td>1</td>
<td>1/60</td>
<td>1/6</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ z_j = -8 \cdot -3 \cdot 17/300 - 17/300 = -62,000 \]
\[ c_j - z_j \]

Total Risk = 62,000

b. The dual price for the second constraint is \(-13/6 = -2.167\). So, every $1 increase in the annual income requirement increases the total risk of the portfolio by 2.167.

c. \[ 7000 - \Delta b_2 (1/6) \geq 0 \rightarrow \Delta b_2 \leq 42,000 \]
\[ 4000 - \Delta b_2 (-1/3) \geq 0 \rightarrow \Delta b_2 \geq -12,000 \]
\[ 10,000 - \Delta b_2 (1/6) \geq 0 \rightarrow \Delta b_2 \leq 60,000 \]

So, \(-12,000 \leq \Delta b_2 \leq 42,000\)

and \(48,000 \leq b_2 \leq 102,000\)

d. The new optimal solution and its value are

\[ s_3 = 7000 - 5000(1/6) = 37000/6 = 6,166.667 \]
\[ x_1 = 4000 - 5000(-1/3) = 17,000/3 = 5,666.667 \]
\[ x_2 = 10,000 - 5000(1/6) = 55,000/6 = 9,166.67 \]

Value = -62,000 - 5000(13/6) = -437,000/6 = -72,833.33

Since, this is a min problem being solved as a max, the new optimal value is 72,833.33
e. There is no upper limit in the range of optimality for the objective function coefficient of the stock fund. Therefore, the solution will not change. But, its value will increase to:

\[ 9(4,000) + 3(10,000) = 66,000 \]

17. a. The dual is given by:

\[
\begin{align*}
\text{Min} & \quad 550u_1 + 700u_2 + 200u_3 \\
\text{s.t.} & \quad 1.5u_1 + 4u_2 + 2u_3 \geq 4 \\
& \quad 2u_1 + 1u_2 + 3u_3 \geq 6 \\
& \quad 4u_1 + 2u_2 + 1u_3 \geq 3 \\
& \quad 3u_1 + 1u_2 + 2u_3 \geq 1 \\
& \quad u_1, u_2, u_3 \geq 0
\end{align*}
\]

b. Optimal solution: \( u_1 = 3/10, u_2 = 0, u_3 = 54/30 \)

The \( z_j \) values for the four surplus variables of the dual show \( x_1 = 0, x_2 = 25, x_3 = 125, \) and \( x_4 = 0. \)

c. Since \( u_1 = 3/10, u_2 = 0, \) and \( u_3 = 54/30, \) machines A and C \( (u_j > 0) \) are operating at capacity. Machine C is the priority machine since each hour is worth \( 54/30. \)

18. The dual is given by:

\[
\begin{align*}
\text{Max} & \quad 5u_1 + 5u_2 + 24u_3 \\
\text{s.t.} & \quad 15u_1 + 4u_2 + 12u_3 \leq 2800 \\
& \quad 15u_1 + 8u_2 \leq 6000 \\
& \quad u_1 + 8u_3 \leq 1200 \\
& \quad u_1, u_2, u_3 \geq 0
\end{align*}
\]

19. The canonical form is

\[
\begin{align*}
\text{Max} & \quad 3x_1 + x_2 + 5x_3 + 3x_4 \\
\text{s.t.} & \quad 3x_1 + 1x_2 + 2x_3 \leq 30 \\
& \quad -3x_1 - 1x_2 - 2x_3 \leq -30 \\
& \quad -2x_1 - 3x_3 - x_4 \leq -15 \\
& \quad 2x_2 + 3x_4 \leq 25 \\
& \quad x_1, x_2, x_3, x_4 \geq 0.
\end{align*}
\]

The dual is

\[
\begin{align*}
\text{Max} & \quad 30u_1^* - 30u_1^- - 15u_2 + 25u_3 \\
\text{s.t.} & \quad 3u_1^* - 3u_1^- - 2u_2 \geq 3 \\
& \quad u_1^* - u_1^- - u_2 + 2u_3 \geq 1 \\
& \quad 2u_1^* - 20u_1^- - 3u_2 \geq 5 \\
& \quad u_1^*, u_1^-, u_2, u_3 \geq 0
\end{align*}
\]
20. a. 

Max 30u_1 + 20u_2 + 80u_3 

\[ \begin{align*}
u_1 + u_3 & \leq 1 \\
u_2 + 2u_3 & \leq 1 \\
u_1, u_2, u_3 & \geq 0
\end{align*} \]

b. The final simplex tableau for the dual problem is given by

<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th>u_1</th>
<th>u_2</th>
<th>u_3</th>
<th>s_1</th>
<th>s_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>30</td>
<td>1</td>
<td>-1/2</td>
<td>0</td>
<td>1</td>
<td>-1/2</td>
</tr>
<tr>
<td>u_3</td>
<td>80</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>z_j</td>
<td>30</td>
<td>25</td>
<td>80</td>
<td>30</td>
<td>25</td>
<td>55</td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>-30</td>
<td>-25</td>
<td></td>
</tr>
</tbody>
</table>

The z_j values for the two slack variables indicate x_1 = 30 and x_2 = 25.

c. With u_3 = 1/2, the relaxation of that constraint by one unit would reduce costs by $.50.

21. a. 

Max 15u_1 + 30u_2 + 20u_3 

\[ \begin{align*}
u_1 + u_3 & \leq 4 \\
0.5u_1 + 2u_2 + u_3 & \leq 3 \\
u_1 + u_2 + 2u_3 & \leq 6 \\
u_1, u_2, u_3 & \geq 0
\end{align*} \]

b. The optimal simplex tableau for the dual is

<table>
<thead>
<tr>
<th>Basis</th>
<th>c_B</th>
<th>u_1</th>
<th>u_2</th>
<th>u_3</th>
<th>s_1</th>
<th>s_2</th>
<th>s_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>15</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u_2</td>
<td>30</td>
<td>0</td>
<td>1</td>
<td>1/4</td>
<td>-1/4</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>s_3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3/4</td>
<td>-3/4</td>
<td>-1/2</td>
<td>1</td>
</tr>
<tr>
<td>z_j</td>
<td>15</td>
<td>30</td>
<td>15/2</td>
<td>15/2</td>
<td>15</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>c_j - z_j</td>
<td>0</td>
<td>-5/2</td>
<td>-15/2</td>
<td>-15</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. From the z_j values for the surplus variables we see that the optimal primal solution is x_1 = 15/2, x_2 = 15, and x_3 = 0.

d. The optimal value for the dual is shown in part b to equal 75. Substituting x_1 = 15/2 and x_2 = 15 into the primal objective function, we find that it gives the same value.

\[ 4(15/2) + 3(15) = 75 \]
22. a. 

\[
\begin{align*}
\text{Max} & \quad 10x_1 + 5x_2 \\
\text{s.t.} & \quad x_1 \geq 20 \\
& \quad x_2 \geq 20 \\
& \quad x_1 \leq 100 \\
& \quad x_2 \leq 100 \\
& \quad 3x_1 + x_2 \leq 175 \\
\end{align*}
\]

\[x_1, x_2 \geq 0\]

b. The dual problem is

\[
\begin{align*}
\text{Min} & \quad -20u_1 - 20u_2 + 100u_3 + 100u_4 + 175u_5 \\
\text{s.t.} & \quad -u_1 + u_3 + 3u_4 \geq 10 \\
& \quad -u_2 + u_4 + u_5 \geq 5 \\
\end{align*}
\]

\[u_1, u_2, u_3, u_4, u_5 \geq 0\]

The optimal solution to this problem is given by:

\[u_1 = 0, \quad u_2 = 0, \quad u_3 = 0, \quad u_4 = 5/3, \quad \text{and} \quad u_5 = 10/3.\]

c. The optimal number of calls is given by the negative of the dual prices for the dual:

\[x_1 = 25 \quad \text{and} \quad x_2 = 100.\]

Commission = $750.

d. \[u_4 = 5/3: \quad \text{$1.67 commission increase for an additional call for product 2.}\]

\[u_5 = 10/3: \quad \text{$3.33 commission increase for an additional hour of selling time per month.}\]

23. a. Extreme point 1: \[x_1 = 0, x_2 = 0 \quad \text{value} = 0\]

Extreme point 2: \[x_1 = 5, x_2 = 0 \quad \text{value} = 15\]

Extreme point 3: \[x_1 = 4, x_2 = 2 \quad \text{value} = 16\]

b. Dual problem:

\[
\begin{align*}
\text{Min} & \quad 8u_1 + 10u_2 \\
\text{s.t.} & \quad u_1 + 2u_2 \geq 3 \\
& \quad 2u_1 + u_2 \geq 2 \\
\end{align*}
\]

\[u_1, u_2 \geq 0\]
c. Extreme Point 1: $u_1 = 3, u_2 = 0$ value = 24  
   Extreme Point 2: $u_1 = \frac{1}{3}, u_2 = \frac{4}{3}$ value = 16  
   Extreme Point 3: $u_1 = 0, u_2 = 2$ value = 20  

d. Each dual extreme point solution yields a value greater-than-or-equal-to each primal extreme point solution.  

e. No. The value of any feasible solution to the dual problem provides an upper bound on the value of any feasible primal solution.  

24. a. If the current optimal solution satisfies the new constraints, it is still optimal. Checking, we find  
   
   $6(10) + 4(30) - 15 = 165 \leq 170 \text{ ok}$  
   $\frac{1}{4}(10) + 30 = 32.5 \geq 25 \text{ ok}$  

   Both of the omitted constraints are satisfied.  
   Therefore, the same solution is optimal.
Chapter 7
Transportation, Assignment, and Transshipment Problems

Learning Objectives

1. Be able to identify the special features of the transportation problem.
2. Become familiar with the types of problems that can be solved by applying a transportation model.
3. Be able to develop network and linear programming models of the transportation problem.
4. Know how to handle the cases of (1) unequal supply and demand, (2) unacceptable routes, and (3) maximization objective for a transportation problem.
5. Be able to identify the special features of the assignment problem.
6. Become familiar with the types of problems that can be solved by applying an assignment model.
7. Be able to develop network and linear programming models of the assignment problem.
8. Be familiar with the special features of the transshipment problem.
9. Become familiar with the types of problems that can be solved by applying a transshipment model.
10. Be able to develop network and linear programming models of the transshipment problem.
11. Be able to utilize the minimum-cost method to find an initial feasible solution to a transportation problem.
12. Be able to utilize the transportation simplex method to find the optimal solution to a transportation problem.
13. Be able to utilize the Hungarian algorithm to solve an assignment problem.
14. Understand the following terms.

- transportation problem
- origin
- destination
- network flow problem
- transportation tableau
- minimum cost method
- stepping-stone path
- modified distribution (MODI) method
- assignment problem
- Hungarian method
- opportunity loss
- transshipment problem
- capacitated transshipment problem
Solutions:

1. The network model is shown.

2. a. Let $x_{11}$ : Amount shipped from Jefferson City to Des Moines
   $x_{12}$ : Amount shipped from Jefferson City to Kansas City
   $x_{23}$ : Amount shipped from Omaha to St. Louis

   \[
   \text{Min } 14x_{11} + 9x_{12} + 7x_{13} + 8x_{21} + 10x_{22} + 5x_{23}
   \]
   s.t.
   \[
   \begin{align*}
   x_{11} + x_{12} + x_{13} & \leq 30 \\
   x_{21} + x_{22} + x_{23} & \leq 20 \\
   x_{11} + x_{21} & = 25 \\
   x_{12} + x_{22} & = 15 \\
   x_{13} + x_{23} & = 10
   \end{align*}
   \]

   $x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \geq 0$

   b. Optimal Solution:

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jefferson City - Des Moines</td>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>Jefferson City - Kansas City</td>
<td>15</td>
<td>135</td>
</tr>
<tr>
<td>Jefferson City - St. Louis</td>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>Omaha - Des Moines</td>
<td>20</td>
<td>160</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>435</td>
</tr>
</tbody>
</table>
3. a. & b.

The linear programming formulation and optimal solution as printed by The Management Scientist are shown below. The first two letters in the variable names identify the “from” node for the shipping route and the last two identify the “to” node. Also, The Management Scientist prints ‘<‘ for ‘≤’.

LINEAR PROGRAMMING PROBLEM

\[ \text{MIN } 2PHAT + 6PHDA + 6PHCO + 2PHBO + 1NOAT + 2NODA + 5NOCO + 7NOBO \]

S.T.

1) \[ PHAT + PHDA + PHCO + PHBO < 5000 \]
2) \[ NOAT + NODA + NOCO + NOBO < 3000 \]
3) \[ PHAT + NOAT = 1400 \]
4) \[ PHDA + NODA = 3200 \]
5) \[ PHCO + NOCO = 2000 \]
6) \[ PHBO + NOBO = 1400 \]

OPTIMAL SOLUTION

Objective Function Value = 24800.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHAT</td>
<td>1400.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PHDA</td>
<td>200.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PHCO</td>
<td>2000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>PHBO</td>
<td>1400.000</td>
<td>0.000</td>
</tr>
<tr>
<td>NOAT</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>NODA</td>
<td>3000.000</td>
<td>0.000</td>
</tr>
<tr>
<td>NOCO</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>NOBO</td>
<td>0.000</td>
<td>9.000</td>
</tr>
</tbody>
</table>

Note that the Philadelphia port satisfies all the demand at Atlanta, Columbus, and Boston as well as the portion of the Dallas demand exceeding the New Orleans capacity.

4. a.
b. Let \( x_{ij} \) = Amount shipped from plant \( i \) to warehouse \( j \)

\[
\begin{align*}
\text{Min} & \quad 20x_{11} + 16x_{12} + 24x_{13} + 10x_{21} + 10x_{22} + 8x_{23} + 12x_{31} + 18x_{32} + 10x_{33} \\
\text{s.t.} & \quad x_{11} + x_{12} + x_{13} \leq 300 \\
& \quad x_{21} + x_{22} + x_{23} \leq 500 \\
& \quad x_{11} + x_{12} + x_{13} + x_{31} + x_{32} + x_{33} \leq 100 \\
& \quad x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} = 200 \\
& \quad x_{13} + x_{23} + x_{33} = 300 \\
& \quad x_{ij} \geq 0 \quad i = 1, 2, 3; \quad j = 1, 2, 3
\end{align*}
\]

Optimal Solution:

<table>
<thead>
<tr>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 - W_2 )</td>
<td>300</td>
</tr>
<tr>
<td>( P_2 - W_1 )</td>
<td>100</td>
</tr>
<tr>
<td>( P_2 - W_2 )</td>
<td>100</td>
</tr>
<tr>
<td>( P_2 - W_3 )</td>
<td>300</td>
</tr>
<tr>
<td>( P_3 - W_1 )</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
10,400
\]

c. The only change necessary, if the data are profit values, is to change the objective to one of maximization.

5. a.
b. Let $x_{ij} = \text{amount shipped from supply node } i \text{ to demand node } j$.

$$\begin{align*}
\text{Min} & \quad 10x_{11} + 20x_{12} + 15x_{13} + 12x_{21} + 15x_{22} + 18x_{23} \\
\text{s.t.} & \quad x_{11} + x_{12} + x_{13} \leq 500 \\
& \quad x_{11} + x_{21} + x_{23} \leq 400 \\
& \quad x_{12} + x_{21} = 400 \\
& \quad x_{13} + x_{22} = 200 \\
& \quad x_{13} + x_{23} = 300 \\
& \quad x_{ij} \geq 0 \text{ for all } i, j
\end{align*}$$

\[x_{ij} \geq 0 \text{ for all } i, j\]

c. Optimal Solution

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern - Hamilton</td>
<td>200</td>
<td>$2000</td>
</tr>
<tr>
<td>Southern - Clermont</td>
<td>300</td>
<td>4500</td>
</tr>
<tr>
<td>Northwest - Hamilton</td>
<td>200</td>
<td>2400</td>
</tr>
<tr>
<td>Northwest - Butler</td>
<td>200</td>
<td>3000</td>
</tr>
<tr>
<td>Total Cost</td>
<td></td>
<td>$11,900</td>
</tr>
</tbody>
</table>

d. To answer this question the simplest approach is to increase the Butler County demand to 300 and to increase the supply by 100 at both Southern Gas and Northwest Gas.

The new optimal solution is:

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern - Hamilton</td>
<td>300</td>
<td>$3000</td>
</tr>
<tr>
<td>Southern - Clermont</td>
<td>300</td>
<td>4500</td>
</tr>
<tr>
<td>Northwest - Hamilton</td>
<td>100</td>
<td>1200</td>
</tr>
<tr>
<td>Northwest - Butler</td>
<td>300</td>
<td>4500</td>
</tr>
<tr>
<td>Total Cost</td>
<td></td>
<td>$13,200</td>
</tr>
</tbody>
</table>

From the new solution we see that Tri-County should contract with Southern Gas for the additional 100 units.
b. The linear programming formulation and optimal solution as printed by *The Management Scientist* are shown. The first two letters of the variable name identify the “from” node and the second two letters identify the “to” node. Also, *The Management Scientist* prints “<” for “≤.”

**LINEAR PROGRAMMING PROBLEM**

\[
\text{MIN } 10SEPI + 20SEMO + 5SEDE + 9SELA + 10SEWA + 2COPI + 10COMO + 8CODE + 30COLA + 6COWA + 1NYPI + 20NYMO + 7NYDE + 10NYLA + 4NYWA
\]
S.T.

1) SEPI + SEMO + SEDE + SELA + SEWA < 9000
2) COPI + COMO + CODE + COLA + COWA < 4000
3) NYPI + NYMO + NYDE + NYLA + NYWA < 8000
4) SEPI + COPI + NYPI = 3000
5) SEMO + COMO + NYMO = 5000
6) SEDE + CODE + NYDE = 4000
7) SELA + COLA + NYLA = 6000
8) SEWA + COWA + NYWA = 3000

OPTIMAL SOLUTION

Objective Function Value = 150000.000

Variable | Value | Reduced Costs
----------|-------|------------------
SEPI      | 0.000 | 10.000
SEMO      | 0.000 | 1.000
SEDE      |  4000 |   0.000
SELA      |  5000 |   0.000
SEWA      |  0.000|   7.000
COPI      |  0.000|  11.000
COMO      |  4000 |   0.000
CODE      |  0.000|  12.000
COLA      |  0.000|  30.000
COWA      |  0.000|  12.000
NYPI      |  3000 |   0.000
NYMO      |  1000 |   0.000
NYDE      |  0.000|   1.000
NYLA      |  1000 |   0.000
NYWA      |  3000 |   0.000

The new optimal solution actually shows a decrease of $9000 in shipping cost. It is summarized.

<table>
<thead>
<tr>
<th>Optimal Solution</th>
<th>Units</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seattle - Denver</td>
<td>4000</td>
<td>$20,000</td>
</tr>
<tr>
<td>Seattle - Los Angeles</td>
<td>5000</td>
<td>45,000</td>
</tr>
<tr>
<td>Columbus - Mobile</td>
<td>5000</td>
<td>50,000</td>
</tr>
<tr>
<td>New York - Pittsburgh</td>
<td>4000</td>
<td>4,000</td>
</tr>
<tr>
<td>New York - Los Angeles</td>
<td>1000</td>
<td>10,000</td>
</tr>
<tr>
<td>New York - Washington</td>
<td>3000</td>
<td>12,000</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td><strong>$141,000</strong></td>
</tr>
</tbody>
</table>
7. a. Let $x_{ij} = \text{number of hours from consultant } i \text{ assigned to client } j$.

Max $100x_{11} + 125x_{12} + 115x_{13} + 100x_{14} + 120x_{21} + 135x_{22} + 115x_{23}$

s.t. $x_{11} + x_{12} + x_{13} + x_{14} \leq 160$
     $x_{21} + x_{22} + x_{23} + x_{24} \leq 160$
     $x_{31} + x_{32} + x_{33} + x_{34} \leq 140$
     $x_{11} + x_{21} + x_{31} = 180$
     $x_{12} + x_{22} + x_{32} = 75$
     $x_{13} + x_{23} + x_{33} = 100$
     $x_{14} + x_{24} + x_{34} = 85$

$x_{ij} \geq 0$ for all $i, j$
Optimal Solution

<table>
<thead>
<tr>
<th>Hours Assigned</th>
<th>Billing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avery - Client B</td>
<td>40</td>
</tr>
<tr>
<td>Avery - Client C</td>
<td>100</td>
</tr>
<tr>
<td>Baker - Client A</td>
<td>40</td>
</tr>
<tr>
<td>Baker - Client B</td>
<td>35</td>
</tr>
<tr>
<td>Baker - Client D</td>
<td>85</td>
</tr>
<tr>
<td>Campbell - Client A</td>
<td>140</td>
</tr>
<tr>
<td><strong>Total Billing:</strong></td>
<td><strong>$57,925</strong></td>
</tr>
</tbody>
</table>

c. New Optimal Solution

<table>
<thead>
<tr>
<th>Hours Assigned</th>
<th>Billing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avery - Client A</td>
<td>40</td>
</tr>
<tr>
<td>Avery - Client C</td>
<td>100</td>
</tr>
<tr>
<td>Baker - Client B</td>
<td>75</td>
</tr>
<tr>
<td>Baker - Client D</td>
<td>85</td>
</tr>
<tr>
<td>Campbell - Client A</td>
<td>140</td>
</tr>
<tr>
<td><strong>Total Billing:</strong></td>
<td><strong>$57,525</strong></td>
</tr>
</tbody>
</table>

8. The network model, the linear programming formulation, and the optimal solution are shown. Note that the third constraint corresponds to the dummy origin. The variables $x_{31}, x_{32}, x_{33},$ and $x_{34}$ are the amounts shipped out of the dummy origin; they do not appear in the objective function since they are given a coefficient of zero.
Transportation, Assignment And Transshipment Models

Note: Dummy origin has supply of 4000.

Max \[ 32x_{11} + 34x_{12} + 32x_{13} + 40x_{14} + 34x_{21} + 30x_{22} + 28x_{23} + 38x_{24} \]

s.t. \[ x_{11} + x_{12} + x_{13} + x_{14} \leq 5000 \]
\[ x_{21} + x_{22} + x_{23} + x_{24} \leq 3000 \]
\[ x_{31} + x_{32} + x_{33} + x_{34} \leq 4000 \]
\[ x_{11} + x_{21} + x_{31} = 2000 \]
\[ x_{12} + x_{22} + x_{32} = 5000 \]
\[ x_{13} + x_{23} + x_{33} = 3000 \]
\[ x_{14} + x_{24} + x_{34} = 2000 \]

\[ x_{ij} \geq 0 \quad \text{for all} \quad i, j \]

Optimal Solution

<table>
<thead>
<tr>
<th>Units</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>$136,000</td>
</tr>
<tr>
<td>1000</td>
<td>40,000</td>
</tr>
<tr>
<td>2000</td>
<td>68,000</td>
</tr>
<tr>
<td>1000</td>
<td>38,000</td>
</tr>
<tr>
<td></td>
<td>Total Cost: $282,000</td>
</tr>
</tbody>
</table>

---

Demand

Supply

Dum
Customer 2 demand has a shortfall of 1000

Customer 3 demand of 3000 is not satisfied.

9. We show a linear programming formulation. The cost of shipping from Martinsville is incremented by $29.50 to every destination, the cost of shipping from Plymouth is incremented by $31.20, and the cost of shipping from Franklin is incremented by $30.35.

Let \( x_{ij} \) = amount produced at plant \( i \) and shipped to distributor \( j \)

Note that no variable is included for the unacceptable Plymouth to Dallas route.

Min \( 30.95x_{11} + 31.10x_{12} + 30.90x_{13} + 32.30x_{2} + 31.80x_{23} + 31.55x_{31} + 31.55x_{32} + 32.15x_{33} \)

s.t.

\[
\begin{align*}
& x_{11} + x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} + x_{33} \leq 400 \\
& x_{11} + x_{21} + x_{31} = 400 \\
& x_{13} + x_{23} + x_{33} = 400 \\
& x_{ij} \geq 0 \text{ for all } i, j
\end{align*}
\]

Optimal Plan:

- Martinsville to Chicago: 300
- Martinsville to Dallas: 100
- Plymouth to Chicago: 100
- Plymouth to New York: 400
- Franklin to Dallas: 300

Total Cost = $37,810

Note: Plymouth has excess supply of 100.

10. The linear programming formulation and optimal solution are shown.

Let \( x_{1A} \) = Units of product A on machine 1

\( x_{1B} \) = Units of product B on machine 1

\( x_{3C} \) = Units of product C on machine 3

Max \( x_{1A} + 1.2x_{1B} + 0.9x_{1C} + 1.3x_{2A} + 1.4x_{2B} + 1.2x_{2C} + 1.1x_{3A} + x_{3B} + 1.2x_{3C} \)

s.t.

\[
\begin{align*}
& x_{1A} + x_{1B} + x_{1C} \leq 1500 \\
& x_{2A} + x_{2B} + x_{2C} \leq 1500 \\
& x_{3A} + x_{3B} + x_{3C} \leq 1000 \\
& x_{1A} + x_{2A} + x_{3A} = 2000 \\
& x_{1B} + x_{2B} + x_{3B} = 500 \\
& x_{1C} + x_{2C} + x_{3C} = 1200 \\
& x_{ij} \geq 0 \text{ for all } i, j
\end{align*}
\]
**Optimal Solution**

<table>
<thead>
<tr>
<th>Units</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$300</td>
</tr>
<tr>
<td>1200</td>
<td>1080</td>
</tr>
<tr>
<td>1200</td>
<td>1560</td>
</tr>
<tr>
<td>500</td>
<td>550</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Total:</td>
<td>$3990</td>
</tr>
</tbody>
</table>

Note: There is an unused capacity of 300 units on machine 2.

11. a. 

b. There are alternative optimal solutions.

**Solution #1**
- Denver to St. Paul: 10
- Atlanta to Boston: 50
- Atlanta to Dallas: 50
- Chicago to Dallas: 20
- Chicago to Los Angeles: 60
- Chicago to St. Paul: 70

**Solution #2**
- Denver to St. Paul: 10
- Atlanta to Boston: 50
- Atlanta to Los Angeles: 50
- Chicago to Dallas: 70
- Chicago to Los Angeles: 10
- Chicago to St. Paul: 70

Total Profit: $4240
If solution #1 is used, Forbelt should produce 10 motors at Denver, 100 motors at Atlanta, and 150 motors at Chicago. There will be idle capacity for 90 motors at Denver.

If solution #2 is used, Forbelt should adopt the same production schedule but a modified shipping schedule.

12. a.

b. Min 10x_{11} + 16x_{12} + 32x_{13} + 14x_{21} + 22x_{22} + 40x_{23} + 22x_{31} + 24x_{32} + 34x_{33}

s.t. \[ x_{11} + x_{12} + x_{13} \leq 1 \]
\[ x_{21} + x_{22} + x_{23} \leq 1 \]
\[ x_{31} + x_{32} + x_{33} \leq 1 \]
\[ x_{11} + x_{21} + x_{31} = 1 \]
\[ x_{12} + x_{22} + x_{32} = 1 \]
\[ x_{13} + x_{23} + x_{33} = 1 \]
\[ x_{ij} \geq 0 \text{ for all } i, j \]

Solution \( x_{12} = 1, x_{21} = 1, x_{33} = 1 \) Total completion time = 64

13. a. Optimal assignment: Jackson to 1, Smith to 3, and Burton to 2. Time requirement is 62 days.

b. Considering Burton has saved 2 days.

c. Ellis.
14. a. Minimize:  

\[ \begin{align*} 
30x_{11} + 44x_{12} + 38x_{13} + 47x_{14} + 31x_{15} + 25x_{21} + \cdots + 28x_{55} 
\end{align*} \]

Subject to:

\[ \begin{align*} 
x_{11} + x_{12} + x_{13} + x_{14} + x_{15} & \leq 1 \\
x_{21} + x_{22} + x_{23} + x_{24} + x_{25} & \leq 1 \\
x_{31} + x_{32} + x_{33} + x_{34} + x_{35} & \leq 1 \\
x_{41} + x_{42} + x_{43} + x_{44} + x_{45} & \leq 1 \\
x_{51} + x_{52} + x_{53} + x_{54} + x_{55} & \leq 1 \\
x_{11} + x_{21} + x_{31} + x_{41} + x_{51} & = 1 \\
x_{12} + x_{22} + x_{32} + x_{42} + x_{52} & = 1 \\
x_{13} + x_{23} + x_{33} + x_{43} + x_{53} & = 1 \\
x_{14} + x_{24} + x_{34} + x_{44} + x_{54} & = 1 \\
x_{15} + x_{25} + x_{35} + x_{45} + x_{55} & = 1 \\
\end{align*} \]

Non-negativity constraints:

\[ x_{ij} \geq 0, \quad i = 1, 2, \ldots, 5; \quad j = 1, 2, \ldots, 5 \]

Optimal Solution:

- Green to Job 1: $26
- Brown to Job 2: $34
- Red to Job 3: $38
- Blue to Job 4: $39
- White to Job 5: $25

Total installation cost for the 5 contracts is $16,200.

Since the data is in hundreds of dollars, the total installation cost for the 5 contracts is $16,200.
15. Optimal Solution:

Terry: Client 2 (15 days)
Carle: Client 3 (5 days)
McClymonds: Client 1 (6 days)
Higley: Not accepted
Total time = 26 days

Note: An alternative optimal solution is Terry: Client 2, Carle: unassigned, McClymonds: Client 3, and Higley: Client 1.

16. a.
b. Let \( x_{ij} = \begin{cases} 
1 & \text{if department } i \text{ is assigned location } j \\
0 & \text{otherwise} 
\end{cases} \)

Max \[
10x_{11} + 6x_{12} + 12x_{13} + 8x_{14} + 15x_{21} + 18x_{22} + 5x_{23} + 11x_{24} + 17x_{31} + 10x_{32} + 13x_{33} + 16x_{34} + 14x_{41} + 12x_{42} + 13x_{43} + 10x_{44} + 14x_{51} + 16x_{52} + 6x_{53} + 12x_{54}
\]

s.t.

\[
x_{11} + x_{12} + x_{13} + x_{14} + x_{21} + x_{22} + x_{23} + x_{24} \leq 1
\]

\[
x_{31} + x_{32} + x_{33} + x_{34} + x_{41} + x_{42} + x_{43} + x_{44} \leq 1
\]

\[
x_{51} + x_{52} + x_{53} + x_{54} \leq 1
\]

\[
x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1
\]

\[
x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 1
\]

\[
x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 1
\]

\[
x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 1
\]

\( x_{ij} \geq 0 \) for all \( i, j \)

Optimal Solution:
Toy: Location 2
Auto Parts: Location 4
Housewares: Location 3
Video: Location 1
Profit: 61

17. a. Simply delete 2 arcs from the network representation in the solution to 16 part (a): the arc from Toy to location 2 and the arc from Auto Parts to location 4.

b. Add two constraints to the linear programming model in the solution to problem 16 part (b).

\( x_{22} = 0 \) and \( x_{34} = 0 \)

Revised optimal solution:
Toy: Location 4
Auto Parts: Location 1
Housewares: Location 3
Video: Location 2
Profit: 57

18. a. This is the variation of the assignment problem in which multiple assignments are possible. Each distribution center may be assigned up to 3 customer zones.

The linear programming model of this problem has 40 variables (one for each combination of distribution center and customer zone). It has 13 constraints. There are 5 supply (\( \leq 3 \)) constraints and 8 demand (\( = 1 \)) constraints.
The problem can also be solved using the Transportation module of *The Management Scientist*. The optimal solution is given below.

<table>
<thead>
<tr>
<th>Assignments</th>
<th>Cost ($1000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plano: Kansas City, Dallas</td>
<td>34</td>
</tr>
<tr>
<td>Flagstaff: Los Angeles</td>
<td>15</td>
</tr>
<tr>
<td>Springfield: Chicago, Columbus, Atlanta</td>
<td>70</td>
</tr>
<tr>
<td>Boulder: Newark, Denver</td>
<td>97</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td><strong>$216</strong></td>
</tr>
</tbody>
</table>

b. the Nashville distribution center is not used.

c. All the distribution centers are used. Columbus is switched from Springfield to Nashville. Total cost increases by $11,000 to $227,000.

19. A linear programming formulation and the optimal solution are given. For the variables, we let the first letter of the sales representatives name be the first subscript and the sales territory be the second subscript.

\[
\text{Max} \quad 44x_{WA} + 80x_{WB} + 52x_{WC} + 60x_{WD} + 60x_{BA} + 56x_{BB} + 40x_{BC} \\
+ 72x_{BD} + 36x_{FA} + 60x_{FB} + 48x_{FC} + 48x_{FD} + 52x_{HA} \\
+ 76x_{HB} + 36x_{HC} + 40x_{HD} \\
\text{s.t.} \quad \begin{cases} 
    x_{WA} + x_{WB} + x_{WC} + x_{WD} & \leq 1 \\
    x_{BA} + x_{BB} + x_{BC} + x_{BD} & \leq 1 \\
    x_{FA} + x_{FB} + x_{FC} + x_{FD} & \leq 1 \\
    x_{HA} + x_{HB} + x_{HC} + x_{HD} & \leq 1 \\
    x_{WA} + x_{BA} + x_{FA} + x_{HA} & = 1 \\
    x_{WB} + x_{BB} + x_{FB} + x_{HB} & = 1 \\
    x_{WC} + x_{BC} + x_{FC} + x_{HC} & = 1 \\
    x_{WD} + x_{BD} + x_{FD} + x_{HD} & = 1 \\
    x_{ij} & \geq 0 \text{ for all } i, j
\end{cases}
\]

**Optimal Solution**

<table>
<thead>
<tr>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington - B</td>
</tr>
<tr>
<td>Benson - D</td>
</tr>
<tr>
<td>Fredricks - C</td>
</tr>
<tr>
<td>Hodson - A</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>
20. A linear programming formulation of this problem can be developed as follows. Let the first letter of each variable name represent the professor and the second two the course. Note that a DPH variable is not created because the assignment is unacceptable.

\[
\text{Max } 2.8AUN + 2.2AMB + 3.3AMS + 3.0APH + 3.2BUN + \ldots + 2.5DMS
\]

\[
\text{s.t. }
\begin{align*}
AUN + &\quad AMB +\quad AMS +\quad APH &\leq 1 \\
BUN + &\quad BMB +\quad BMS +\quad BPH &\leq 1 \\
CUN + &\quad CMB +\quad CMS +\quad CPH &\leq 1 \\
DUN + &\quad DMB +\quad DMS &\leq 1 \\
AUN + &\quad BUN +\quad CUN +\quad DUN = 1 \\
AMB + &\quad BMB +\quad CMS +\quad DMB = 1 \\
AMS + &\quad BMS +\quad CMS +\quad DMS = 1 \\
APH + &\quad BPH +\quad CPH = 1 \\
\end{align*}
\]

All Variables \( \geq 0 \)

Optimal Solution:  
Rating
A to MS course 3.3  
B to Ph.D. course 3.6  
C to MBA course 3.2  
D to Undergraduate course 3.2  
Max Total Rating 13.3

21. a.

\[
\text{Min } 150x_{11} + 210x_{12} + 270x_{13} \\
+ 170x_{21} + 230x_{22} + 220x_{23} \\
+ 180x_{31} + 230x_{32} + 225x_{33} \\
+ 160x_{41} + 240x_{42} + 230x_{43}
\]

\[
\text{s.t. }
\begin{align*}
x_{11} + &\quad x_{12} +\quad x_{13} &\leq 1 \\
x_{21} + &\quad x_{22} +\quad x_{23} &\leq 1 \\
x_{31} + &\quad x_{32} +\quad x_{33} &\leq 1 \\
x_{41} + &\quad x_{42} +\quad x_{43} &\leq 1 \\
x_{11} + &\quad x_{21} +\quad x_{31} +\quad x_{41} = 1 \\
x_{12} + &\quad x_{22} +\quad x_{32} +\quad x_{42} = 1 \\
x_{13} + &\quad x_{23} +\quad x_{33} +\quad x_{43} = 1 \\
\end{align*}
\]

\( x_{ij} \geq 0 \) for all \( i, j \)

Optimal Solution: \( x_{12} = 1, x_{23} = 1, x_{41} = 1 \)

Total hours required: 590

Note: statistician 3 is not assigned.

b. The solution will not change, but the total hours required will increase by 5. This is the extra time required for statistician 4 to complete the job for client A.

c. The solution will not change, but the total time required will decrease by 20 hours.
d. The solution will not change; statistician 3 will not be assigned. Note that this occurs because increasing the time for statistician 3 makes statistician 3 an even less attractive candidate for assignment.

22. a. The total cost is the sum of the purchase cost and the transportation cost. We show the calculation for Division 1 - Supplier 1 and present the result for the other Division-Supplier combinations.

**Division 1 - Supplier 1**

<table>
<thead>
<tr>
<th>Purchase Cost (40,000 x $12.60)</th>
<th>$504,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation Cost (40,000 x $2.75)</td>
<td>$110,000</td>
</tr>
<tr>
<td>Total Cost:</td>
<td>$614,000</td>
</tr>
</tbody>
</table>

**Cost Matrix ($1,000s)**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>614</td>
<td>660</td>
<td>534</td>
<td>680</td>
<td>590</td>
<td>630</td>
</tr>
<tr>
<td>2</td>
<td>603</td>
<td>639</td>
<td>702</td>
<td>693</td>
<td>693</td>
<td>630</td>
</tr>
<tr>
<td>3</td>
<td>865</td>
<td>830</td>
<td>775</td>
<td>850</td>
<td>900</td>
<td>930</td>
</tr>
<tr>
<td>4</td>
<td>532</td>
<td>553</td>
<td>511</td>
<td>581</td>
<td>595</td>
<td>553</td>
</tr>
<tr>
<td>5</td>
<td>720</td>
<td>648</td>
<td>684</td>
<td>693</td>
<td>657</td>
<td>747</td>
</tr>
</tbody>
</table>

b. Optimal Solution:

- Supplier 1 - Division 2: $603
- Supplier 2 - Division 5: 648
- Supplier 3 - Division 3: 775
- Supplier 5 - Division 1: 590
- Supplier 6 - Division 4: 553
- Total: $3,169

23. a. Network Model
b. & c.

The linear programming formulation and solution as printed by *The Management Scientist* is shown.

LINEAR PROGRAMMING PROBLEM

\[
\begin{align*}
\text{MIN} & : & 4X_{14} + 7X_{15} + 8X_{24} + 5X_{25} + 5X_{34} + 6X_{35} + 6X_{46} + 4X_{47} + 8X_{48} + 4X_{49} + \\
& & 3X_{56} + 6X_{57} + 7X_{58} + 7X_{59} \\
\text{S.T.} & : & \\
1) & & X_{14} + X_{15} < 450 \\
2) & & X_{24} + X_{25} < 600 \\
3) & & X_{34} + X_{35} < 380 \\
4) & & X_{46} + X_{47} + X_{48} + X_{49} - X_{14} - X_{24} - X_{34} = 0 \\
5) & & X_{56} + X_{57} + X_{58} + X_{59} - X_{15} - X_{25} - X_{35} = 0 \\
6) & & X_{46} + X_{56} = 300 \\
7) & & X_{47} + X_{57} = 300 \\
8) & & X_{48} + X_{58} = 300 \\
9) & & X_{49} + X_{59} = 400 
\end{align*}
\]
OPTIMAL SOLUTION

Objective Function Value = 11850.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>X14</td>
<td>450.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X15</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>X24</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>X25</td>
<td>600.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X34</td>
<td>250.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X35</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>X46</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>X47</td>
<td>300.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X48</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>X49</td>
<td>400.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X56</td>
<td>300.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X57</td>
<td>0.000</td>
<td>2.000</td>
</tr>
<tr>
<td>X58</td>
<td>300.00</td>
<td>0.000</td>
</tr>
<tr>
<td>X59</td>
<td>0.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

There is an excess capacity of 130 units at plant 3.

24. a. Three arcs must be added to the network model in problem 23a. The new network is shown.

b.&c.

The linear programming formulation and optimal solution as printed by The management Scientist follow:
LINEAR PROGRAMMING PROBLEM

MIN 4X14 + 7X15 + 8X24 + 5X25 + 5X34 + 6X35 + 6X46 + 4X47 + 8X48 + 4X49 + 3X56 + 6X57 + 7X58 + 7X59 + 7X39 + 2X45 + 2X54

S.T.

1) X14 + X15 < 450
2) X24 + X25 < 600
3) X34 + X35 + X39 < 380
4) X45 + X46 + X47 + X48 + X49 - X14 - X24 - X34 - X45 - X54 = 0
5) X54 + X56 + X57 + X58 + X59 - X15 - X25 - X35 - X45 = 0
6) X46 + X56 = 300
7) X47 + X57 = 300
8) X48 + X58 = 300
9) X39 + X49 + X59 = 400

OPTIMAL SOLUTION

Objective Function Value = 11220.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>X14</td>
<td>320.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X15</td>
<td>0.000</td>
<td>2.000</td>
</tr>
<tr>
<td>X24</td>
<td>0.000</td>
<td>4.000</td>
</tr>
<tr>
<td>X25</td>
<td>600.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X34</td>
<td>0.000</td>
<td>2.000</td>
</tr>
<tr>
<td>X35</td>
<td>0.000</td>
<td>2.000</td>
</tr>
<tr>
<td>X46</td>
<td>0.000</td>
<td>2.000</td>
</tr>
<tr>
<td>X47</td>
<td>300.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X48</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X49</td>
<td>20.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X56</td>
<td>300.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X57</td>
<td>0.000</td>
<td>3.000</td>
</tr>
<tr>
<td>X58</td>
<td>300.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X59</td>
<td>0.000</td>
<td>4.000</td>
</tr>
<tr>
<td>X39</td>
<td>380.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X45</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>X54</td>
<td>0.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

The value of the solution here is $630 less than the value of the solution for problem 23. The new shipping route from plant 3 to customer 4 has helped (x39 = 380). There is now excess capacity of 130 units at plant 1.

25. a&b

To model, we create a transshipment problem with a supply of one at node 1 and a demand of 1 at node 7.

The linear programming formulation and optimal solution as provided by *The Management Scientist* are shown below.
TRANSPORTATION, ASSIGNMENT AND TRANSSHIPMENT MODELS

LINEAR PROGRAMMING PROBLEM

MIN 35X12 + 30X13 + 12X23 + 18X24 + 39X27 + 15X35 + 12X45 + 16X47 + 9X56 + 18X67

S.T.
1) 1X12 + 1X13 = 1
2) -1X12 + 1X23 + 1X24 + 1X27 = 0
3) -1X13 - 1X23 + 1X35 = 0
4) -1X24 + 1X45 + 1X47 = 0
5) -1X35 - 1X45 + 1X56 = 0
6) -1X56 + 1X67 = 0
7) +1X27 + 1X47 + 1X67 = 1

OPTIMAL SOLUTION

Objective Function Value = 69.000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>X12</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X13</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X23</td>
<td>0.000</td>
<td>17.000</td>
</tr>
<tr>
<td>X24</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X27</td>
<td>0.000</td>
<td>5.000</td>
</tr>
<tr>
<td>X35</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X45</td>
<td>0.000</td>
<td>20.000</td>
</tr>
<tr>
<td>X47</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X56</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>X67</td>
<td>0.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
<th>Dual Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>-1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>34.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>29.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>52.000</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>44.000</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>53.000</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>-68.000</td>
</tr>
</tbody>
</table>

c. Allowing for 8 minutes to get to node 1 and 69 minutes to go from node 1 to node 7, we expect to take 77 minutes for the delivery. With a 10% safety margin, we can guarantee a delivery in 85 minutes - that is at 1:25 p.m.
26. a. 

![Graph showing transportation network with cities and distances]

b. 

Minimize \[ 7x_{13} + 5x_{14} + 3x_{23} + 4x_{24} + 8x_{35} + 5x_{36} + 7x_{37} + 5x_{45} + 6x_{46} + 10x_{47} \]

subject to:

\[ x_{13} + x_{14} \leq 300 \]
\[ x_{23} + x_{24} \leq 100 \]
\[ x_{13} - x_{14} - x_{23} - x_{24} + x_{35} + x_{36} + x_{37} = 0 \]
\[ x_{35} + x_{45} + x_{46} + x_{47} = 0 \]
\[ x_{36} + x_{45} + x_{46} + x_{47} = 150 \]
\[ x_{37} + x_{45} + x_{46} + x_{47} = 150 \]
\[ x_{ij} \geq 0 \text{ for all } i \text{ and } j \]

c. Optimal Solution:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{13})</td>
<td>50</td>
</tr>
<tr>
<td>(x_{14})</td>
<td>250</td>
</tr>
<tr>
<td>(x_{23})</td>
<td>100</td>
</tr>
<tr>
<td>(x_{24})</td>
<td>0</td>
</tr>
<tr>
<td>(x_{35})</td>
<td>0</td>
</tr>
<tr>
<td>(x_{36})</td>
<td>0</td>
</tr>
<tr>
<td>(x_{37})</td>
<td>150</td>
</tr>
<tr>
<td>(x_{45})</td>
<td>150</td>
</tr>
<tr>
<td>(x_{46})</td>
<td>100</td>
</tr>
<tr>
<td>(x_{47})</td>
<td>0</td>
</tr>
</tbody>
</table>

Objective Function: 4300
27. a.

b. Min

\[ 6x_{14} + 8x_{15} + 8x_{24} + 12x_{25} + 10x_{34} + 5x_{35} + 9x_{46} + 7x_{47} + 6x_{48} + 10x_{49} + 7x_{56} + 9x_{57} + 6x_{58} + 8x_{59} \]

s.t.

\[
\begin{align*}
    x_{14} + x_{15} & \leq 400 \\
    x_{24} + x_{25} & \leq 450 \\
    x_{34} + x_{35} & \leq 350 \\
    -x_{14} - x_{24} - x_{34} + x_{46} + x_{47} + x_{48} + x_{49} - x_{56} - x_{57} - x_{58} - x_{59} & = 0 \\
    +x_{46} & = 200 \\
    +x_{47} & = 500 \\
    +x_{48} & = 300 \\
    +x_{49} & = 200 \\

x_{ij} & \geq 0 \text{ for all } i, j
\end{align*}
\]
c. Optimal Solution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{14}$</td>
<td>400</td>
</tr>
<tr>
<td>$x_{15}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{24}$</td>
<td>450</td>
</tr>
<tr>
<td>$x_{25}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{34}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{35}$</td>
<td>350</td>
</tr>
<tr>
<td>$x_{46}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{47}$</td>
<td>500</td>
</tr>
<tr>
<td>$x_{48}$</td>
<td>300</td>
</tr>
<tr>
<td>$x_{49}$</td>
<td>50</td>
</tr>
<tr>
<td>$x_{56}$</td>
<td>200</td>
</tr>
<tr>
<td>$x_{57}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{58}$</td>
<td>0</td>
</tr>
<tr>
<td>$x_{59}$</td>
<td>150</td>
</tr>
</tbody>
</table>

Value of optimal solution: 16150

28.
A linear programming model is

Min
\[ 8x_{14} + 6x_{15} + 3x_{24} + 8x_{25} + 9x_{34} + 3x_{35} + 44x_{46} + 34x_{47} + 34x_{48} + 32x_{49} + 57x_{56} + 35x_{57} + 28x_{58} + 24x_{59} \]

s.t.
\[ x_{14} + x_{15} \leq 3 \]
\[ x_{24} + x_{25} \leq 6 \]
\[ x_{34} + x_{35} \leq 5 \]
\[ -x_{14} - x_{24} - x_{34} + x_{46} + x_{47} + x_{48} + x_{49} = 0 \]
\[ -x_{15} - x_{25} - x_{35} + x_{46} + x_{56} + x_{57} + x_{58} + x_{59} = 0 \]
\[ x_{46} + x_{56} = 2 \]
\[ x_{47} + x_{57} = 4 \]
\[ x_{48} + x_{58} = 3 \]
\[ x_{49} + x_{59} = 3 \]
\[ x_{ij} \geq 0 \text{ for all } i, j \]

Optimal Solution | Units Shipped | Cost
---|---|---
Muncie to Cincinnati | 1 | 6
Cincinnati to Concord | 3 | 84
Brazil to Louisville | 6 | 18
Louisville to Macon | 2 | 88
Louisville to Greenwood | 4 | 136
Xenia to Cincinnati | 5 | 15
Cincinnati to Chatham | 3 | 72

Two rail cars must be held at Muncie until a buyer is found.

29. a.
Min
\[ 20x_{12} + 25x_{15} + 30x_{25} + 45x_{27} + 20x_{31} + 35x_{36} \]
\[ + 30x_{42} + 25x_{53} + 15x_{54} + 28x_{56} + 12x_{67} + 27x_{74} \]

s.t.
\[ x_{31} - x_{12} - x_{15} = 8 \]
\[ x_{25} + x_{27} - x_{12} - x_{42} = 5 \]
\[ x_{31} + x_{36} - x_{53} = 3 \]
\[ x_{54} + x_{74} - x_{42} = 3 \]
\[ x_{53} + x_{54} + x_{56} - x_{15} - x_{25} = 2 \]
\[ x_{36} + x_{56} - x_{67} = 5 \]
\[ x_{74} - x_{27} - x_{67} = 6 \]
\[ x_{ij} \geq 0 \text{ for all } i, j \]
b. \( x_{12} = 0 \quad x_{53} = 5 \)
\( x_{15} = 0 \quad x_{54} = 0 \)
\( x_{35} = 8 \quad x_{56} = 5 \)
\( x_{27} = 0 \quad x_{67} = 0 \)
\( x_{31} = 8 \quad x_{74} = 6 \)
\( x_{36} = 0 \)
\( x_{42} = 3 \)

Total cost of redistributing cars = $917

30.

The positive numbers by nodes indicate the amount of supply at that node. The negative numbers by nodes indicate the amount of demand at the node.

31. a. Modify Figure 7.12 by adding two nodes and two arcs. Let node 0 be a beginning inventory node with a supply of 50 and an arc connecting it to node 5 (period 1 demand). Let node 9 be an ending inventory node with a demand of 100 and an arc connecting node 8 (period 4 demand to it).

b. Min
\[
\begin{align*}
\text{Min} & \quad 2x_{15} + 5x_{26} + 3x_{37} + 3x_{48} + 0.25x_{56} + 0.25x_{67} + 0.25x_{78} + 0.25x_{89} \\
\text{s.t.} & \quad x_{05} = 50 \\
& \quad x_{15} \leq 600 \\
& \quad x_{26} \leq 300 \\
& \quad x_{37} \leq 500 \\
& \quad x_{48} \leq 400 \\
& \quad x_{05} + x_{15} = 400 \\
& \quad x_{26} + x_{56} - x_{67} = 500 \\
& \quad x_{37} + x_{67} - x_{78} = 400 \\
& \quad x_{48} + x_{78} - x_{89} = 400 \\
& \quad x_{89} = 100 \\
& \quad x_{ij} \geq 0 \text{ for all } i \text{ and } j
\end{align*}
\]

Optimal Solution:
\[ x_{05} = 50 \quad x_{56} = 250 \]
\[ x_{15} = 600 \quad x_{67} = 0 \]
\[ x_{26} = 250 \quad x_{78} = 100 \]
\[ x_{37} = 500 \quad x_{89} = 100 \]
\[ x_{48} = 400 \]

Total Cost = $5262.50

32. a. Let \( R1, R2, R3 \) represent regular time production in months 1, 2, 3
\( O1, O2, O3 \) represent overtime production in months 1, 2, 3
\( D1, D2, D3 \) represent demand in months 1, 2, 3

Using these 9 nodes, a network model is shown.
b. Use the following notation to define the variables: first two letters designates the "from node" and the second two letters designates the "to node" of the arc. For instance, R1D1 is amount of regular time production available to satisfy demand in month 1, O1D1 is amount of overtime production in month 1 available to satisfy demand in month 1, D1D2 is the amount of inventory carried over from month 1 to month 2, and so on.

\[
\text{MIN } 50R1D1 + 80O1D1 + 20D1D2 + 50R2D2 + 80O2D2 + 20D2D3 + 60R3D3 + 100O3D3
\]

subject to:
\[
\begin{align*}
1) & \quad R1D1 \leq 275 \\
2) & \quad O1D1 \leq 100 \\
3) & \quad R2D2 \leq 200 \\
4) & \quad O2D2 \leq 50 \\
5) & \quad R3D3 \leq 100 \\
6) & \quad O3D3 \leq 50 \\
7) & \quad R1D1 + O1D1 - D1D2 = 150 \\
8) & \quad R2D2 + O2D2 + D1D2 - D2D3 = 250 \\
9) & \quad R3D3 + O3D3 + D2D3 = 300
\end{align*}
\]

c. Optimal Solution:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1D1</td>
<td>275.000</td>
</tr>
<tr>
<td>O1D1</td>
<td>25.000</td>
</tr>
<tr>
<td>D1D2</td>
<td>150.000</td>
</tr>
<tr>
<td>R2D2</td>
<td>200.000</td>
</tr>
<tr>
<td>O2D2</td>
<td>50.000</td>
</tr>
<tr>
<td>D2D3</td>
<td>150.000</td>
</tr>
<tr>
<td>R3D3</td>
<td>100.000</td>
</tr>
<tr>
<td>O3D3</td>
<td>50.000</td>
</tr>
</tbody>
</table>

Value = $46,750

Note: Slack variable for constraint 2 = 75.

d. The values of the slack variables for constraints 1 through 6 represent unused capacity. The only nonzero slack variable is for constraint 2; its value is 75. Thus, there are 75 units of unused overtime capacity in month 1.
33. a. 

<table>
<thead>
<tr>
<th>( u_i )</th>
<th>( v_j )</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>-2</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

This is the minimum cost solution since \( e_{ij} \geq 0 \) for all \( i,j \).

Solution:

<table>
<thead>
<tr>
<th>Shipping Route (Arc)</th>
<th>Units</th>
<th>Unit Cost</th>
<th>Arc Shipping Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1 - D1</td>
<td>25</td>
<td>5</td>
<td>$125</td>
</tr>
<tr>
<td>O1 - D3</td>
<td>50</td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td>O2 - D3</td>
<td>100</td>
<td>8</td>
<td>800</td>
</tr>
<tr>
<td>O2 - D4</td>
<td>75</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>O3 - D1</td>
<td>100</td>
<td>6</td>
<td>600</td>
</tr>
<tr>
<td>O4 - D2</td>
<td>100</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td>O4 - D4</td>
<td>50</td>
<td>4</td>
<td>200</td>
</tr>
</tbody>
</table>

Total Transportation Cost: $2875

b. Yes, \( e_{32} = 0 \). This indicates that we can ship over route O3 - D2 without increasing the cost. To find the alternative optimal solution identify cell (3, 2) as the incoming cell and make appropriate adjustments on the stepping stone path.

The increasing cells on the path are O4 - D4, O2 - D3, and O1 - D1. The decreasing cells on the path are O4 - D2, O2 - D4, O1 - D3, and O3 - D1. The decreasing cell with the smallest number of units is O1 - D3 with 50 units. Therefore, 50 units is assigned to O3 - D2. After making the appropriate increases and decreases on the stepping stone path the following alternative optimal solution is identified.
Note that all $e_{ij} \geq 0$ indicating that this solution is also optimal. Also note that $e_{13} = 0$ indicating there is an alternative optimal solution with cell (1, 3) in solution. This is the solution we found in part (a).

34. a. An initial solution is given below.

b. Note that the initial solution is degenerate. A zero is assigned to the cell in row 3 and column 1 so that the row and column indices can be computed.

Total Cost: $7800
Cell in row 3 and column 3 is identified as an incoming cell.

Stepping-stone path shows cycle of adjustments. Outgoing cell is in row 3 and column 1.

Solution is recognized as optimal. It is degenerate.

Thus, the initial solution turns out to be the optimal solution; total cost = $7800.
c. To begin with we reduce the supply at Tucson by 100 and the demand at San Diego by 100; the new solution is shown below:
Optimal Solution: recall, however, that the 100 units shipped from Tucson to San Diego must be added to obtain the total cost.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>San Jose to San Francisco:</td>
<td>100</td>
</tr>
<tr>
<td>Las Vegas to Los Angeles:</td>
<td>200</td>
</tr>
<tr>
<td>Las Vegas to San Diego:</td>
<td>100</td>
</tr>
<tr>
<td>Tucson to San Francisco:</td>
<td>200</td>
</tr>
<tr>
<td>Tucson to San Diego:</td>
<td>100</td>
</tr>
<tr>
<td><strong>Total Cost:</strong></td>
<td><strong>$7800</strong></td>
</tr>
</tbody>
</table>

Note that this total cost is the same as for part (a); thus, we have alternative optima.

d. The final transportation tableau is shown below. The total transportation cost is $8,000, an increase of $200 over the solution to part (a).

<table>
<thead>
<tr>
<th>2</th>
<th>10</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>200</td>
</tr>
</tbody>
</table>

35. a.
b.

This is an initial feasible solution with a total cost of $475.

36. An initial feasible solution found by the minimum cost method is given below.
Computing row and column indexes and evaluating the unoccupied cells one identifies the cell in row 2 and column 1 as the incoming cell.

\[
\begin{array}{ccc}
  & u_i & v_j \\
0 & 100 & 200 \\
-6 & 100 & 200 \\
-8 & 100 & 200 \\
\end{array}
\]

The +’s and -’s above show the cycle of adjustments necessary on the stepping-stone path as flow is allocated to the cell in row 2 and column 1. The cell in row 1 and column 1 is identified as corresponding to the outgoing arc. The new solution follows.

\[
\begin{array}{ccc}
  & u_i & v_j \\
0 & 16 & 20 \\
-6 & 100 & 200 \\
-8 & 100 & 200 \\
\end{array}
\]

Since all per-unit costs are \( \geq 0 \), this solution is optimal. However, an alternative optimal solution can be found by shipping 100 units over the \( P_3 - W_3 \) route.

37. a. Initial Solution:

\[
\begin{array}{ccc}
  & D_1 & D_2 & D_3 \\
O_1 & 150 & 100 & 8 \\
O_2 & 18 & 12 & 14 \\
O_3 & 8 & 10 & 100 \\
\end{array}
\]

Total Cost: $4600
b.

\[ u_i \quad \begin{array}{ccc} \multicolumn{3}{c}{v_j} \\ \hline 6 & 8 & 10 \\ \hline 0 & 150 & 100 \quad 8 \quad 8 \\ 4 & 18 & 100 \quad + \quad 12 \quad - \quad 14 \\ 0 & 0 \quad 8 \quad 12 \quad 100 \quad 10 \end{array} \]

Incoming arc: \( O_1 - D_3 \)

\[ \begin{array}{ccc} \multicolumn{3}{c}{v_j} \\ \hline 6 & 8 & 8 \\ \hline 100 & 100 \quad 12 \quad 14 \\ 18 \quad + \quad 100 \quad 12 \quad 50 \end{array} \]

Outgoing arc: \( O_2 - D_3 \)

\[ \begin{array}{ccc} \multicolumn{3}{c}{v_j} \\ \hline 6 & 8 & 8 \\ \hline 0 & 150 & 50 \quad 50 \\ 4 & 18 & 100 \quad 12 \quad 14 \\ 2 & 8 \quad 12 \quad 100 \quad 10 \end{array} \]

Since all cell evaluations are non-negative, the solution is optimal; Total Cost: $4500.
c. At the optimal solution found in part (b), the cell evaluation for $O_3 - D_1 = 0$. Thus, additional units can be shipped over the $O_3 - D_1$ route with no increase in total cost.

Thus, an alternative optimal solution is

38. a.
We note that the net effect is the same as the per-unit cost change obtained using the MODI method.

b.

Again the net effect is the same as $e_{34} = +7$ computed using the MODI method.
b. All of the cells corresponding to production in one period being used to satisfy demand in a previous period are assigned a "big M" cost.

The initial solution found using the minimum cost method is optimal.

\[
\begin{array}{cccc}
400 & 200 & 2.25 & 2.50 & 2.75 & 600 \\
300 & 5 & 5.25 & 5.50 \\
400 & 3 & 3.25 & 500 \\
400 & 400 & 400 & 400 \\
\end{array}
\]

40. Subtract 10 from row 1, 14 from row 2, and 22 from row 3 to obtain:

\[
\begin{array}{cccc}
1 & 2 & 3 \\
Jackson & 0 & 6 & 22 \\
Ellis & 0 & 8 & 26 \\
Smith & 0 & 2 & 12 \\
\end{array}
\]

Subtract 0 from column 1, 2 from column 2, and 12 from column 3 to obtain:

\[
\begin{array}{cccc}
1 & 2 & 3 \\
Jackson & 0 & 4 & 10 \\
Ellis & 0 & 6 & 14 \\
Smith & 0 & 0 & 0 \\
\end{array}
\]
Two lines cover the zeros. The minimum unlined element is 4. Step 3 yields:

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\hline
Jackson & 0 & \boxed{0} & 6 \\
Ellis & 0 & 2 & 10 \\
Smith & 0 & 0 & \boxed{0} \\
\end{array}
\]

Optimal Solution:

Jackson - 2
Ellis - 1
Smith - 3

Time requirement is 64 days.

41. Subtract 30 from row 1, 25 from row 2, 23 from row 3, 26 from row 4, and 26 from row 5 to obtain:

\[
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
\hline
Red & 0 & 14 & 8 & 17 & 1 \\
White & 0 & 7 & 20 & 19 & 0 \\
Blue & 0 & 17 & 14 & 16 & 6 \\
Green & 0 & 12 & 11 & 19 & 2 \\
Brown & 0 & 8 & 18 & 17 & 2 \\
\end{array}
\]

Subtract 0 from column 1, 7 from column 2, 8 from column 3, 16 from column 4, and 0 from column 5 to obtain:

\[
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
\hline
Red & 0 & 7 & 0 & 1 & 1 \\
White & 0 & 0 & 12 & 3 & 0 \\
Blue & 0 & 10 & 6 & 0 & 6 \\
Green & 0 & 5 & 3 & 3 & 2 \\
Brown & 0 & 1 & 10 & 1 & 2 \\
\end{array}
\]

Four lines cover the zeroes. The minimum unlined element is 1. Step 3 of the Hungarian algorithm yields:
Transportation, Assignment And Transshipment Models

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>1</td>
<td>7</td>
<td>[0]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>White</td>
<td>1</td>
<td>0</td>
<td>12</td>
<td>3</td>
<td>[0]</td>
</tr>
<tr>
<td>Blue</td>
<td>1</td>
<td>10</td>
<td>6</td>
<td>[0]</td>
<td>6</td>
</tr>
<tr>
<td>Green</td>
<td>[0]</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Brown</td>
<td>0</td>
<td>[0]</td>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Optimal Solution:

- Green to Job 1 $26
- Brown to Job 2 34
- Red to Job 3 38
- Blue to Job 4 39
- White to Job 5 25

Total cost is $162.

42. After adding a dummy column, we get an initial assignment matrix.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
<td>9</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>3</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>6</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Applying Steps 1 and 2 we obtain:

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>1</th>
<th>6</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Applying Step 3 followed by Step 2 results in:
Finally, application of Step's 3 and 2 lead to the optimal solution shown below.

```
  3  0  5  0
  2  3  1  0
  0  0  0  0
  1  1  2  0
```

Terry: Client 2 (15 days)
Carle: Client 3 (5 days)
McClymonds: Client 1 (6 days)
Higley: Not accepted

Total time = 26 days

Note: An alternative optimal solution is Terry: Client 2, Carle: unassigned, McClymonds: Client 3, and Higley: Client 1.

43. We start with the opportunity loss matrix.

```
  7 12 1 8 0
  0 8 5 0 0
  0 3 0 0 0
3 6 0 6 0
3 2 7 4 0
```

```
  5 12 1 6 0
  0 0 8 3 0
  0 10 2 0 2
  1 6 0 4 0
1 2 7 2 0
```
### Transportation, Assignment And Transshipment Models

#### Optimal Solution Profit

<table>
<thead>
<tr>
<th></th>
<th>Toy</th>
<th>Auto</th>
<th>Houseware</th>
<th>Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoe</td>
<td>2</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toy</td>
<td></td>
<td>4</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Auto</td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Houseware</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Video</td>
<td></td>
<td></td>
<td></td>
<td>61</td>
</tr>
</tbody>
</table>

44. Subtracting each element from the largest element in its column leads to the opportunity loss matrix.

#### Opportunity Loss Matrix

<table>
<thead>
<tr>
<th></th>
<th>Shoe</th>
<th>Toy</th>
<th>Auto</th>
<th>Houseware</th>
<th>Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toy</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Auto</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Houseware</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Video</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Shoe</th>
<th>Toy</th>
<th>Auto</th>
<th>Houseware</th>
<th>Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toy</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Auto</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Houseware</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Video</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
45. Original problem:

\[
\begin{array}{cccc}
44 & 80 & 52 & 60 \\
60 & 56 & 40 & 72 \\
36 & 60 & 48 & 48 \\
52 & 76 & 36 & 40 \\
\end{array}
\]

Opportunity loss matrix:

\[
\begin{array}{cccc}
14 & 0 & 0 & 12 \\
0 & 24 & 12 & 0 \\
24 & 20 & 4 & 24 \\
8 & 4 & 16 & 32 \\
\end{array}
\]

Step 1 (row reduction) and lining out zeros.

\[
\begin{array}{cccc}
16 & 0 & 0 & 12 \\
0 & 24 & 12 & 0 \\
20 & 16 & 0 & 20 \\
4 & 0 & 12 & 28 \\
\end{array}
\]

Step 3 followed by Step 2 results in the optimal solution

\[
\begin{array}{cccc}
12 & 0 & 0 & 8 \\
0 & 28 & 16 & 0 \\
16 & 16 & 0 & 16 \\
0 & 0 & 12 & 24 \\
\end{array}
\]
Optimal Solution:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington to B</td>
<td>80</td>
</tr>
<tr>
<td>Benson to D</td>
<td>72</td>
</tr>
<tr>
<td>Fredricks to C</td>
<td>48</td>
</tr>
<tr>
<td>Hodson to A</td>
<td>52</td>
</tr>
<tr>
<td>Total Sales</td>
<td>252</td>
</tr>
</tbody>
</table>
Chapter 8
Integer Linear Programming

Learning Objectives

1. Be able to recognize the types of situations where integer linear programming problem formulations are desirable.

2. Know the difference between all-integer and mixed integer linear programming problems.

3. Be able to solve small integer linear programs with a graphical solution procedure.

4. Be able to formulate and solve fixed charge, capital budgeting, distribution system, and product design problems as integer linear programs.

5. See how zero-one integer linear variables can be used to handle special situations such as multiple choice, $k$ out of $n$ alternatives, and conditional constraints.

6. Be familiar with the computer solution of MILPs.

7. Understand the following terms:

   - all-integer
   - mixed integer
   - zero-one variables
   - LP relaxation
   - multiple choice constraint
   - mutually exclusive constraint
   - $k$ out of $n$ alternatives constraint
   - conditional constraint
   - co-requisite constraint
Solutions:

1. a. This is a mixed integer linear program. Its LP Relaxation is

   \[
   \text{Max } \quad 30x_1 + 25x_2 \\
   \text{s.t. } \quad 3x_1 + 1.5x_2 \leq 400 \\
   \quad 1.5x_1 + 2x_2 \leq 250 \\
   \quad x_1 + x_2 \leq 150 \\
   \quad x_1, x_2 \geq 0
   \]

   b. This is an all-integer linear program. Its LP Relaxation just requires dropping the words "and integer" from the last line.

2. a. The optimal solution to the LP Relaxation is given by \(x_1 = 1.43, x_2 = 4.29\) with an objective function value of 41.47.

   Rounding down gives the feasible integer solution \(x_1 = 1, x_2 = 4\). Its value is 37.

   c.
The optimal solution is given by $x_1 = 0, x_2 = 5$. Its value is 40. This is not the same solution as that found by rounding down. It provides a 3 unit increase in the value of the objective function.

3. a.
b. The optimal solution to the LP Relaxation is shown on the above graph to be \( x_1 = 4, x_2 = 1 \). Its value is 5.

c. The optimal integer solution is the same as the optimal solution to the LP Relaxation. This is always the case whenever all the variables take on integer values in the optimal solution to the LP Relaxation.

4. a. 

The value of the optimal solution to the LP Relaxation is 36.7 and it is given by \( x_1 = 3.67, x_2 = 0.0 \).

Since we have all less-than-or-equal-to constraints with positive coefficients, the solution obtained by "rounding down" the values of the variables in the optimal solution to the LP Relaxation is feasible. The solution obtained by rounding down is \( x_1 = 3, x_2 = 0 \) with value 30.

Thus a lower bound on the value of the optimal solution is given by this feasible integer solution with value 30. An upper bound is given by the value of the LP Relaxation, 36.7. (Actually an upper bound of 36 could be established since no integer solution could have a value between 36 and 37.)
The optimal solution to the ILP is given by $x_1 = 3, x_2 = 2$. Its value is 36. The solution found by "rounding down" the solution to the LP relaxation had a value of 30. A 20% increase in this value was obtained by finding the optimal integer solution - a substantial difference if the objective function is being measured in thousands of dollars.
The optimal solution to the LP Relaxation is $x_1 = 0, x_2 = 5.71$ with value $= 34.26$. The solution obtained by "rounding down" is $x_1 = 0, x_2 = 5$ with value $30$. These two values provide an upper bound of 34.26 and a lower bound of 30 on the value of the optimal integer solution.

There are alternative optimal integer solutions given by $x_1 = 0, x_2 = 5$ and $x_1 = 2, x_2 = 4$; value is 30. In this case rounding the LP solution down does provide the optimal integer solution.
The feasible mixed integer solutions are indicated by the boldface vertical lines in the graph above.

b. The optimal solution to the LP relaxation is given by $x_1 = 3.14, x_2 = 2.60$. Its value is 14.08. Rounding the value of $x_1$ down to find a feasible mixed integer solution yields $x_1 = 3, x_2 = 2.60$ with a value of 13.8. This solution is clearly not optimal. With $x_1 = 3$ we can see from the graph that $x_2$ can be made larger without violating the constraints.

c. The optimal solution to the MILP is given by $x_1 = 3, x_2 = 2.67$. Its value is 14.
6. a.

b. The optimal solution to the LP Relaxation is given by \( x_1 = 1.96, x_2 = 5.48 \). Its value is 7.44. Thus an upper bound on the value of the optimal is given by 7.44.

Rounding the value of \( x_2 \) down yields a feasible solution of \( x_1 = 1.96, x_2 = 5 \) with value 6.96. Thus a lower bound on the value of the optimal solution is given by 6.96.
The optimal solution to the MILP is $x_1 = 1.29, x_2 = 6$. Its value is 7.29.

The solution $x_1 = 2.22, x_2 = 5$ is almost as good. Its value is 7.22.

7. a. $x_1 + x_3 + x_5 + x_6 = 2$

b. $x_3 - x_5 = 0$

c. $x_1 + x_4 = 1$

d. $x_4 \leq x_1$

$\begin{align*}
x_4 &\leq x_3 \\
x_4 &\leq x_3
\end{align*}$

e. $x_4 \leq x_1$

$\begin{align*}
x_4 &\leq x_3 \\
x_4 &\geq x_1 + x_3 - 1
\end{align*}$
8. a. Let $x_i = \begin{cases} 1 & \text{if investment alternative } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$

$$\text{max} \quad 4000x_1 + 6000x_2 + 10500x_3 + 4000x_4 + 8000x_5 + 3000x_6$$

$$\text{s.t.} \quad \begin{align*}
3000x_1 + 2500x_2 + 6000x_3 + 2000x_4 + 5000x_5 + 1000x_6 & \leq 10,500 \\
1000x_1 + 3500x_2 + 4000x_3 + 1500x_4 + 1000x_5 + 500x_6 & \leq 7,000 \\
4000x_1 + 3500x_2 + 5000x_3 + 1800x_4 + 4000x_5 + 900x_6 & \leq 8,750 \\
\end{align*}$$

$x_1, x_2, x_3, x_4, x_5, x_6 = 0, 1$

Optimal Solution found using *The Management Scientist* or LINDO

$$x_3 = 1$$
$$x_4 = 1$$
$$x_6 = 1$$

Value = 17,500

b. The following mutually exclusive constraint must be added to the model.

$$x_1 + x_2 \leq 1 \quad \text{No change in optimal solution.}$$

c. The following co-requisite constraint must be added to the model in b.

$$x_3 - x_4 = 0. \quad \text{No change in optimal solution.}$$

9. a. $x_4 \leq 8000 s_4$

b. $x_6 \leq 6000 s_6$

c. $x_4 \leq 8000 s_4$

$$x_6 \leq 6000 s_6$$

$s_4 + s_6 = 1$

d. Min $15x_4 + 18x_6 + 2000 s_4 + 3500 s_6$
10. a. Let $x_i = 1$ if a substation is located at site $i$, 0 otherwise

$$\begin{align*}
\text{min} & \quad x_A + x_B + x_C + x_D + x_E + x_F + x_G \\
\text{s.t.} & \quad x_A + x_B + x_C + x_E + x_G \geq \text{(area 1 covered)} \\
& \quad x_B + x_D \geq \text{(area 2 covered)} \\
& \quad x_C + x_E \geq \text{(area 3 covered)} \\
& \quad x_D + x_F + x_G \geq \text{(area 4 covered)} \\
& \quad x_A + x_B + x_C + x_D + x_F + x_G \geq \text{(area 5 covered)} \\
& \quad x_A + x_B + x_C + x_E + x_F + x_G \geq \text{(area 6 covered)} \\
& \quad x_A + x_B \geq \text{(area 7 covered)}
\end{align*}$$

b. Choose locations B and E.

11. a. Let $P_i = \text{units of product } i \text{ produced}$

$$\begin{align*}
\text{Max} & \quad 25P_1 + 28P_2 + 30P_3 \\
\text{s.t.} & \quad 1.5P_1 + 3P_2 + 2P_3 \leq 450 \\
& \quad 2P_1 + 1P_2 + 2.5P_3 \leq 350 \\
& \quad .25P_1 + .25P_2 + .25P_3 \leq 50 \\
P_1, P_2, P_3 & \geq 0
\end{align*}$$

b. The optimal solution is

$P_1 = 60$
$P_2 = 80$     Value = 5540
$P_3 = 60$

This solution provides a profit of $5540.$

c. Since the solution in part (b) calls for producing all three products, the total setup cost is

$$1550 = 400 + 550 + 600.$$  

Subtracting the total setup cost from the profit in part (b), we see that

$$\text{Profit} = 5540 - 1550 = 3990.$$ 

d. We introduce a 0-1 variable $y_i$ that is one if any quantity of product $i$ is produced and zero otherwise.

With the maximum production quantities provided by management, we obtain 3 new constraints:

$$\begin{align*}
P_1 & \leq 175y_1 \\
P_2 & \leq 150y_2 \\
P_3 & \leq 140y_3
\end{align*}$$
Bringing the variables to the left-hand side of the constraints, we obtain the following fixed charge formulation of the Hart problem.

\[
\begin{align*}
\text{Max} & \quad 25P_1 + 28P_2 + 30P_3 - 400y_1 - 550y_2 - 600y_3 \\
\text{s.t.} & \quad 1.5P_1 + 3P_2 + 2P_3 \leq 450 \\
& \quad 2P_1 + 1P_2 + 2.5P_3 \leq 350 \\
& \quad 0.25P_1 + 0.25P_2 + 0.25P_3 \leq 50 \\
& \quad P_1 - 175y_1 \leq 0 \\
& \quad P_2 - 150y_2 \leq 0 \\
& \quad P_3 - 140y_3 \leq 0 \\
& \quad P_1, P_2, P_3 \geq 0; \quad y_1, y_2, y_3 = 0, 1
\end{align*}
\]

\[e. \quad \text{The optimal solution using The Management Scientist is}
\]
\[
\begin{align*}
P_1 &= 100 \quad y_1 = 1 \\
P_2 &= 100 \quad y_2 = 1 \quad \text{Value} = 4350 \\
P_3 &= 0 \quad y_3 = 0
\end{align*}
\]

The profit associated with this solution is $4350. This is an improvement of $360 over the solution in part (c).

12. a. Constraints

\[
\begin{align*}
P &\leq 15 + 15Y_p \\
D &\leq 15 + 15Y_D \\
J &\leq 15 + 15Y_J \\
Y_p + Y_D + Y_J &\leq 1
\end{align*}
\]

b. We must add a constraint requiring 60 tons to be shipped and an objective function.

\[
\begin{align*}
\text{Min} & \quad 100Y_p + 85Y_D + 50Y_J \\
\text{s.t.} & \quad P + D + J = 60 \\
& \quad P \leq 15 + 15Y_p \\
& \quad D \leq 15 + 15Y_D \\
& \quad J \leq 15 + 15Y_J \\
& \quad Y_p + Y_D + Y_J \leq 1 \\
& \quad P, D, J \geq 0 \\
& \quad Y_p, Y_D, Y_J = 0, 1
\end{align*}
\]

Optimal Solution: \( P = 15, D = 15, J = 30 \)
\( Y_p = 0, Y_D = 0, Y_J = 1 \)
\( \text{Value} = 50 \)
13. a. One just needs to add the following multiple choice constraint to the problem.

\[ y_1 + y_2 = 1 \]

New Optimal Solution: \( y_1 = 1, \ y_3 = 1, \ x_{12} = 10, \ x_{31} = 30, \ x_{52} = 10, \ x_{53} = 20 \)

Value = 940

b. Since one plant is already located in St. Louis, it is only necessary to add the following constraint to the model

\[ y_3 + y_4 \leq 1 \]

New Optimal Solution: \( y_4 = 1, \ x_{42} = 20, \ x_{43} = 20, \ x_{51} = 30 \)

Value = 860

14. a. Let 1 denote the Michigan plant
2 denote the first New York plant
3 denote the second New York plant
4 denote the Ohio plant
5 denote the California plant

It is not possible to meet needs by modernizing only one plant.

The following table shows the options which involve modernizing two plants.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Transmission</th>
<th>Engine Block</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Capacity</td>
<td>Capacity</td>
</tr>
<tr>
<td>1</td>
<td>700</td>
<td>1300</td>
</tr>
<tr>
<td>2</td>
<td>1100</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>900</td>
<td>1400</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>5</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1700</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>1400</td>
<td>1300</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>900</td>
<td>1100</td>
</tr>
</tbody>
</table>

b. Modernize plants 1 and 3 or plants 4 and 5.
c. Let $x_i = \begin{cases} 
1 & \text{if plant } i \text{ is modernized} \\
0 & \text{if plant } i \text{ is not modernized} 
\end{cases}$

\[ \begin{align*}
\text{Min} & \quad 25x_1 + 35x_2 + 35x_3 + 40x_4 + 25x_5 \\
\text{s.t.} & \quad 300x_1 + 400x_2 + 800x_3 + 600x_4 + 300x_5 \geq 900 \text{ Transmissions} \\
& \quad 500x_1 + 800x_2 + 400x_3 + 900x_4 + 200x_5 \geq 900 \text{ Engine Blocks} 
\end{align*} \]

d. Optimal Solution: $x_1 = x_3 = 1$.

15. a. Let $x_i = \begin{cases} 
1 & \text{if a principal place of business in in county } i \\
0 & \text{otherwise} 
\end{cases}$

$y_i = \begin{cases} 
1 & \text{if county } i \text{ is not served} \\
0 & \text{if county } i \text{ is served} 
\end{cases}$

The objective function for an integer programming model calls for minimizing the population not served.

\[ \text{min } 195y_1 + 96y_2 + \cdots + 175y_{13} \]

There are 13 constraints needed; each is written so that $y_1$ will be forced to equal one whenever it is not possible to do business in county $i$.

Constraint 1: $x_1 + x_2 + x_3 + y_1 \geq 1$
Constraint 2: $x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + y_2 \geq 1$
Constraint 13: $x_{11} + x_{12} + x_{13} + y_{13} \geq 1$

One more constraint must be added to reflect the requirement that only one principal place of business may be established.

\[ x_1 + x_2 + \cdots + x_{13} = 1 \]

The optimal solution has a principal place of business in County 11 with an optimal value of 739,000. A population of 739,000 cannot be served by this solution. Counties 1-5 and 10 will not be served.

b. The only change necessary in the integer programming model for part a is that the right-hand side of the last constraint is increased from 1 to 2.

\[ x_1 + x_2 + \cdots + x_{13} = 2. \]

The optimal solution has principal places of business in counties 3 and 11 with an optimal value of 76,000. Only County 10 with a population of 76,000 is not served.
c. It is not the best location if only one principal place of business can be established; 1,058,000 customers in the region cannot be served. However, 642,000 can be served and if there is no opportunity to obtain a principal place of business in County 11, this may be a good start. Perhaps later there will be an opportunity in County 11.

16. a. 

\[
\begin{align*}
\text{min} & \quad 105x_9 + 105x_{10} + 105x_{11} + 32y_9 + 32y_{10} + 32y_{11} + 32y_{12} + 32y_1 + 32y_2 + 32y_3 \\
x_9 + & \quad y_9 \geq 6 \\
x_9 + x_{10} + & \quad y_9 + y_{10} \geq 4 \\
x_9 + x_{10} + x_{11} + & \quad y_9 + y_{10} + y_{11} \geq 8 \\
x_9 + x_{10} + x_{11} + & \quad y_9 + y_{10} + y_{11} + y_{12} \geq 10 \\
x_{10} + x_{11} + & \quad y_{10} + y_{11} + y_{12} + y_1 \geq 9 \\
x_9 + x_{11} + & \quad y_{11} + y_{12} + y_1 + y_2 \geq 6 \\
x_9 + & \quad x_{10} + y_{12} + y_1 + y_2 + y_3 \geq 4 \\
x_9 + x_{10} + & \quad x_{11} + y_1 + y_2 + y_3 \geq 7 \\
x_{10} + x_{11} + & \quad y_2 + y_3 \geq 6 \\
x_{11} & \quad + y_3 \geq 6 \\
\end{align*}
\]

\(x_i, y_j \geq 0\) and integer for \(i = 9, 10, 11\) and \(j = 9, 10, 11, 12, 1, 2, 3\)

b. Solution to LP Relaxation obtained using LINDO/PC:

\[
\begin{align*}
y_9 &= 6 \\
y_{12} &= 6 \\
y_3 &= 6 \\
y_{11} &= 2 \\
y_1 &= 1 \\
\text{Cost: $672.}
\end{align*}
\]

\[
\begin{align*}
y_9 &= 5 \\
x_{11} &= 4 \\
y_{12} &= 5 \\
y_3 &= 2
\end{align*}
\]

c. The solution to the LP Relaxation is integral therefore it is the optimal solution to the integer program.

A difficulty with this solution is that only part-time employees are used; this may cause problems with supervision, etc. The large surpluses from 5, 12-1 (4 employees), and 3-4 (9 employees) indicate times when the tellers are not needed for customer service and may be reassigned to other tasks.

d. Add the following constraints to the formulation in part (a).

\[
\begin{align*}
x_9 \geq 1 \\
x_{11} \geq 1 \\
x_9 + x_{10} + x_{11} \geq 5
\end{align*}
\]

The new optimal solution, which has a daily cost of $909 is

\[
\begin{align*}
x_9 &= 1 \\
y_9 &= 5 \\
x_{11} &= 4 \\
y_{12} &= 5 \\
y_3 &= 2
\end{align*}
\]
There is now much less reliance on part-time employees. The new solution uses 5 full-time employees and 12 part-time employees; the previous solution used no full-time employees and 21 part-time employees.

17. a. \( x_1 = 1 \) if PPB is Lorain, 0 otherwise
\[ x_2 = 1 \] if PPB is Huron, 0 otherwise
\[ x_3 = 1 \] if PPB is Richland, 0 otherwise
\[ x_4 = 1 \] if PPB is Ashland, 0 otherwise
\[ x_5 = 1 \] if PPB is Wayne, 0 otherwise
\[ x_6 = 1 \] if PPB is Medina, 0 otherwise
\[ x_7 = 1 \] if PPB is Knox, 0 otherwise

Min \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \)
s.t.
\[ x_1 + x_2 + x_4 + x_6 \geq 1 \] (Lorain)
\[ x_1 + x_2 + x_3 + x_4 \geq 1 \] (Huron)
\[ x_2 + x_3 + x_4 + x_7 \geq 1 \] (Richland)
\[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 1 \] (Ashland)
\[ x_4 + x_5 + x_6 \geq 1 \] (Wayne)
\[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 1 \] (Medina)
\[ x_3 + x_4 + x_7 \geq 1 \] (Knox)

b. Locating a principal place of business in Ashland county will permit Ohio Trust to do business in all 7 counties.

18. a. Add the part-worths for Antonio’s Pizza for each consumer in the Salem Foods’ consumer panel.

<table>
<thead>
<tr>
<th>Consumer</th>
<th>Overall Preference for Antonio’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 + 6 + 17 + 27 = 52</td>
</tr>
<tr>
<td>2</td>
<td>7 + 15 + 26 + 1 = 49</td>
</tr>
<tr>
<td>3</td>
<td>5 + 8 + 7 + 16 = 36</td>
</tr>
<tr>
<td>4</td>
<td>20 + 20 + 14 + 29 = 83</td>
</tr>
<tr>
<td>5</td>
<td>8 + 6 + 20 + 5 = 39</td>
</tr>
<tr>
<td>6</td>
<td>17 + 11 + 30 + 12 = 70</td>
</tr>
<tr>
<td>7</td>
<td>19 + 12 + 25 + 23 = 79</td>
</tr>
<tr>
<td>8</td>
<td>9 + 4 + 16 + 30 = 59</td>
</tr>
</tbody>
</table>

b. Let \( l_{ij} = 1 \) if level \( i \) is chosen for attribute \( j \), 0 otherwise
\[ y_k = 1 \] if consumer \( k \) chooses the Salem brand, 0 otherwise

Max \( y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 \)
s.t.
\( l_{i1} + 2l_{j1} + 6l_{i2} + 7l_{j2} + 3l_{i3} + 17l_{j3} + 26l_{i4} + 27l_{j4} + 8l_{i4} - 52y_1 \geq 1 \)
\( 11l_{i1} + 7l_{j1} + 15l_{i2} + 17l_{j2} + 16l_{i3} + 26l_{j3} + 14l_{i4} + 11l_{j4} + 10l_{i4} - 49y_2 \geq 1 \)
\( 7l_{i1} + 5l_{j1} + 8l_{i2} + 14l_{j2} + 16l_{i3} + 7l_{j3} + 29l_{i4} + 16l_{j4} + 19l_{i4} - 36y_3 \geq 1 \)
\( 13l_{i1} + 20l_{j1} + 20l_{i2} + 17l_{j2} + 17l_{i3} + 14l_{j3} + 25l_{i4} + 29l_{i4} + 10l_{i4} - 83y_4 \geq 1 \)
\( 2l_{i1} + 8l_{j1} + 6l_{i2} + 11l_{j2} + 30l_{i3} + 20l_{j3} + 15l_{i4} + 5l_{j4} + 12l_{i4} - 39y_5 \geq 1 \)
\( 12l_{i1} + 17l_{j1} + 11l_{i2} + 9l_{j2} + 2l_{i3} + 30l_{i3} + 22l_{j3} + 12l_{j4} + 20l_{i4} - 70y_6 \geq 1 \)
\( 9l_{i1} + 19l_{j1} + 12l_{i2} + 16l_{j2} + 16l_{i3} + 25l_{j3} + 30l_{i4} + 23l_{j4} + 19l_{j4} - 79y_7 \geq 1 \)
\( 5l_{i1} + 9l_{j1} + 4l_{i2} + 14l_{j2} + 23l_{i3} + 16l_{i3} + 16l_{i4} + 30l_{i4} + 3l_{j4} - 59y_8 \geq 1 \)
\( l_{i1} + l_{j1} = 1 \)
\( l_{i2} + l_{j2} = 1 \)
The optimal solution shows $l_{21} = l_{22} = l_{23} = 1$. This calls for a pizza with a thick crust, a cheese blend, a chunky sauce, and medium sausage. With $y_1 = y_2 = y_3 = y_7 = y_8 = 1$, we see that 6 of the 8 people in the consumer panel will prefer this pizza to Antonio's.

19. a. Let $l_{ij} = 1$ if level $i$ is chosen for attribute $j$, 0 otherwise.

The share of choices problem to solve is given below:

Max $y_1 + y_2 + y_3 + y_4 + y_5 + y_6$

s.t.

The optimal solution obtained using LINDO on Excel shows $l_{12} = l_{32} = l_{13} = 1$. This indicates that a cereal with a low wheat/corn ratio, artificial sweetener, and no flavor bits will maximize the share of choices.

The optimal solution also has $y_4 = y_5 = y_7 = 1$ which indicates that children 4, 5, and 7 will prefer this cereal.

b. The coefficients for the $y_i$ variable must be changed to -70 in constraints 1-4 and to -80 in constraints 5-7.

The new optimal solution has $l_{21} = l_{12} = l_{23} = 1$. This is a cereal with a high wheat/corn ratio, a sugar sweetener, and no flavor bits. Four children will prefer this design: 1, 2, 4, and 5.

20. a. Objective function changes to

$$\text{Min } 25x_1 + 40x_2 + 40x_3 + 40x_4 + 25x_5$$

b. $x_4 = x_5 = 1$; modernize the Ohio and California plants.

c. Add the constraint $x_2 + x_3 = 1$

d. $x_1 = x_3 = 1$; modernize the Michigan plant and the first New York plant.
21. a. Let \( x_i = \begin{cases} 1 \text{ if a camera is located at opening } i \\ 0 \text{ if not} \end{cases} \)

\[
\begin{align*}
\text{min } & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \\
\text{s.t. } & \quad x_1 + x_4 + x_6 \geq 1 \quad \text{Room 1} \\
& \quad x_6 + x_8 + x_{12} \geq 1 \quad \text{Room 2} \\
& \quad x_1 + x_2 + x_3 \geq 1 \quad \text{Room 3} \\
& \quad x_3 + x_4 + x_5 + x_7 \geq 1 \quad \text{Room 4} \\
& \quad x_7 + x_8 + x_9 + x_{10} \geq 1 \quad \text{Room 5} \\
& \quad x_{10} + x_{12} + x_{13} \geq 1 \quad \text{Room 6} \\
& \quad x_2 + x_5 + x_9 + x_{11} \geq 1 \quad \text{Room 7} \\
& \quad x_{11} + x_{13} \geq 1 \quad \text{Room 8} \\
\end{align*}
\]

b. \( x_1 = x_5 = x_8 = x_{13} = 1 \). Thus, cameras should be located at 4 openings: 1, 5, 8, and 13.

An alternative optimal solution is \( x_1 = x_7 = x_{11} = x_{12} = 1 \).

c. Change the constraint for room 7 to \( x_2 + x_5 + x_9 + x_{11} \geq 2 \)

d. \( x_3 = x_6 = x_9 = x_{11} = x_{12} = 1 \). Thus, cameras should be located at openings 3, 6, 9, 11, and 12.

An alternate optimal solution is \( x_2 = x_4 = x_6 = x_{10} = x_{11} = 1 \). Optimal Value = 5

22. Note that Team Size = \( x_1 + x_2 + x_3 \)

The following two constraints will guarantee that the team size will be 3, 5, or 7.

\[
\begin{align*}
x_1 + x_2 + x_3 &= 3y_1 + 5y_2 + 7y_3 \\
y_1 + y_2 + y_3 &= 1
\end{align*}
\]

Of course, the variables in the first constraint will need to be brought to the left hand side if a computer solution is desired.

23. a. A mixed integer linear program can be set up to solve this problem. Binary variables are used to indicate whether or not we setup to produce the subassemblies.

Let  
\[
\begin{align*}
\text{SB} &= 1 \text{ if bases are produced; 0 if not} \\
\text{STVC} &= 1 \text{ if TV cartridges are produced; 0 if not} \\
\text{SVCRC} &= 1 \text{ if VCR cartridges are produced; 0 if not} \\
\text{STVP} &= 1 \text{ if TV keypads are produced; 0 if not} \\
\text{SVCRP} &= 1 \text{ if VCR keypads are produced; 0 if not} \\
\text{BM} &= \text{No. of bases manufactured} \\
\text{BP} &= \text{No. of bases purchased} \\
\text{TVC} &= \text{No. of TV cartridges made} \\
\text{VCRPP} &= \text{No. of VCR keypads purchased}
\end{align*}
\]
A mixed integer linear programming model for solving this problem follows. There are 11 constraints. Constraints (1) to (5) are to satisfy demand. Constraint (6) reflects the limitation on manufacturing time. Finally, constraints (7) - (11) are constraints not allowing production unless the setup variable equals 1. Variables SB, STVC, SVCRC, STVP, and SVCRP must be specified as 0/1.

LINEAR PROGRAMMING PROBLEM

MIN
0.4BM+2.9TVCM+3.15VCRCM+0.3TVPM+0.55VCRPM+0.65BP+3.45TVCP+3.7VCRCP+0.5TVPP+0.7VCRPP+1000SB+1200STVC+1900SVCRC+1500STVP+1500SVCRP

S.T.

1) 1BM+1BP=12000
2) +1TVCM+1TVCP=7000
3) +1VCRCM+1VCRCP=5000
4) +1TVPM+1TVPP=7000
5) +1VCRPM+1VCRPP=5000
6) 0.9BM+2.2TVCM+3VCRCM+0.8TVPM+1VCRPM<30000
7) 1BM-12000SB<0
8) +1TVCM-7000STVC<0
9) +1VCRCM-5000SVCRC<0
10) +1TVPM-7000STVP<0
11) +1VCRPM-5000SVCRP<0

OPTIMAL SOLUTION

Objective Function Value = 52800.00

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>12000.00</td>
</tr>
<tr>
<td>TVCM</td>
<td>7000.00</td>
</tr>
<tr>
<td>VCRCM</td>
<td>0.000</td>
</tr>
<tr>
<td>TVPM</td>
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</tr>
<tr>
<td>VCRPM</td>
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</tr>
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<td>BP</td>
<td>0.000</td>
</tr>
<tr>
<td>TVCP</td>
<td>0.000</td>
</tr>
<tr>
<td>VCRCP</td>
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</tr>
<tr>
<td>TVPP</td>
<td>7000.00</td>
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<tr>
<td>VCRPP</td>
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<td>SB</td>
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</tr>
<tr>
<td>STVC</td>
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<tr>
<td>SVCRC</td>
<td>0.000</td>
</tr>
<tr>
<td>STVP</td>
<td>0.000</td>
</tr>
<tr>
<td>SVCRP</td>
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</tr>
</tbody>
</table>
b. This part can be solved by changing appropriate coefficients in the formulation for part (a). The coefficient of SVCRC becomes 3000 and the coefficient of VCRCM becomes 2.6 in the objective function. Also, the coefficient of VCRCM becomes 2.5 in constraint (6). The new optimal solution is shown below.

OPTIMAL SOLUTION

Objective Function Value = 52300.00

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>TVCM</td>
<td>7000.00</td>
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<tr>
<td>VCRCM</td>
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<tr>
<td>BP</td>
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<td>VCRCP</td>
<td>0.000</td>
</tr>
<tr>
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<td>SB</td>
<td>0.000</td>
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</tr>
<tr>
<td>SVCRC</td>
<td>1.000</td>
</tr>
<tr>
<td>STVP</td>
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<tr>
<td>SVCRCP</td>
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</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Slack/Surplus</th>
</tr>
</thead>
<tbody>
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<td>3</td>
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<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>2100.000</td>
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<td>7</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
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<tr>
<td>9</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>0.000</td>
</tr>
</tbody>
</table>
24. a. Variable for movie 1: \(x_{111}, x_{112}, x_{121}\)

   b. Only 1 schedule for movie 1: \(x_{111} + x_{112} + x_{121} \leq 1\)

   c. Only 1 schedule for movie 5: \(x_{531} + x_{532} + x_{533} + x_{541} + x_{542} + x_{551} + x_{552} + x_{561} \leq 1\)

   d. Only 2-screens are available at the theater.

     Week 1 constraint: \(x_{111} + x_{112} + x_{211} + x_{212} + x_{311} \leq 2\)

   e. Week 3 constraint:

     \[x_{213} + x_{222} + x_{231} + x_{422} + x_{531} + x_{532} + x_{533} + x_{631} + x_{632} + x_{633} \leq 2\]

25. a. Let \(x_i = \begin{cases} 1 & \text{if a service facility is located in city } i \\ 0 & \text{otherwise} \end{cases}\)

   \[
   \begin{align*}
   \text{min} & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \\
   \text{s.t.} & \quad \geq 1 \\
   \text{(Boston)} & \quad x_1 + x_2 + x_3 \\
   \text{(New York)} & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\
   \text{(Philadelphia)} & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
   \text{(Baltimore)} & \quad x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
   \text{(Washington)} & \quad x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
   \text{(Richmond)} & \quad x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \\
   \text{(Raleigh)} & \quad x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \\
   \text{(Florence)} & \quad x_6 + x_7 + x_8 + x_9 + x_{10} \\
   \text{(Savannah)} & \quad x_7 + x_8 + x_9 + x_{10} + x_{11} \\
   \text{(Jacksonville)} & \quad x_8 + x_9 + x_{10} + x_{11} \\
   \text{(Tampa)} & \quad x_9 + x_{10} + x_{11} + x_{12} \\
   \text{(Miami)} & \quad x_{11} + x_{12} \geq 1
   \end{align*}
   \]

b. 3 service facilities: Philadelphia, Savannah and Tampa.

   Note: alternate optimal solution is New York, Richmond and Tampa.

   c. 4 service facilities: New York, Baltimore, Savannah and Tampa.

   Note: alternate optimal solution: Boston, Philadelphia, Florence and Tampa.
Chapter 9
Network Models

Learning Objectives

1. Know the basic characteristics of the shortest route problem.
2. Know the basic characteristics of the minimal spanning tree problem.
3. Know the basic characteristics of the maximal flow problem.
4. Be able to use network-based algorithms to solve shortest route, and minimal spanning tree problems.
5. Be able to formulate and solve a maximal flow problem as a linear program.
6. Understand the following terms:
   - shortest route
   - tentative label
   - permanent label
   - spanning tree
   - minimal spanning tree
   - maximal flow
   - source node
   - sink node
   - arc flow capacities
**Network Models**

**Solutions:**

1. **Node Shortest Route From Node 1 Distance**

<table>
<thead>
<tr>
<th>Node</th>
<th>Shortest Route From Node 1</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1-2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>1-2-5-64</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>1-2-5</td>
<td>12</td>
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<tr>
<td>6</td>
<td>1-2-5-6</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>1-2-5-67</td>
<td>17</td>
</tr>
</tbody>
</table>

2. **Node Shortest Route From Node 7 Distance**

<table>
<thead>
<tr>
<th>Node</th>
<th>Shortest Route From Node 7</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7-6-5-21</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>7-6-5-2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>7-6-5-3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>7-6-4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7-6-5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7-6</td>
<td>3</td>
</tr>
</tbody>
</table>

3. **Node Shortest Route From Node 1 Time**

<table>
<thead>
<tr>
<th>Node</th>
<th>Shortest Route From Node 1</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>1-2-4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>1-3-5</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>1-3-5-6</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>1-2-4-7</td>
<td>43</td>
</tr>
</tbody>
</table>

4. **Node Shortest Route From Node 1 Distance**

<table>
<thead>
<tr>
<th>Node</th>
<th>Shortest Route From Node 1</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
</tr>
<tr>
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<td>1-4-5</td>
<td>6</td>
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<tr>
<td>6</td>
<td>1-4-5-6</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>1-4-5-67</td>
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</tr>
<tr>
<td>8</td>
<td>1-4-5-6-8</td>
<td>10</td>
</tr>
</tbody>
</table>

5. **Shortest route: 1-3-5-8-10**

**Total Distance: 19.**
6. | Node | Shortest Route From Node C | Distance |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>35</td>
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<tr>
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<td>C-2</td>
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<td>7</td>
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<td>8</td>
<td>C-3-8</td>
<td>80</td>
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<td>9</td>
<td>C-4-109</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>C-4-10</td>
<td>70</td>
</tr>
</tbody>
</table>

7. Shortest route: 1-5-4-6-7-10
Time = $10 + 4 + 3 + 4 + 4 = 25$ minutes

8. Shortest route: 1-2-8-10-11
Value = 15
Note: an alternative optimal solution is: 1-4-3-7-6-9-11

9. Shortest route or minimum-cost policy: 0-2-3-4
Total cost is $2500

10. | Start Node | End Node | Distance |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>3</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>4</td>
</tr>
</tbody>
</table>

Total length = 37
11.

Minimum length of connections = 2 + 0.5 + 1 + 1 + 2 + 0.5 + 1 = 8
8000 feet

12.

Minimum length of connections = 6 + 5 + 3 + 4 + 2 + 5 + 4 = 29

13.

Minimum length of connections = 2 + 0.5 + 1 + 1 + 2 + 0.5 + 1 = 8
8000 feet
Minimum length of cable lines = 2 + 2 + 2 + 3 + 2 + 3 + 3 + 4 + 4 = 28 miles

15. The capacitated transshipment problem to solve is given:

Max $x_{61}$

subject to

\[ x_{12} + x_{13} + x_{14} - x_{61} = 0 \]
\[ x_{24} + x_{25} - x_{12} - x_{42} = 0 \]
\[ x_{34} + x_{36} - x_{13} - x_{43} = 0 \]
\[ x_{42} + x_{43} + x_{45} + x_{14} + x_{24} + x_{34} + x_{44} = 0 \]
\[ x_{54} + x_{56} - x_{34} - x_{45} = 0 \]
\[ x_{61} - x_{36} + x_{46} = 0 \]

\[ x_{ij} \geq 0 \text{ for all } i, j \]

The system cannot accommodate a flow of 10,000 vehicles per hour.
16. 

The maximum number of messages that may be sent is 10,000.

18. a. 10,000 gallons per hour or 10 hours

b. Flow reduced to 9,000 gallons per hour; 11.1 hours.


With arc 3-4 at a 3,000 unit/hour flow capacity, total system flow is increased to 8,000 vehicles/hour. Increasing arc 3-4 to 2,000 units/hour will also increase system to 8,000 vehicles/hour. Thus a 2,000 unit/hour capacity is recommended for this arc.

20. Maximal Flow = 23 gallons/minute. Five gallons will flow from node 3 to node 5.
Learning Objectives

1. Understand the role and application of PERT/CPM for project scheduling.
2. Learn how to define a project in terms of activities such that a network can be used to describe the project.
3. Know how to compute the critical path and the project completion time.
4. Know how to convert optimistic, most probable, and pessimistic time estimates into expected activity time estimates.
5. With uncertain activity times, be able to compute the probability of the project being completed by a specific time.
6. Understand the concept and need for crashing.
7. Be able to formulate the crashing problem as a linear programming model.
8. Learn how to schedule and control project costs with PERT/Cost.
9. Understand the following terms:

<table>
<thead>
<tr>
<th>term</th>
<th>definition</th>
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<tbody>
<tr>
<td>network</td>
<td>beta distribution</td>
</tr>
<tr>
<td>PERT/CPM</td>
<td>path</td>
</tr>
<tr>
<td>activities</td>
<td>critical path</td>
</tr>
<tr>
<td>event</td>
<td>critical activities</td>
</tr>
<tr>
<td>optimistic time</td>
<td>slack</td>
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<tr>
<td>most probable time</td>
<td>crashing</td>
</tr>
<tr>
<td>pessimistic time</td>
<td></td>
</tr>
</tbody>
</table>
Solutions:

1.

2.

3.

4. a.
5.

Critical Path: A-D-G

b. The critical path activities require 15 months to complete. Thus the project should be completed in 1-1/2 years.

6.

a. Critical path: A-D-F-H

b. 22 weeks

c. No, it is a critical activity
d. Yes, 2 weeks.

e. Schedule for activity E:

| Earliest Start | 3 |
| Latest Start  | 4 |
| Earliest Finish| 10 |
| Latest Finish | 11 |

7. a. 

b. B-D-E-F-H

c. 21 weeks

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
<th>Slack</th>
<th>Critical Activity</th>
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<tbody>
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<td>1</td>
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<td>4</td>
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<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>0</td>
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<tr>
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<td>D</td>
<td>6</td>
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<td>11</td>
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<tr>
<td>E</td>
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<tr>
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<tr>
<td>G</td>
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<td>H</td>
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</table>

8. a. 

b. B-C-E-F-H
c.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
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d. Yes. Project Completion Time 49 weeks.

9. a. A-C-E-H-I

b.

<table>
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<tr>
<th>Activity</th>
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c. Project completion 24 weeks. The park can open within the 6 months (26 weeks) after the project is started.

10. a.

<table>
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<tr>
<th>Activity</th>
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b. Critical activities: B-D-F

Expected project completion time: 9.00 + 8.83 + 6.00 = 23.83.

Variance of projection completion time: 0.11 + 0.25 + 0.11 = 0.47
11. 

12. a. 

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<tr>
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<th>Variance</th>
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Critical Path: A-D-H-I

b. \( E(T) = t_A + t_D + t_H + t_I \)
\[ = 4.83 + 8.83 + 8 + 4 = 25.66 \text{ days} \]

c. \( \sigma^2 = \sigma_A^2 + \sigma_D^2 + \sigma_H^2 + \sigma_I^2 \)
\[ = 0.25 + 0.25 + 0.44 + 0.11 = 1.05 \]

Using the normal distribution,
\[ z = \frac{25 - E(T)}{\sigma} = \frac{25 - 25.66}{\sqrt{1.05}} = -0.65 \]

From Appendix, area for \( z = -0.65 \) is 0.2422.

Probability of 25 days or less = 0.5000 - 0.2422 = 0.2578
13.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Expected Time</th>
<th>Variance</th>
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<tr>
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</table>

From problem 6, A-D-F-H is the critical path.

\[
E(T) = 5 + 6 + 3 + 8 = 22
\]

\[
\sigma^2 = 0.11 + 0.44 + 0.11 + 1.78 = 2.44
\]

\[
z = \frac{\text{Time} - E(T)}{\sigma} = \frac{\text{Time} - 22}{\sqrt{2.44}}
\]

a. From Appendix

<table>
<thead>
<tr>
<th>Time = 21</th>
<th>( z = -0.64 )</th>
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<tr>
<td>Area</td>
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<tr>
<td>( P(21 \text{ weeks}) = 0.5000 - 0.2389 = 0.2611 )</td>
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b. From Appendix

<table>
<thead>
<tr>
<th>Time = 22</th>
<th>( z = 0 )</th>
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<tr>
<td>Area</td>
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<td>( P(22 \text{ weeks}) = 0.5000 )</td>
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c. From Appendix

<table>
<thead>
<tr>
<th>Time = 25</th>
<th>( z = +1.92 )</th>
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<tr>
<td>Area</td>
<td>( 0.4726 )</td>
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<tr>
<td>( P(22 \text{ weeks}) = 0.5000 + 0.4726 = 0.9726 )</td>
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14. a.

<table>
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<tr>
<th>Activity</th>
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</table>
Critical Path: A-D-F-G  Time = 29.5

b. Activity C:
   Slack = LS - ES = 7.5 - 6 = 1.5 days

c. $E(T) = t_A + t_D + t_F + t_G$
   \[ E(T) = 6 + 9 + 7.5 + 7 = 29.5 \text{ days} \]

   \[ \sigma^2 = \sigma_A^2 + \sigma_D^2 + \sigma_F^2 + \sigma_G^2 \]
   \[ \sigma^2 = 0.11 + 1.00 + 0.25 + 1.00 = 2.36 \]

d. Area
   \[ z = \frac{30 - E(T)}{\sigma} = \frac{30 - 29.5}{\sqrt{2.36}} = 0.33 \]

   \[ P(30 \text{ days}) = 0.5000 + 0.1293 = 0.6293 \]

15. a. 

\[
\begin{array}{ccccc}
\text{Activity} & \text{Expected Time} & \text{Variance} \\
A & 2 & 0.03 \\
B & 3 & 0.44 \\
C & 2 & 0.11 \\
D & 2 & 0.03 \\
E & 1 & 0.03 \\
F & 2 & 0.11 \\
G & 4 & 0.44 \\
H & 4 & 0.11 \\
I & 2 & 0.03 \\
\end{array}
\]
## Project Scheduling: PERT/CPM

<table>
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<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
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c. Critical Path: A-B-G-H-I  
\[ E(T) = 2 + 3 + 4 + 4 + 2 = 15 \text{ weeks} \]

d. Variance on critical path  
\[ \sigma^2 = 0.03 + 0.44 + 0.44 + 0.11 + 0.03 = 1.05 \]

From Appendix, we find 0.99 probability occurs at \( z = +2.33 \). Thus  
\[
\begin{align*}
    z &= \frac{T - E(T)}{\sigma} = \frac{15 - 15}{\sqrt{1.05}} = 2.33 \\
    
    \text{or} \\
    T &= 15 + 2.33 \sqrt{1.05} = 17.4 \text{ weeks}
\end{align*}
\]

16. a. A-D-G-J  
\[
\begin{align*}
    E(T) &= 6 + 5 + 3 + 2 = 16 \\
    \sigma^2 &= 1.78 + 1.78 + 0.25 + 0.11 = 3.92
\end{align*}
\]

A-C-F-J  
\[
\begin{align*}
    E(T) &= 6 + 3 + 2 + 2 = 13 \\
    \sigma^2 &= 1.78 + 0.11 + 0.03 + 0.11 = 2.03
\end{align*}
\]

B-H-I-J  
\[
\begin{align*}
    E(T) &= 2 + 4 + 2 + 2 = 10 \\
    \sigma^2 &= 0.44 + 0.69 + 0.03 + 0.11 = 1.27
\end{align*}
\]

b. A-D-G-J  
\[
\begin{align*}
    z &= \frac{20 - 16}{\sqrt{3.92}} = 2.02 \\
    \text{Area} &= 0.4783 + 0.5000 = 0.9783
\end{align*}
\]

A-C-F-J  
\[
\begin{align*}
    z &= \frac{20 - 13}{\sqrt{2.03}} = 4.91 \\
    \text{Area is approximately} &= 1.0000
\end{align*}
\]
B-H-I-J

\[ z = \frac{20 - 10}{\sqrt{1.27}} = 8.87 \quad \text{Area is approximately 1.0000} \]

c. Critical path is the longest path and generally will have the lowest probability of being completed by the desired time. The noncritical paths should have a higher probability of being completed on time.

It may be desirable to consider the probability calculation for a noncritical path if the path activities have little slack, if the path completion time is almost equal to the critical path completion time, or if the path activity times have relatively high variances. When all of these situations occur, the noncritical path may have a probability of completion on time that is less than the critical path.

17. a. 

<table>
<thead>
<tr>
<th>Start</th>
<th>A</th>
<th>B</th>
<th>D</th>
<th>Finish</th>
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</thead>
</table>

b. Critical Path A-B-D

Expected Time = 4.5 + 8.0 + 6.0 = 18.5 weeks

c. Material Cost = $3000 + $5000 = $8000

Best Cost (Optimistic Times) 3 + 5 + 2 + 4 = 14 days
Total Cost = $8000 + 14($400) = $12,800

Worst Case (Pessimistic Times) 8 + 11 + 6 + 12 = 37 days
Total Cost = $8000 + 37($400) = $22,800

d. Bid Cost = $8000 + 18.5($400) = $15,400

.50 probability time and cost will exceed the expected time and cost.

e. \[ \sigma = \sqrt{3.47} = 1.86 \]

\[
\text{Bid} = \frac{16,800}{400 \text{ Days}} = \frac{8,000 + \text{Days (400)}}{400 \text{ Days}} = \frac{16,800 - 8000}{8,800} = \frac{8,800}{22} = 188
\]

The project must be completed in 22 days or less.

The probability of a loss = \( P(T > 22) \)

\[ z = \frac{22 - 18.5}{1.86} = 1.88 \]

From Appendix, Area = .5000 - .4699 = .0301
18. a. 

![Network Diagram]

b. 

<table>
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<tr>
<th>Activity</th>
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<th>Variance</th>
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<td>3.17</td>
<td>13.17</td>
<td>10.00</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>9.17</td>
<td>9.17</td>
<td>11.17</td>
<td>11.17</td>
<td>0.00</td>
<td>Yes</td>
</tr>
<tr>
<td>H</td>
<td>11.17</td>
<td>11.17</td>
<td>13.17</td>
<td>13.17</td>
<td>0.00</td>
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</tr>
<tr>
<td>I</td>
<td>13.17</td>
<td>13.17</td>
<td>14.17</td>
<td>14.17</td>
<td>0.00</td>
<td>Yes</td>
</tr>
</tbody>
</table>

c. Critical Path: A-B-D-G-H-I  
Expected Project Completion Time = 1.17 + 6 + 2 + 2 + 2 + 1 = 14.17 weeks

d. Compute the probability of project completion in 13 weeks or less.

\[
\sigma^2 = \sigma_A^2 + \sigma_B^2 + \sigma_D^2 + \sigma_G^2 + \sigma_H^2 + \sigma_I^2 \\
= 0.03 + 0.44 + 0.11 + 0.11 + 0.11 + 0.00 = 0.80 \\
z = \frac{13 - E(T)}{\sigma} = \frac{13 - 14.17}{\sqrt{0.80}} = -1.31 \quad \text{Area} \quad 0.4049 \cdot P(13 \text{ weeks}) = 0.5000 - 0.4049 = 0.0951
\]

With this low probability, the manager should start prior to February 1.
19. a.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Expected Time</th>
<th>Variance</th>
</tr>
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<tbody>
<tr>
<td>A</td>
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<td>0.11</td>
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<tr>
<td>B</td>
<td>4</td>
<td>0.44</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>0.11</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>0.11</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>1.78</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>0.69</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>H</td>
<td>7</td>
<td>1.78</td>
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<tr>
<td>I</td>
<td>3</td>
<td>0.44</td>
</tr>
<tr>
<td>J</td>
<td>5</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Critical Path: B-E-H-J
b. 

\[ E(T) = t_B + t_E + t_H + t_I = 4 + 10 + 7 + 5 = 26 \]

\[ \sigma^2 = \sigma_B^2 + \sigma_E^2 + \sigma_H^2 + \sigma_I^2 = 0.44 + 1.78 + 1.78 + 0.11 = 4.11 \]

\[ z = \frac{T - E(T)}{\sigma} \]

\[ z = \frac{25 - 26}{\sqrt{4.11}} = -0.49 \quad P \text{ (25 weeks) = 0.5000 - 0.1879 = 0.3121} \]

\[ z = \frac{30 - 26}{\sqrt{4.11}} = 1.97 \quad P \text{ (30 weeks) = 0.5000 + 0.756 = 0.9756} \]

20. a. 

<table>
<thead>
<tr>
<th>Activity</th>
<th>Maximum Crash</th>
<th>Crash Cost/Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>667</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>350</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>450</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>360</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>1000</td>
</tr>
</tbody>
</table>

Min \[ 400Y_A + 667Y_B + 500Y_C + 300Y_D + 350Y_E + 450Y_F + 360Y_G + 1000Y_H \]

s.t.

\[ x_A + y_A \geq 3 \]
\[ x_B + y_B \geq 6 \]
\[ x_C + y_C \cdot x_A \geq 2 \]
\[ x_D + y_D \cdot x_C \geq 5 \]
\[ x_G + y_G \cdot x_D \geq 5 \]
\[ x_H + y_H \cdot x_G \geq 3 \]

Maximum Crashing:

\[ y_A \leq 2 \]
\[ y_B \leq 3 \]
\[ y_C \leq 1 \]
\[ y_D \leq 2 \]
\[ y_E \leq 1 \]
\[ y_F \leq 2 \]
\[ y_G \leq 5 \]
\[ y_H \leq 1 \]
b. Linear Programming Solution

<table>
<thead>
<tr>
<th>Activity</th>
<th>Crash Time</th>
<th>New Time</th>
<th>Crash Cost</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>3</td>
<td>—</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>5</td>
<td>667</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>2</td>
<td>—</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>3</td>
<td>350</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>2</td>
<td>450</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>8</td>
<td>360</td>
</tr>
<tr>
<td>H</td>
<td>0</td>
<td>3</td>
<td>—</td>
</tr>
</tbody>
</table>

Total Crashing Cost $2,427

c.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
<th>Slack</th>
<th>Critical Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>8</td>
<td>11</td>
<td>11</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>11</td>
<td>13</td>
<td>13</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>G</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>14</td>
<td>2</td>
<td></td>
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<tr>
<td>H</td>
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<td>13</td>
<td>16</td>
<td>16</td>
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</tr>
</tbody>
</table>

All activities are critical.

21. a.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
<th>Slack</th>
<th>Critical Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>8</td>
<td>14</td>
<td>14</td>
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</tr>
<tr>
<td>F</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>14</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Critical Path: A-C-E

Project Completion Time = $t_A + t_C + t_E = 3 + 5 + 6 = 14$ days

b. Total Cost = $8,400
22. a.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Max Crash Days</th>
<th>Crash Cost/Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>600</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>700</td>
</tr>
<tr>
<td>C</td>
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<td>400</td>
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<tr>
<td>D</td>
<td>2</td>
<td>400</td>
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<tr>
<td>E</td>
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<td>500</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>500</td>
</tr>
</tbody>
</table>

Min $600y_A + 700y_B + 400y_C + 400y_D + 500y_E + 400y_F + 400y_G$

s.t.

- $x_A + y_A \geq 3$
- $x_B + y_B \geq 2$
- $x_C + y_C - x_A \geq 5$
- $x_D + y_D - x_B \geq 5$
- $x_E + y_E - x_C \geq 6$
- $x_E + y_E - x_D \geq 6$
- $x_F + y_F - x_C \geq 2$
- $x_F + y_F - x_D \geq 2$
- $x_G + y_G - x_F \geq 2$
- $x_{FIN} - x_E \geq 0$
- $x_{FIN} - x_D \geq 0$

$y_A \leq 1$
$y_B \leq 1$
$y_C \leq 2$
$y_D \leq 2$
$y_E \leq 2$
$y_F \leq 1$
$y_G \leq 1$

All $x, y \geq 0$

b.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Crash</th>
<th>Crashing Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1 day</td>
<td>$400$</td>
</tr>
<tr>
<td>E</td>
<td>1 day</td>
<td>500</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>$900$</td>
</tr>
</tbody>
</table>

c. Total Cost = Normal Cost + Crashing Cost

= $8,400 + 900 = $9,300$
23. a. This problem involves the formulation of a linear programming model that will determine the length of the critical path in the network. Since \( x_I \), the completion time of activity I, is the project completion time, the objective function is:

\[
\text{Min } x_I
\]

Constraints are needed for the completion times for all activities in the project. The optimal solution will determine \( x_I \) which is the length of the critical path.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Constraints</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>( x_A \geq \tau_A )</td>
</tr>
<tr>
<td>B</td>
<td>( x_B \geq \tau_B )</td>
</tr>
<tr>
<td>C</td>
<td>( x_C - x_A \geq \tau_C )</td>
</tr>
<tr>
<td>D</td>
<td>( x_D - x_A \geq \tau_D )</td>
</tr>
<tr>
<td>E</td>
<td>( x_E - x_A \geq \tau_E )</td>
</tr>
<tr>
<td>F</td>
<td>( x_F - x_E \geq \tau_F )</td>
</tr>
<tr>
<td>G</td>
<td>( x_G - x_D \geq \tau_G )</td>
</tr>
<tr>
<td>H</td>
<td>( x_H - x_B \geq \tau_H )</td>
</tr>
<tr>
<td>I</td>
<td>( x_I - x_G \geq \tau_I )</td>
</tr>
</tbody>
</table>

All \( x \geq 0 \)

24. a.

![Project Scheduling Diagram](image)

b.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>10</td>
<td>10</td>
<td>10</td>
<td>0</td>
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<tr>
<td>B</td>
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<tr>
<td>C</td>
<td>18</td>
<td>18</td>
<td>28</td>
<td>28</td>
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<tr>
<td>D</td>
<td>10</td>
<td>11</td>
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<td>17</td>
<td>18</td>
<td>27</td>
<td>28</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>28</td>
<td>28</td>
<td>31</td>
<td>31</td>
<td>0</td>
</tr>
</tbody>
</table>

c. Activities A, B, C, and F are critical. The expected project completion time is 31 weeks.
d.

<table>
<thead>
<tr>
<th>Crash Activities</th>
<th>Number of Weeks</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>$ 40</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>12.5</td>
</tr>
</tbody>
</table>

$ 112.5

e.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Earliest Start</th>
<th>Latest Start</th>
<th>Earliest Finish</th>
<th>Latest Finish</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
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<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
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<tr>
<td>F</td>
<td>23</td>
<td>23</td>
<td>26</td>
<td>26</td>
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</tr>
</tbody>
</table>

All activities are critical.

f. Total added cost due to crashing $112,500 (see part d.)
Chapter 12
Waiting Line Models

Learning Objectives

1. Be able to identify where waiting line problems occur and realize why it is important to study these problems.

2. Know the difference between single-channel and multiple-channel waiting lines.

3. Understand how the Poisson distribution is used to describe arrivals and how the exponential distribution is used to describe services times.

4. Learn how to use formulas to identify operating characteristics of the following waiting line models:
   a. Single-channel model with Poisson arrivals and exponential service times
   b. Multiple-channel model with Poisson arrivals and exponential service times
   c. Single-channel model with Poisson arrivals and arbitrary service times
   d. Multiple-channel model with Poisson arrivals, arbitrary service times, and no waiting
   e. Single-channel model with Poisson arrivals, exponential service times, and a finite calling population

5. Know how to incorporate economic considerations to arrive at decisions concerning the operation of a waiting line.

6. Understand the following terms:
   - queuing theory
   - queue
   - single-channel
   - multiple-channel
   - mean arrival rate
   - mean service rate
   - queue discipline
   - steady state
   - utilization factor
   - operating characteristics
   - blocking
   - infinite calling population
   - finite calling population
Solutions:

1. a. \( \lambda = 5(0.4) = 2 \) per five minute period

b. \( P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{2^x e^{-2}}{x!} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1353</td>
</tr>
<tr>
<td>1</td>
<td>0.2707</td>
</tr>
<tr>
<td>2</td>
<td>0.2707</td>
</tr>
<tr>
<td>3</td>
<td>0.1804</td>
</tr>
</tbody>
</table>

c. \( P(Delay \ Problems) = P(x > 3) = 1 - P(x \leq 3) = 1 - 0.8571 = 0.1429 \)

2. a. \( \mu = 0.6 \) customers per minute

\[ P(service \ time \leq 1) = 1 - e^{-(0.6)1} = 0.4512 \]

b. \[ P(service \ time \leq 2) = 1 - e^{-(0.6)2} = 0.6988 \]

c. \[ P(service \ time > 2) = 1 - 0.6988 = 0.3012 \]

3. a. \( P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{0.4}{0.6} = 0.3333 \)

b. \( L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(0.4)^2}{0.6 (0.6 - 0.4)} = 1.3333 \)

c. \( L = L_q + \frac{\lambda}{\mu} = 1.3333 + \frac{0.4}{0.6} = 2 \)

d. \( W_q = \frac{L_q}{\lambda} = \frac{1.3333}{0.4} = 3.3333 \) min.

e. \( W = W_q + \frac{1}{\mu} = 3.3333 + \frac{1}{0.6} = 5 \) min.

f. \( P_w = \frac{\lambda}{\mu} = \frac{0.4}{0.6} = 0.6667 \)

4. \( P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 = \left(\frac{0.4}{0.6}\right)^n (0.3333) \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( P_n )</th>
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<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>0.2222</td>
</tr>
<tr>
<td>2</td>
<td>0.1481</td>
</tr>
<tr>
<td>3</td>
<td>0.0988</td>
</tr>
</tbody>
</table>

\[ P(n > 3) = 1 - P(n \leq 3) = 1 - 0.8024 = 0.1976 \]

5. a. \( P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{10}{12} = 0.1667 \)

b. \( L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{12 (12 - 10)} = 4.1667 \)

\( W_q = \frac{L_q}{\lambda} = 0.4167 \) hours (25 minutes)
d. $W = W_q + \frac{1}{\mu} = .5 \text{ hours (30 minutes)}$

e. $P_w = \frac{\lambda}{\mu} \approx \frac{10}{12} = 0.8333$

6. a. $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{1.25}{2} = 0.375$

b. $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{1.25^2}{2(2 - 1.25)} = 1.0417$

c. $W_q = \frac{L_q}{\lambda} = \frac{1.0417}{1.25} = 0.8333 \text{ minutes (50 seconds)}$

d. $P_w = \frac{\lambda}{\mu} \approx \frac{1.25}{2} = 0.625$

e. Average one customer in line with a 50 second average wait appears reasonable.

7. a. $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(2.5)^2}{5(5 - 2.5)} = 0.5000$

\[
L = L_q + \frac{\lambda}{\mu} = 0.5000 + \frac{2.5}{5} = 1
\]

b. $W_q = L_q = \frac{0.5000}{2.5} = 0.20 \text{ hours (12 minutes)}$

c. $W = W_q + \frac{1}{\mu} = 0.20 + \frac{1}{5} = 0.40 \text{ hours (24 minutes)}$

d. $P_w = \frac{\lambda}{\mu} \approx \frac{2.5}{5} = 0.50$
8. \( \lambda = 1 \) and \( \mu = 1.25 \)

\[
P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{1}{1.25} = 0.20
\]

\[
L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{1}{1.25(0.25)} = 3.2
\]

\[
L = L_q + \frac{\lambda}{\mu} = 3.2 + \frac{1}{1.25} = 4
\]

\[
W_q = \frac{L_q}{\lambda} = \frac{3.2}{1} = 3.2 \text{ minutes}
\]

\[
W = W_q + \frac{1}{\mu} = 3.2 + \frac{1}{1.25} = 4 \text{ minutes}
\]

\[
P_\infty = \frac{\lambda}{\mu} = \frac{1}{1.25} = 0.80
\]

Even though the services rate is increased to \( \mu = 1.25 \), this system provides slightly poorer service due to the fact that arrivals are occurring at a higher rate. The average waiting times are identical, but there is a higher probability of waiting and the number waiting increases with the new system.

9. a. \( P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2.2}{5} = 0.56 \)

b. \( P_1 = \left(\frac{\lambda}{\mu}\right)P_0 = \frac{2.2}{5} \cdot 0.56 = 0.2464 \)

c. \( P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0 = \left(\frac{2.2}{5}\right)^2 \cdot 0.56 = 0.1084 \)

d. \( P_3 = \left(\frac{\lambda}{\mu}\right)^3 P_0 = \left(\frac{2.2}{5}\right)^3 \cdot 0.56 = 0.0477 \)

e. \( P(\text{More than 2 waiting}) = P(\text{More than 3 are in system}) = 1 - (P_0 + P_1 + P_2 + P_3) = 1 - 0.9625 = 0.0375 \)

f. \( L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{2.2^2}{5(5 - 2.2)} = 0.3457 \)

\[
W_q = \frac{L_q}{\lambda} = 0.157 \text{ hours} \quad (9.43 \text{ minutes})
\]
10. a. 

<table>
<thead>
<tr>
<th></th>
<th>( \lambda = 2 )</th>
<th>( \mu = 3 )</th>
<th>( \mu = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_q )</td>
<td>1.3333</td>
<td>0.5000</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>2.0000</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>( W_q )</td>
<td>0.6667</td>
<td>0.2500</td>
<td></td>
</tr>
<tr>
<td>( W )</td>
<td>1.0000</td>
<td>0.5000</td>
<td></td>
</tr>
<tr>
<td>( P_w )</td>
<td>0.6667</td>
<td>0.5000</td>
<td></td>
</tr>
</tbody>
</table>

b. New mechanic = \( 30(L) + 14 \) 
= 30(2) + 14 = $74 per hour
Experienced mechanic = \( 30(L) + 20 \)
= 30(1) + 20 = $50 per hour

\[ \therefore \] Hire the experienced mechanic

11. a. \( \lambda = 2.5 \) \( \mu = 60/10 = 6 \) customers per hour

\[ L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(2.5)^2}{6(6 - 2.5)} = 0.2976 \]
\[ L = L_q + \frac{\lambda}{\mu} = 0.7143 \]
\[ W_q = \frac{L_q}{\lambda} = 0.1190 \text{ hours (7.14 minutes)} \]
\[ W = W_q + \frac{1}{\mu} = 0.2857 \text{ hours} \]
\[ P_w = \frac{\lambda}{\mu} = \frac{2.5}{6} = 0.4167 \]

b. No; \( W_q = 7.14 \text{ minutes} \). Firm should increase the mean service rate \( (\mu) \) for the consultant or hire a second consultant.

c. \( \mu = 60/8 = 7.5 \) customers per hour

\[ L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(2.5)^2}{7.5(7.5 - 2.5)} = 0.1667 \]
\[ W_q = \frac{L_q}{\lambda} = 0.0667 \text{ hours (4 minutes)} \]

The service goal is being met.
12. 

\[ P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{20} = 0.25 \]

\[ L_q = \frac{\lambda^2}{\mu (\mu - \lambda)} = \frac{15^2}{20 (20 - 15)} = 2.25 \]

\[ L = L + \frac{\lambda}{\mu} = 3 \]

\[ W_q = \frac{L_q}{\lambda} = 0.15 \text{ hours} \quad (9 \text{ minutes}) \]

\[ W = W_q + \frac{1}{\mu} = 0.20 \text{ hours} \quad (12 \text{ minutes}) \]

\[ P_w = \frac{\lambda}{\mu} = \frac{15}{20} = 0.75 \]

With \( W_q = 9 \) minutes, the checkout service needs improvements.

13. Average waiting time goal: 5 minutes or less.

a. One checkout counter with 2 employees

\[ \lambda = 15, \mu = 30 \text{ per hour} \]

\[ L_q = \frac{\lambda^2}{\mu (\mu - \lambda)} = \frac{15^2}{30 (30 - 15)} = 0.50 \]

\[ W_q = \frac{L_q}{\lambda} = 0.0333 \text{ hours} \quad (2 \text{ minutes}) \]

b. Two channel-two counter system

\[ \lambda = 15, \mu = 20 \text{ per hour for each} \]

From Table, \( P_0 = 0.4545 \)

\[ L_q = \frac{(\lambda / \mu)^2}{1! (2 (20) - 15)^2} P_0 = \frac{(15 / 20)^2 (15) (20)}{(40 - 15)^2} (0.4545) = 0.1227 \]

\[ W_q = \frac{L_q}{\lambda} = 0.0082 \text{ hours} \quad (0.492 \text{ minutes}) \]

Recommend one checkout counter with two people. This meets the service goal with \( W_q = 2 \) minutes. The two counter system has better service, but has the added cost of installing a new counter.

14. a. \( \mu = \frac{60}{7.5} = 8 \text{ customers per hour} \)

b. \( P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{5}{8} = 0.3750 \)

c. \( L_q = \frac{\lambda^2}{\mu (\mu - \lambda)} = \frac{5^2}{8 (8 - 5)} = 1.0417 \)
d. $W_q = \frac{L_q}{\lambda} = \frac{10.417}{5} = 0.2083$ hours (12.5 minutes)

e. $P_w = \frac{\lambda}{\mu} = \frac{5}{8} = 0.6250$

f. 62.5% of customers have to wait and the average waiting time is 12.5 minutes. Ocala needs to add more consultants to meet its service guidelines.

15. $k = 2, \lambda = 5, \mu = 8$

Using the equation for $P_0$, $P_0 = 0.5238$

\[
L_q = \frac{b! \mu^2}{(k\mu - \lambda)^2} P_0 = 0.0676
\]

$W_q = \frac{L_q}{\lambda} = \frac{0.0676}{5} = 0.0135$ hours (0.81 minutes)

\[
P_0 = 0.5238 \quad P_i = \frac{b! \mu^i}{i!} P_0 = \frac{5}{8} \times (0.5238) = 0.3274
\]

\[
P_w = P(n \geq 2) = 1 - P(n \leq 1)
\]

\[
= 1 - 0.5238 - 0.3274 = 0.1488
\]

Two consultants meet service goals with only 14.88% of customers waiting with an average waiting time of 0.81 minutes (49 seconds).

16. a. $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{5}{10} = 0.50$

b. $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{5^2}{10(10-5)} = 0.50$

c. $W_q = \frac{L_q}{\lambda} = 0.1$ hours (6 minutes)

d. $W = W_q + \frac{1}{\mu} = 0.2$ hours (12 minutes)

e. Yes, unless $W_q = 6$ minutes is considered too long.

17. a. From Table, $P_0 = 0.60$

b. $L_q = \frac{(\frac{\lambda}{\mu})^2 \lambda \mu}{1! (k \mu - \lambda)^2} P_0 = 0.0333$

c. $W_q = \frac{L_q}{\lambda} = 0.0067$ hours (24.12 seconds)

d. $W = W_q + \frac{1}{\mu} = 0.1067$ (6.4 minutes)
12 - 8

Waiting Line Models

18. a. \( k = 2 \) \( \lambda/\mu = 14/10 = 1.4 \)

From Table, \( P_0 = 0.1765 \)

b. \( L_q = \frac{b / \mu g^1 \mu}{1!(2\mu - \lambda)^2} P_0 = \frac{(14)^2(14)(10)}{(20-14)^2}(0.1765) = 1.3451 \)

\( L = L_q + \frac{\lambda}{\mu} = 1.3451 + \frac{14}{10} = 2.7451 \)

c. \( W_q = \frac{L_q}{\lambda} = \frac{1.3453}{14} = 0.0961 \text{ hours (5.77 minutes)} \)

d. \( W = W_q + \frac{1}{\mu} = 0.0961 + \frac{1}{10} = 0.196 \text{ hours (11.77 minutes)} \)

e. \( P_0 = 0.1765 \)

\[ P_1 = 1 - \frac{(\lambda / \mu)^1}{1!} P_0 = \frac{14}{10} (0.1765) = 0.2470 \]

\( P(\text{wait}) = P(n \geq 2) = 1 - P(n \leq 1) = 1 - 0.4235 = 0.5765 \)

19. a. From Table, \( P_0 = 0.2360 \)

\[ L_q = \frac{b / \mu g^1 \mu}{2!(3\mu - \lambda)^2} P_0 = \frac{(14)^2(14)(10)}{2(30-14)^2}(0.2360) = 0.1771 \]

\( L = L_q + \frac{\lambda}{\mu} = 1.5771 \)

\( W_q = \frac{L_q}{\lambda} = \frac{0.1771}{14} = 0.0126 \text{ hours (0.76 minutes)} \)

\( W = W_q + \frac{1}{\mu} = 0.126 + \frac{1}{10} = 0.1126 \text{ hours (6.76 minutes)} \)

b. \( k = 2 \) \( P(\text{wait}) = 0.5765 \)

\( k = 3 \) \( P_0 = 0.2360 \)
\[ P_1 = 1 - \frac{b}{\mu} \cdot g_{1!} P_2 = (1.4)(0.236) = 0.3304 \]

\[ P_2 = 1 - \frac{b}{\mu} \cdot g_{2!} P_0 = \frac{(1.4)^2}{2}(0.236) = 0.2312 \]

\[ P(\text{wait}) = P(n \geq 3) = 1 - P(n \leq 2) = 1 - 0.7976 = 0.2024 \]

∴ Prefer the three-channel system.

20. a. Note \( \frac{\lambda}{\mu} = \frac{1.2}{0.75} = 1.60 > 1 \). Thus, one postal clerk cannot handle the arrival rate.

Try \( k = 2 \) postal clerks

From Table with \( \frac{\lambda}{\mu} = 1.60 \) and \( k = 2 \), \( P_0 = 0.1111 \)

\[ L_q = \frac{(\lambda / \mu)^2 \cdot \lambda \mu}{1!(2\mu - \lambda)^2} P_0 = 2.8444 \]

\[ L = L_q + \frac{\lambda}{\mu} = 4.4444 \]

\[ W_q = \frac{L_q}{\lambda} = 2.3704 \text{ minutes} \]

\[ W = W_q + \frac{1}{\mu} = 3.7037 \text{ minutes} \]

\[ P_w = 0.7111 \]

Use 2 postal clerks with average time in system 3.7037 minutes. No need to consider \( k = 3 \).

b. Try \( k = 3 \) postal clerks.

From Table with \( \frac{\lambda}{\mu} = \frac{2.1}{0.75} = 2.80 \) and \( k = 3 \), \( P_0 = 0.0160 \)

\[ L_q = \frac{(\lambda / \mu)^3 \cdot \lambda \mu}{2(3\mu - \lambda)^3} P_0 = 12.2735 \]

\[ L = L_q + \frac{\lambda}{\mu} = 15.0735 \]

\[ W_q = \frac{L_q}{\lambda} = 5.8445 \text{ minutes} \]
Waiting Line Models

\[ W = W_q + \frac{1}{\mu} = 7.1778 \text{ minutes} \]

\[ P_w = 0.8767 \]

Three postal clerks will not be enough in two years. Average time in system of 7.1778 minutes and an average of 15.0735 customers in the system are unacceptable levels of service. Post office expansion to allow at least four postal clerks should be considered.

21. From question 11, a service time of 8 minutes has \( \mu = 60/8 = 7.5 \)

\[ L_q = \frac{\lambda^2}{\mu (\mu - \lambda)} = \frac{(2.5)^2}{7.5 (7.5 - 2.5)} = 0.1667 \]

\[ L = L_q + \frac{\lambda}{\mu} = 0.50 \]

Total Cost = \$25L + \$16 = 25(0.50) + 16 = \$28.50

Two channels: \( \lambda = 2.5 \quad \mu = 60/10 = 6 \)

Using equation, \( P_0 = 0.6552 \)

\[ L_q = \frac{(\lambda / \mu)^2 \lambda \mu P_0}{1! (2 \mu - \lambda)^2} = 0.0189 \]

\[ L = L_q + \frac{\lambda}{\mu} = 0.4356 \]

Total Cost = 25(0.4356) + 2(16) = \$42.89

Use the one consultant with an 8 minute service time.

22. \( \lambda = 24 \)

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>System A ((k = 1, \mu = 30))</th>
<th>System B ((k = 1, \mu = 48))</th>
<th>System C ((k = 2, \mu = 30))</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( P_0 )</td>
<td>0.2000</td>
<td>0.5000</td>
<td>0.4286</td>
</tr>
<tr>
<td>b. ( L_q )</td>
<td>3.2000</td>
<td>0.5000</td>
<td>0.1524</td>
</tr>
<tr>
<td>c. ( W_q )</td>
<td>0.1333</td>
<td>0.0200</td>
<td>0.0063</td>
</tr>
<tr>
<td>d. ( W )</td>
<td>0.1667</td>
<td>0.0417</td>
<td>0.0397</td>
</tr>
<tr>
<td>e. ( L )</td>
<td>4.0000</td>
<td>1.0000</td>
<td>0.9524</td>
</tr>
<tr>
<td>f. ( P_w )</td>
<td>0.8000</td>
<td>0.5000</td>
<td>0.2286</td>
</tr>
</tbody>
</table>

System C provides the best service.

23. Service Cost per Channel

\[
\begin{align*}
\text{System A:} & \quad 6.50 \quad + \quad 20.00 \quad = \quad 26.50/\text{hour} \\
\text{System B:} & \quad 2(6.50) \quad + \quad 20.00 \quad = \quad 33.00/\text{hour} \\
\text{System C:} & \quad 6.50 \quad + \quad 20.00 \quad = \quad 26.50/\text{hour}
\end{align*}
\]

Total Cost = \( c_wL + c_kk \)
System B is the most economical.

24. \( \lambda = 2.8, \mu = 3.0, W_q = 30 \text{ minutes} \)
   a. \( \lambda = \frac{2.8}{60} = 0.0466 \)
   \( \mu = \frac{3}{60} = 0.0500 \)
   b. \( L_q = \lambda W_q = (0.0466)(30) = 1.4 \)
   c. \( W = W_q + \frac{1}{\mu} = 30 + \frac{1}{0.05} = 50 \text{ minutes} \)
   \( \therefore 11:00 \text{ a.m.} \)

25. \( \lambda = 4, W = 10 \text{ minutes} \)
   a. \( \mu = \frac{1}{2} = 0.5 \)
   b. \( W_q = W - \frac{1}{\mu} = 10 - \frac{1}{0.5} = 8 \text{ minutes} \)
   c. \( L = \lambda W = 4(10) = 40 \)

26. a. Express \( \lambda \) and \( \mu \) in mechanics per minute
   \( \lambda = \frac{4}{60} = 0.0667 \text{ mechanics per minute} \)
   \( \mu = \frac{1}{6} = 0.1667 \text{ mechanics per minute} \)
   \( L_q = \lambda W_q = 0.0667(4) = 0.2668 \)
   \( W = W_q + \frac{1}{\mu} = 4 + \frac{1}{0.1667} = 10 \text{ minutes} \)
   \( L = \lambda W = (0.0667)(10) = 0.6667 \)
   b. \( L_q = 0.0667(1) = 0.0667 \)
   \( W = 1 + \frac{1}{0.1667} = 7 \text{ minutes} \)
   \( L = \lambda W = (0.0667)(7) = 0.4669 \)
   c. One-Channel
   Total Cost = 20(0.6667) + 12(1) = $25.33

Two-Channel
   Total Cost = 20(0.4669) + 12(2) = $33.34
   One-Channel is more economical.
27. a. \( \frac{2}{8} \text{ hours} = 0.25 \text{ per hour} \)
   
b. \( \frac{1}{3.2} \text{ hours} = 0.3125 \text{ per hour} \)
   
c. \( L_q = \frac{\lambda^2 \sigma^2 + (\lambda / \mu)^2}{2(1 - \lambda / \mu)} = \frac{(0.25)^2(2)^2 + (0.25 / 0.3125)^2}{2(1 - 0.25 / 0.3125)} = 2.225 \)
   
d. \( W_q = L_q / \lambda = \frac{2.225}{0.25} = 8.9 \text{ hours} \)
   
e. \( W = W_q + \frac{1}{\mu} = 8.9 + \frac{1}{1.3125} = 12.1 \text{ hours} \)
   
f. Same at \( P_w = \frac{\lambda}{\mu} = \frac{0.25}{0.3125} = 0.80 \)
     
80% of the time the welder is busy.

28. \( \lambda = 5 \)
   
a. 
   
\[ 
\begin{array}{c|c|c}
   \text{Design} & \mu & \\
   \hline
   A & 60/6 = 10 \\
   B & 60/6.25 = 9.6 \\
\end{array}
\]
   
b. Design A with \( \mu = 10 \) jobs per hour.
   
c. \( \frac{3}{60} = 0.05 \) for A \( \frac{0.6}{60} = 0.01 \) for B
   
d. 
   
\[ 
\begin{array}{l|c|c}
   \text{Characteristic} & \text{Design A} & \text{Design B} \\
   \hline
   P_0 & 0.5000 & 0.4792 \\
   L_q & 0.3125 & 0.2857 \\
   L & 0.8125 & 0.8065 \\
   W_q & 0.0625 & 0.0571 \\
   W & 0.1625 & 0.1613 \\
   P_w & 0.5000 & 0.5208 \\
\end{array}
\]
   
e. Design B is slightly better due to the lower variability of service times.

29. a. \( \lambda = \frac{3}{8} = .375 \)
   
\( \mu = \frac{1}{2} = .5 \)
   
b. \( L_q = \frac{\lambda^2 \sigma^2 + (\lambda / \mu)^2}{2(1 - \lambda / \mu)} = \frac{(0.375)^2(1.5)^2 + (.375 / .5)^2}{2(1 - .375 / .5)} = 1.7578 \)
   
\( L = L_q + \lambda / \mu = 1.7578 + .375 / .5 = 2.5078 \)
   
\( TC = c_wL + c_sk = 35 (2.5078) + 28 (1) = $115.71 \)
c.

<table>
<thead>
<tr>
<th>Current System (σ = 1.5)</th>
<th>New System (σ = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_q = 1.7578$</td>
<td>$L_q = 1.125$</td>
</tr>
<tr>
<td>$L = 2.5078$</td>
<td>$L = 1.875$</td>
</tr>
<tr>
<td>$W_q = 4.6875$</td>
<td>$W_q = 3.00$</td>
</tr>
<tr>
<td>$W = 6.6875$</td>
<td>$W = 5.00$</td>
</tr>
</tbody>
</table>

$TC = cWL + c_{sk} = 35 (1.875) + 32 (1) = 97.63$

d. Yes; Savings = 40 ($115.77 - 97.63) = 725.60

Note: Even with the advantages of the new system, $W_q = 3$ shows an average waiting time of 3 hours. The company should consider a second channel or other ways of improving the emergency repair service.

30. a. $\lambda = 42$ $\mu = 20$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$(\lambda/\mu)^i \cdot i!$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>2.1000</td>
</tr>
<tr>
<td>2</td>
<td>2.2050</td>
</tr>
<tr>
<td>3</td>
<td>1.5435 6.8485</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$j$</th>
<th>$P_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1/6.8485 = 0.1460$</td>
</tr>
<tr>
<td>1</td>
<td>$2.1/6.8485 = 0.3066$</td>
</tr>
<tr>
<td>2</td>
<td>$2.2050/6.8485 = 0.3220$</td>
</tr>
<tr>
<td>3</td>
<td>$1.5435/6.8485 = 0.2254$</td>
</tr>
</tbody>
</table>

b. 0.2254

c. $L = \lambda/\mu(1 - P_k) = 42/20 (1 - 0.2254) = 1.6267$

d. Four lines will be necessary. The probability of denied access is 0.1499.

31. a. $\lambda = 20$ $\mu = 12$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$(\lambda/\mu)^i \cdot i!$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>1.6667</td>
</tr>
<tr>
<td>2</td>
<td>1.3889 4.0556</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$j$</th>
<th>$P_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1/4.0556 = 0.2466$</td>
</tr>
<tr>
<td>1</td>
<td>$1.6667/4.0556 = 0.4110$</td>
</tr>
<tr>
<td>2</td>
<td>$1.3889/4.0556 = 0.3425$</td>
</tr>
</tbody>
</table>
$P_2 = 0.3425 \quad 34.25\%$

b. $k = 3 \quad P_3 = 0.1598$

$k = 4 \quad P_4 = 0.0624 \quad \text{Must go to } k = 4.$

c. $L = \frac{\lambda}{\mu}(1 - P_4) = \frac{20}{12}(1 - 0.0624) = 1.5626$

32. a. $\lambda = 40 \quad \mu = 30$

\[
\begin{array}{c|c|c}
\hline
i & (\frac{\lambda}{\mu})^i / i! & \\
\hline
0 & 1.0000 & \\
1 & 1.3333 & \\
2 & 0.8888 & 3.2221 \\
\hline
\end{array}
\]

$P_0 = 1.0000/3.2221 = 0.3104 \quad 31.04\%$

b. $P_2 = 0.8888/3.2221 = 0.2758 \quad 27.58\%$

c. $P_2 = 0.2758$

$P_3 = 0.3951/(3.2221 + 0.3951) = 0.1092$

$P_4 = 0.1317/(3.2221 + 0.3951 + 0.1317) = 0.0351$

d. $k = 3 \quad 10.92\% \text{ of calls receiving a busy signal.}$

33. a. $\lambda = 0.05 \quad \mu = 0.50 \quad \lambda/\mu = 0.10 \quad N = 8$

\[
\begin{array}{c|c|c}
\hline
n & \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n & \\
\hline
0 & 1.0000 & \\
1 & 0.8000 & \\
2 & 0.5600 & \\
3 & 0.3360 & \\
4 & 0.1680 & \\
5 & 0.0672 & \\
6 & 0.0202 & \\
7 & 0.0040 & \\
8 & 0.0004 & 2.9558 \\
\hline
\end{array}
\]
\[ P_0 = 1/2.9558 = 0.3383 \]

\[ L_q = N - \left( \frac{\lambda + \mu}{\lambda} \right) (1 - P_0) = 8 - \left( \frac{0.225}{0.05} \right) (1 - 0.3383) = 0.7215 \]

\[ L = L_q + (1 - P_0) = 0.7213 + (1 - 0.3383) = 1.3832 \]

\[ W_q = \frac{L_q}{(N - L)\lambda} = \frac{0.7215}{(8 - 1.3832)(0.05)} = 2.1808 \text{ hours} \]

\[ W = W_q + \frac{1}{\mu} = 2.1808 + \frac{1}{0.50} = 4.1808 \text{ hours} \]

\( b. \quad P_0 = 0.4566 \)

\[ L_q = 0.0646 \]

\[ L = 0.7860 \]

\[ W_q = 0.1791 \text{ hours} \]

\[ W = 2.1791 \text{ hours} \]

\( c. \quad \text{One Employee} \)

\[ \text{Cost} = 80L + 20 \]

\[ = 80(1.3832) + 20 = $130.65 \]

\( \text{Two Employees} \)

\[ \text{Cost} = 80L + 20(2) \]

\[ = 80(0.7860) + 40 = $102.88 \]

\( \text{Use two employees.} \)

\begin{align*}
N &= 5 & \lambda &= 0.025 & \mu &= 0.20 & \lambda/\mu &= 0.125 \\
\end{align*}

\( a. \)

\[
\begin{array}{c|c|c}
 n & \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^n & \\
0 & 1.0000 & \\
1 & 0.6250 & \\
2 & 0.3125 & \\
3 & 0.1172 & \\
4 & 0.0293 & \\
5 & 0.0037 & \\
\hline & & 2.0877 & \\
\end{array}
\]

\[ P_0 = 1/2.0877 = 0.4790 \]

\[ L_q = N - \left( \frac{\lambda + \mu}{\lambda} \right) (1 - P_0) = 5 - \left( \frac{0.225}{0.025} \right) (1 - 0.4790) = 0.3110 \]
c. \[ L = L_q + (1 - P_0) = 0.3110 + (1 - 0.4790) = 0.8321 \]

d. \[ W_q = \frac{L_q}{(N - L)\lambda} = \frac{0.3110}{(5 - 0.8321)(0.025)} = 2.9854 \text{ min} \]

e. \[ W = W_q + \frac{1}{\mu} = 2.9854 + \frac{1}{0.20} = 7.9854 \text{ min} \]

f. Trips/Days = (8 hours)(60 min/hour)(\lambda) = (8)(60)(0.025) = 12 trips

Time at Copier: 12 x 7.9854 = 95.8 minutes/day
Wait Time at Copier: 12 x 2.9854 = 35.8 minutes/day

g. Yes. Five administrative assistants x 35.8 = 179 min. (3 hours/day)
3 hours per day are lost to waiting.

(35.8/480)(100) = 7.5% of each administrative assistant's day is spent waiting for the copier.

35. \[ N = 10 \quad \lambda = 0.25 \quad \mu = 4 \quad \lambda/\mu = 0.0625 \]

<table>
<thead>
<tr>
<th>n</th>
<th>( \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.6250</td>
</tr>
<tr>
<td>2</td>
<td>0.3516</td>
</tr>
<tr>
<td>3</td>
<td>0.1758</td>
</tr>
<tr>
<td>4</td>
<td>0.0769</td>
</tr>
<tr>
<td>5</td>
<td>0.0288</td>
</tr>
<tr>
<td>6</td>
<td>0.0090</td>
</tr>
<tr>
<td>7</td>
<td>0.0023</td>
</tr>
<tr>
<td>8</td>
<td>0.0004</td>
</tr>
<tr>
<td>9</td>
<td>0.0001</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>2.2698</td>
</tr>
</tbody>
</table>

\[ P_0 = 1/2.2698 = 0.4406 \]
b. \[ L_q = N - \frac{\mu}{\lambda} (1 - P_0) = 10 - \frac{2.5}{0.25} (1 - 0.4406) = 0.4895 \]

c. \[ L = L_q + (1 - P_0) = 0.4895 + (1 - 0.4406) = 1.0490 \]

d. \[ W_q = \frac{L_q}{(N - L)\lambda} = \frac{0.4895}{(10 - 1.0490)(0.25)} = 0.2188 \]

e. \[ W = W_q + \frac{1}{\mu} = 0.2188 + \frac{1}{4} = 0.4688 \]

f. \[ TC = c_w L + c_v k \\
    = 50 (1.0490) + 30 (1) = $82.45 \]

g. \[ k = 2 \]
\[ TC = c_w L + c_v k \\
    = 50L + 30(2) = $82.45 \]
\[ 50L = 22.45 \]
\[ L = 0.4490 \text{ or less.} \]

h. Using *The Management Scientist* with \( k = 2 \),
\[ L = 0.6237 \]
\[ TC = c_w L + c_v k \\
    = 50 (1.6237) + 30 (2) = $91.18 \]

The company should not expand to the two-channel truck dock.
Chapter 14
Decision Analysis

Learning Objectives

1. Learn how to describe a problem situation in terms of decisions to be made, chance events and consequences.

2. Be able to analyze a simple decision analysis problem from both a payoff table and decision tree point of view.

3. Be able to develop a risk profile and interpret its meaning.

4. Be able to use sensitivity analysis to study how changes in problem inputs affect or alter the recommended decision.

5. Be able to determine the potential value of additional information.

6. Learn how new information and revised probability values can be used in the decision analysis approach to problem solving.

7. Understand what a decision strategy is.

8. Learn how to evaluate the contribution and efficiency of additional decision making information.

9. Be able to use a Bayesian approach to computing revised probabilities.

10. Know what is meant by utility.

11. Understand why utility could be preferred to monetary value in some situations.

12. Be able to use expected utility to select a decision alternative.

13. Be able to use TreePlan software for decision analysis problems.

14. Understand the following terms:

- decision alternatives
- chance events
- states of nature
- influence diagram
- payoff table
- decision tree
- optimistic approach
- conservative approach
- minimax regret approach
- opportunity loss or regret
- expected value approach
- expected value of perfect information (EVPI)

- decision strategy
- risk profile
- sensitivity analysis
- prior probabilities
- posterior probabilities
- expected value of sample information (EVSI)
- efficiency of sample information
- Bayesian revision
- utility
- lottery
- expected utility
Solutions:

1. a. 

   ![Diagram](image)

   Decision Analysis

b. 

<table>
<thead>
<tr>
<th>Decision</th>
<th>Maximum Profit</th>
<th>Minimum Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>250</td>
<td>25</td>
</tr>
<tr>
<td>$d_2$</td>
<td>100</td>
<td>75</td>
</tr>
</tbody>
</table>

   Optimistic approach: select $d_1$

   Conservative approach: select $d_2$

   Regret or opportunity loss table:

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>$d_2$</td>
<td>150</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

   Maximum Regret: 50 for $d_1$ and 150 for $d_2$; select $d_1$

2. a. 

<table>
<thead>
<tr>
<th>Decision</th>
<th>Maximum Profit</th>
<th>Minimum Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>$d_2$</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>$d_3$</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>$d_4$</td>
<td>13</td>
<td>8</td>
</tr>
</tbody>
</table>

   Optimistic approach: select $d_1$

   Conservative approach: select $d_3$

   Regret or Opportunity Loss Table with the Maximum Regret
Minimax regret approach: select \( d_3 \)

b. The choice of which approach to use is up to the decision maker. Since different approaches can result in different recommendations, the most appropriate approach should be selected before analyzing the problem.

c. | Decision | Minimum Cost | Maximum Cost |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>( d_4 )</td>
<td>8</td>
<td>13</td>
</tr>
</tbody>
</table>

Optimistic approach: select \( d_1 \)
Conservative approach: select \( d_2 \) or \( d_3 \)

Regret or Opportunity Loss Table

<table>
<thead>
<tr>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>Maximum Regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>( d_4 )</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Minimax regret approach: select \( d_2 \)

3. a. The decision to be made is to choose the best plant size. There are 2 alternatives to choose from: a small plant or a large plant.

The chance event is the market demand for the new product line. It is viewed as having 3 possible outcomes (states of nature): low, medium and high.

b. Influence Diagram:
4. a. The decision is to choose the best lease option; there are three alternatives. The chance event is the number of miles Amy will drive per year. There are three possible outcomes.

b. The payoff table for Amy's problem is shown below. To illustrate how the payoffs were computed, we show how to compute the total cost of the Forno Saab lease assuming Amy drives 15,000 miles per year.

Total Cost = (Total Monthly Charges) + (Total Additional Mileage Cost)
= 36($299) + $0.15(45,000 - 36,000)
= $10,764 + $1350
= $12,114

<table>
<thead>
<tr>
<th>Annual Miles Driven</th>
<th>12,000</th>
<th>15,000</th>
<th>18,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forno Saab</td>
<td>$10,764</td>
<td>$12,114</td>
<td>$13,464</td>
</tr>
<tr>
<td>Midtown Motors</td>
<td>$11,160</td>
<td>$11,160</td>
<td>$12,960</td>
</tr>
<tr>
<td>Hopkins Automotive</td>
<td>$11,700</td>
<td>$11,700</td>
<td>$11,700</td>
</tr>
</tbody>
</table>

c. | Decision Alternative | Minimum Cost | Maximum Cost |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Forno Saab</td>
<td>$10,764</td>
<td>$13,464</td>
</tr>
<tr>
<td>Midtown Motors</td>
<td>$11,160</td>
<td>$12,960</td>
</tr>
<tr>
<td>Hopkins Automotive</td>
<td>$11,700</td>
<td>$11,700</td>
</tr>
</tbody>
</table>

Optimistic Approach: Forno Saab ($10,764)

Conservative Approach: Hopkins Automotive ($11,160)
Opportunity Loss or Regret Table

<table>
<thead>
<tr>
<th>Decision Alternative</th>
<th>Actual Miles Driven</th>
<th>Maximum Regret</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36,000</td>
<td>45,000</td>
</tr>
<tr>
<td>Forno Saab</td>
<td>0</td>
<td>$954</td>
</tr>
<tr>
<td>Midtown Motors</td>
<td>$396</td>
<td>0</td>
</tr>
<tr>
<td>Hopkins Automotive</td>
<td>$936</td>
<td>$540</td>
</tr>
</tbody>
</table>

Minimax Regret Approach: Hopkins Automotive

d. EV (Forno Saab) = 0.5($10,764) + 0.4($12,114) + 0.1($13,464) = $11,574
EV (Midtown Motors) = 0.5($11,160) + 0.4($11,160) + 0.1($12,960) = $11,340
EV (Hopkins Automotive) = 0.5($11,700) + 0.4($11,700) + 0.1($11,700) = $11,700

Best Decision: Midtown Motors
e.

The most likely cost is $11,160 with a probability of 0.9. There is a probability of 0.1 of incurring a cost of $12,960.

f. EV (Forno Saab) = 0.3($10,764) + 0.4($12,114) + 0.3($13,464) = $12,114
EV (Midtown Motors) = 0.3($11,160) + 0.4($11,160) + 0.3($12,960) = $11,700
EV (Hopkins Automotive) = 0.3($11,700) + 0.4($11,700) + 0.3($11,700) = $11,700

Best Decision: Midtown Motors or Hopkins Automotive

With these probabilities, Amy would be indifferent between the Midtown Motors and Hopkins Automotive leases. However, if the probability of driving 18,000 miles per year goes up any further, the Hopkins Automotive lease will be the best.

5. \[ EV(d_1) = 0.65(250) + 0.15(100) + 0.20(25) = 182.5 \]
\[ EV(d_2) = 0.65(100) + 0.15(100) + 0.20(75) = 95 \]

The optimal decision is \( d_1 \)

6. \[ EV(d_1) = 0.5(14) + 0.2(9) + 0.2(10) + 0.1(5) = 11.3 \]
\[ EV(d_2) = 0.5(11) + 0.2(10) + 0.2(8) + 0.1(7) = 9.8 \]
\[ EV(d_3) = 0.5(9) + 0.2(10) + 0.2(10) + 0.1(11) = 9.6 \]
\[ EV(d_4) = 0.5(8) + 0.2(10) + 0.2(11) + 0.1(13) = 9.5 \]

Recommended decision: \( d_1 \)
b. The best decision in this case is the one with the smallest expected value; thus, \( d_4 \), with an expected cost of 9.5, is the recommended decision.

7.  a. \[
\begin{align*}
EV(\text{own staff}) &= 0.2(650) + 0.5(650) + 0.3(600) = 635 \\
EV(\text{outside vendor}) &= 0.2(900) + 0.5(600) + 0.3(300) = 570 \\
EV(\text{combination}) &= 0.2(800) + 0.5(650) + 0.3(500) = 635 
\end{align*}
\]

The optimal decision is to hire an outside vendor with an expected annual cost of $570,000.

b. The risk profile in tabular form is shown.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.3</td>
</tr>
<tr>
<td>600</td>
<td>0.5</td>
</tr>
<tr>
<td>900</td>
<td>0.2</td>
</tr>
</tbody>
</table>

A graphical representation of the risk profile is also shown:

8.  a. \[
\begin{align*}
EV(d_1) &= p(10) + (1 - p)(1) = 9p + 1 \\
EV(d_2) &= p(4) + (1 - p)(3) = 1p + 3 
\end{align*}
\]
$d_2$ is optimal for $p \leq 0.25$; $d_1$ is optimal for $p \geq 0.25$.

b. The best decision is $d_2$ since $p = 0.20 < 0.25$.

$EV(d_1) = 0.2(10) + 0.8(1) = 2.8$
$EV(d_2) = 0.2(4) + 0.8(3) = 3.2$

c. The best decision in part (b) is $d_2$ with $EV(d_2) = 3.2$. Decision $d_2$ will remain optimal as long as its expected value is higher than that for $d_1$ ($EV(d_1) = 2.8$).

Let $s =$ payoff for $d_1$ under state of nature $s_1$. Decision $d_2$ will remain optimal provided that

$EV(d_2) = 0.2(s) + 0.8(3) \geq 2.8$

$0.2s \geq 2.8 - 2.4$

$0.2s \geq 0.4$

$s \geq 2$

As long as the payoff for $s_1$ is $\geq 2$, then $d_2$ will be optimal.

9. a. The decision to be made is to choose the type of service to provide. The chance event is the level of demand for the Myrtle Air service. The consequence is the amount of quarterly profit. There are two decision alternatives (full price and discount service). There are two outcomes for the chance event (strong demand and weak demand).

b.

<table>
<thead>
<tr>
<th>Type of Service</th>
<th>Maximum Profit</th>
<th>Minimum Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Price</td>
<td>$960</td>
<td>-$490</td>
</tr>
<tr>
<td>Discount</td>
<td>$670</td>
<td>$320</td>
</tr>
</tbody>
</table>

Optimistic Approach: Full price service
Conservative Approach: Discount service

Opportunity Loss or Regret Table

<table>
<thead>
<tr>
<th></th>
<th>High Demand</th>
<th>Low Demand</th>
<th>Maximum Regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Service</td>
<td>0</td>
<td>810</td>
<td>810</td>
</tr>
<tr>
<td>Discount Service</td>
<td>290</td>
<td>0</td>
<td>290</td>
</tr>
</tbody>
</table>

Minimax Regret Approach: Discount service

e. $EV($Full$) = 0.7(960) + 0.3(-490) = 525$

$EV($Discount$) = 0.7(670) + 0.3(320) = 565$

Optimal Decision: Discount service

d. $EV($Full$) = 0.8(960) + 0.2(-490) = 670$

$EV($Discount$) = 0.8(670) + 0.2(320) = 600$

Optimal Decision: Full price service

e. Let $p =$ probability of strong demand

$EV($Full$) = p(960) + (1-p)(-490) = 1450p - 490$
EV (Discount) = \( p(670) + (1-p)(320) = 350p + 320 \)

EV (Full) = EV(Discout)  
\[ 1450p - 490 = 350p + 320 \]
\[ 1100p = 810 \]
\[ p = \frac{810}{1100} = 0.7364 \]

If \( p = 0.7364 \), the two decision alternatives provide the same expected value.

For values of \( p \) below 0.7364, the discount service is the best choice. For values of \( p \) greater than 0.7364, the full price service is the best choice.
b. \[ \text{EV(node 2)} = 0.2(1000) + 0.5(700) + 0.3(300) = 640 \]

\[ \text{EV(node 4)} = 0.3(800) + 0.4(400) + 0.3(200) = 460 \]

\[ \text{EV(node 5)} = 0.5(1600) + 0.3(800) + 0.2(400) = 1120 \]

\[ \text{EV(node 3)} = 0.6\text{EV(node 4)} + 0.4\text{EV(node 5)} = 0.6(460) + 0.4(1120) = 724 \]

Space Pirates is recommended. Expected value of $724,000 is $84,000 better than Battle Pacific.

c. Risk Profile for Space Pirates

Outcome:

\[ \begin{align*}
1600 & \quad (0.4)(0.5) = 0.20 \\
800 & \quad (0.6)(0.3) + (0.4)(0.3) = 0.30 \\
400 & \quad (0.6)(0.4) + (0.4)(0.2) = 0.32 \\
200 & \quad (0.6)(0.3) = 0.18 \\
\end{align*} \]

\[ p = 0 \quad \text{EV(node 5)} = 1120 \]
\[ p = 1 \quad \text{EV(node 4)} = 460 \]
The probability of competition would have to be greater than 0.7273 before we would change to the Battle Pacific video game.

11. a. Currently, the large complex decision is optimal with $EV(d_3) = 0.8(20) + 0.2(-9) = 14.2$. In order for $d_3$ to remain optimal, the expected value of $d_2$ must be less than or equal to 14.2.

Let $s =$ payoff under strong demand

$$EV(d_2) = 0.8(s) + 0.2(5) \leq 14.2$$

$$0.8s + 1 \leq 14.2$$

$$0.8s \leq 13.2$$

$$s \leq 16.5$$

Thus, if the payoff for the medium complex under strong demand remains less than or equal to $16.5$ million, the large complex remains the best decision.

b. A similar analysis is applicable for $d_1$

$$EV(d_1) = 0.8(s) + 0.2(7) \leq 14.2$$

$$0.8s + 1.4 \leq 14.2$$

$$0.8s \leq 12.8$$

$$s \leq 16$$

If the payoff for the small complex under strong demand remains less than or equal to $16$ million, the large complex remains the best decision.

12. a. There is only one decision to be made: whether or not to lengthen the runway. There are only two decision alternatives. The chance event represents the choices made by Air Express and DRI concerning whether they locate in Potsdam. Even though these are decisions for Air Express and DRI, they are chance events for Potsdam.
The payoffs and probabilities for the chance event depend on the decision alternative chosen. If Potsdam lengthens the runway, there are four outcomes (both, Air Express only, DRI only, neither). The probabilities and payoffs corresponding to these outcomes are given in the tables of the problem statement. If Potsdam does not lengthen the runway, Air Express will not locate in Potsdam so we only need to consider two outcomes: DRI and no DRI. The approximate probabilities and payoffs for this case are given in the last paragraph of the problem statements.

The consequence is the estimated annual revenue.

b. Runway is Lengthened

<table>
<thead>
<tr>
<th>New Air Express Center</th>
<th>New DRI Plant</th>
<th>Probability</th>
<th>Annual Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>0.3</td>
<td>$600,000</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>0.1</td>
<td>$150,000</td>
</tr>
<tr>
<td>No</td>
<td>Yes</td>
<td>0.4</td>
<td>$250,000</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>0.2</td>
<td>-$200,000</td>
</tr>
</tbody>
</table>

\[ EV \text{ (Runway is Lengthened)} = 0.3(600,000) + 0.1(150,000) + 0.4(250,000) - 0.2(200,000) = 255,000 \]

c. EV (Runway is Not Lengthened) = 0.6($450,000) + 0.4($0) = $270,000

d. The town should not lengthen the runway.

e. EV (Runway is Lengthened) = 0.4(600,000) + 0.1(150,000) + 0.3(250,000) - 0.2(200,000) = 290,000

The revised probabilities would lead to the decision to lengthen the runway.

13. a. The decision is to choose what type of grapes to plant, the chance event is demand for the wine and the consequence is the expected annual profit contribution. There are three decision alternatives (Chardonnay, Riesling and both). There are four chance outcomes: (W,W); (W,S); (S,W); and (S,S). For instance, (W,S) denotes the outcomes corresponding to weak demand for Chardonnay and strong demand for Riesling.

b. In constructing a decision tree, it is only necessary to show two branches when only a single grape is planted. But, the branch probabilities in these cases are the sum of two probabilities. For example, the probability that demand for Chardonnay is strong is given by:

\[ P \text{ (Strong demand for Chardonnay)} = P(S,W) + P(S,S) = 0.25 + 0.20 = 0.45 \]
c. \[ \text{EV (Plant Chardonnay)} = 0.55(20) + 0.45(70) = 42.5 \]
\[ \text{EV (Plant both grapes)} = 0.05(22) + 0.50(40) + 0.25(26) + 0.20(60) = 39.6 \]
\[ \text{EV (Plant Riesling)} = 0.30(25) + 0.70(45) = 39 \]
Optimal decision: Plant Chardonnay grapes only.

d. This changes the expected value in the case where both grapes are planted and when Riesling only is planted.
\[ \text{EV (Plant both grapes)} = 0.05(22) + 0.50(40) + 0.05(26) + 0.40(60) = 46.4 \]
\[ \text{EV (Plant Riedling)} = 0.10(25) + 0.90(45) = 43.0 \]
We see that the optimal decision is now to plant both grapes. The optimal decision is sensitive to this change in probabilities.

e. Only the expected value for node 2 in the decision tree needs to be recomputed.
\[ \text{EV (Plant Chardonnay)} = 0.55(20) + 0.45(50) = 33.5 \]
This change in the payoffs makes planting Chardonnay only less attractive. It is now best to plant both types of grapes. The optimal decision is sensitive to a change in the payoff of this magnitude.
14. a. If \( s_1 \) then \( d_1 \); if \( s_2 \) then \( d_1 \) or \( d_2 \); if \( s_3 \) then \( d_2 \)

b. \( EV_{wPI} = 0.65(250) + 0.15(100) + 0.20(75) = 192.5 \)

c. From the solution to Problem 5 we know that \( EV(d_1) = 182.5 \) and \( EV(d_2) = 95 \); thus, the recommended decision is \( d_1 \). Hence, \( EV_{woPI} = 182.5 \).

d. \( EV_{PI} = EV_{wPI} - EV_{woPI} = 192.5 - 182.5 = 10 \)

15. a. \( EV\) (Small) = 0.1(400) + 0.6(500) + 0.3(660) = 538

\( EV\) (Medium) = 0.1(-250) + 0.6(650) + 0.3(800) = 605

\( EV\) (Large) = 0.1(-400) + 0.6(580) + 0.3(990) = 605

Best decision: Build a medium or large-size community center.

Note that using the expected value approach, the Town Council would be indifferent between building a medium-size community center and a large-size center.

b. Risk profile for medium-size community center:

Given the mayor's concern about the large loss that would be incurred if demand is not large enough to support a large-size center, we would recommend the medium-size center. The large-size center has a
probability of 0.1 of losing $400,000. With the medium-size center, the most the town can lose is $250,000.

c. The Town's optimal decision strategy based on perfect information is as follows:

   If the worst-case scenario, build a small-size center
   If the base-case scenario, build a medium-size center
   If the best-case scenario, build a large-size center

Using the consultant's original probability assessments for each scenario, 0.10, 0.60 and 0.30, the expected value of a decision strategy that uses perfect information is:

   \[ EV_{wPI} = 0.1(400) + 0.5(650) + 0.4(990) = 761 \]

In part (a), the expected value approach showed that \( EV(\text{Medium}) = EV(\text{Large}) = 605 \). Therefore, \( EV_{wPI} = 605 \) and \( EV_{PI} = 761 - 605 = 156 \)

The town should seriously consider additional information about the likelihood of the three scenarios. Since perfect information would be worth $156,000, a good market research study could possibly make a significant contribution.

d. \( EV(\text{Small}) = 0.2(400) + 0.5(500) + 0.3(660) = 528 \)
   \( EV(\text{Medium}) = 0.2(-250) + 0.5(650) + 0.3(800) = 515 \)
   \( EV(\text{Small}) = 0.2(-400) + 0.5(580) + 0.3(990) = 507 \)

   Best decision: Build a small-size community center.

e. If the promotional campaign is conducted, the probabilities will change to 0.0, 0.6 and 0.4 for the worst case, base case and best case scenarios respectively.

   \( EV(\text{Small}) = 0.0(400) + 0.6(500) + 0.4(660) = 564 \)
   \( EV(\text{Medium}) = 0.0(-250) + 0.6(650) + 0.4(800) = 710 \)
   \( EV(\text{Small}) = 0.0(-400) + 0.6(580) + 0.4(990) = 744 \)

In this case, the recommended decision is to build a large-size community center. Compared to the analysis in Part (a), the promotional campaign has increased the best expected value by $744,000 - 605,000 = $139,000. Compared to the analysis in part (d), the promotional campaign has increased the best expected value by $744,000 - 528,000 = $216,000.

Even though the promotional campaign does not increase the expected value by more than its cost ($150,000) when compared to the analysis in part (a), it appears to be a good investment. That is, it eliminates the risk of a loss, which appears to be a significant factor in the mayor's decision-making process.
16. a. 

b. \( \text{EV (node 6)} = 0.57(100) + 0.43(300) = 186 \) 
\( \text{EV (node 7)} = 0.57(400) + 0.43(200) = 314 \) 
\( \text{EV (node 8)} = 0.18(100) + 0.82(300) = 264 \) 
\( \text{EV (node 9)} = 0.18(400) + 0.82(200) = 236 \) 
\( \text{EV (node 10)} = 0.40(100) + 0.60(300) = 220 \) 
\( \text{EV (node 11)} = 0.40(400) + 0.60(200) = 280 \) 

\( \text{EV (node 3)} = \max(186,314) = 314 \quad d_2 \) 
\( \text{EV (node 4)} = \max(264,236) = 264 \quad d_1 \) 
\( \text{EV (node 5)} = \max(220,280) = 280 \quad d_2 \)
17. The decision tree is as shown in the answer to problem 16a. The calculations using the decision tree in problem 16a with the probabilities and payoffs here are as follows:

\[
\begin{align*}
\text{a,b. EV (node 6)} &= 0.18(600) + 0.82(-200) = -56 \\
\text{EV (node 7)} &= 0 \\
\text{EV (node 8)} &= 0.89(600) + 0.11(-200) = 512 \\
\text{EV (node 9)} &= 0 \\
\text{EV (node 10)} &= 0.50(600) + 0.50(-200) = 200 \\
\text{EV (node 11)} &= 0 \\
\text{EV (node 3)} &= \text{Max}(-56,0) = 0 \quad d_2 \\
\text{EV (node 4)} &= \text{Max}(512,0) = 512 \quad d_1 \\
\text{EV (node 5)} &= \text{Max}(200,0) = 200 \quad d_1 \\
\text{EV (node 2)} &= 0.55(0) + 0.45(512) = 230.4
\end{align*}
\]

Without the option, the recommended decision is \(d_1\) purchase with an expected value of $200,000.

With the option, the best decision strategy is

- If high resistance \(H\), \(d_2\) do not purchase
- If low resistance \(L\), \(d_1\) purchase

Expected Value = $230,400

c. \(\text{EVSI} = 230,400 - 200,000 = 30,400\). Since the cost is only $10,000, the investor should purchase the option.

18. a. Outcome 1 ($ in 000s)

\[
\begin{align*}
\text{Bid} & \quad -\$200 \\
\text{Contract} & \quad -2000 \\
\text{Market Research} & \quad -150 \\
\text{High Demand} & \quad +5000 \\
\text{Total} & \quad 2650
\end{align*}
\]

Outcome 2 ($ in 000s)

\[
\begin{align*}
\text{Bid} & \quad -\$200 \\
\text{Contract} & \quad -2000 \\
\text{Market Research} & \quad -150 \\
\text{Moderate Demand} & \quad +3000 \\
\text{Total} & \quad 650
\end{align*}
\]
b. \[ EV (\text{node 8}) = 0.85(2650) + 0.15(650) = 2350 \]
\[ EV (\text{node 5}) = \text{Max}(2350, 1150) = 2350 \quad \text{Decision: Build} \]
\[ EV (\text{node 9}) = 0.225(2650) + 0.775(650) = 1100 \]
\[ EV (\text{node 6}) = \text{Max}(1100, 1150) = 1150 \quad \text{Decision: Sell} \]
\[ EV (\text{node 10}) = 0.6(2800) + 0.4(800) = 2000 \]
\[ EV (\text{node 7}) = \text{Max}(2000, 1300) = 2000 \quad \text{Decision: Build} \]
\[ EV (\text{node 4}) = 0.6 \times EV(\text{node 5}) + 0.4 \times EV(\text{node 6}) = 0.6(2350) + 0.4(1150) = 1870 \]
\[ EV (\text{node 3}) = \text{Max}(EV(\text{node 4}), EV(\text{node 7})) = \text{Max}(1870, 2000) = 2000 \quad \text{Decision: No Market Research} \]
\[ EV (\text{node 2}) = 0.8 \times EV(\text{node 3}) + 0.2 \times (-200) = 0.8(2000) + 0.2(-200) = 1560 \]
\[ EV (\text{node 1}) = \text{Max}(EV(\text{node 2}), 0) = \text{Max}(1560, 0) = 1560 \quad \text{Decision: Bid on Contract} \]

Decision Strategy:

- Bid on the Contract
- Do not do the Market Research
- Build the Complex

Expected Value is $1,560,000

c. Compare Expected Values at nodes 4 and 7.

\[ EV(\text{node 4}) = 1870 \quad \text{Includes $150 cost for research} \]
\[ EV (\text{node 7}) = 2000 \]

Difference is $2000 - 1870 = $130

Market research cost would have to be lowered $130,000 to $20,000 or less to make undertaking the research desirable.

d. Shown below is the reduced decision tree showing only the sequence of decisions and chance events for Dante's optimal decision strategy. If Dante follows this strategy, only 3 outcomes are possible with payoffs of -200, 800, and 2800. The probabilities for these payoffs are found by multiplying the probabilities on the branches leading to the payoffs. A tabular presentation of the risk profile is:

<table>
<thead>
<tr>
<th>Payoff (Smillion)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>-200</td>
<td>.20</td>
</tr>
<tr>
<td>800</td>
<td>(.8)(.4) = .32</td>
</tr>
<tr>
<td>2800</td>
<td>(.8)(.6) = .48</td>
</tr>
</tbody>
</table>

Reduced Decision Tree Showing Only Branches for Optimal Strategy
19. a.

b. Using node 5,

\[
\text{EV (node 10)} = 0.20(-100) + 0.30(50) + 0.50(150) = 70 \\
\text{EV (node 11)} = 100
\]

Decision Sell; Expected Value = $100

c. \[\text{EVwPI} = 0.20(100) + 0.30(100) + 0.50(150) = $125\]

\[\text{EVPI} = $125 - $100 = $25\]

d. \[\text{EV (node 6)} = 0.09(-100) + 0.26(50) + 0.65(150) = 101.5 \]

\[\text{EV (node 7)} = 100 \]

\[\text{EV (node 8)} = 0.45(-100) + 0.39(50) + 0.16(150) = -1.5 \]

\[\text{EV (node 9)} = 100 \]

\[\text{EV (node 3)} = \text{Max}(101.5, 100) = 101.5 \quad \text{Produce} \]

\[\text{EV (node 4)} = \text{Max}(-1.5, 100) = 100 \quad \text{Sell} \]
EV (node 2) = 0.69(101.5) + 0.31(100) = 101.04

If Favorable, Produce
If Unfavorable, Sell     EV = $101.04

e. EVSI = $101.04 - 100 = $1.04 or $1,040.

f. No, maximum Hale should pay is $1,040.

g. No agency; sell the pilot.

20. a.

b. EV (node 7) = 0.75(750) + 0.25(-250) = 500
EV (node 8) = 0.417(750) + 0.583(-250) = 167

Decision (node 4) → Accept EV = 500
Decision (node 5) → Accept EV = 167

EV(node 2) = 0.7(500) + 0.3(167) = $400
Note: Regardless of the review outcome $F$ or $U$, the recommended decision alternative is to accept the manuscript.

$\text{EV(node 3)} = .65(750) + .35(-250) = $400

The expected value is $400,000 regardless of review process. The company should accept the manuscript.

c. The manuscript review cannot alter the decision to accept the manuscript. Do not do the manuscript review.

d. Perfect Information.

If $s_1$, accept manuscript $750$
If $s_2$, reject manuscript -$250$

$\text{EVwPI} = 0.65(750) + 0.35(0) = 487.5$

$\text{EVwoPI} = 400$

$\text{EVPI} = 487.5 - 400 = 87.5$ or $87,500$.

A better procedure for assessing the market potential for the textbook may be worthwhile.

21. a. $\text{EV (1 lot)} = 0.3(60) + 0.3(60) + 0.4(50) = 56$
$\text{EV (2 lots)} = 0.3(80) + 0.3(80) + 0.4(30) = 60$
$\text{EV (3 lots)} = 0.3 (100) + 0.3(70) + 0.4(10) = 55$

Decision: Order 2 lots   Expected Value $60,000
b. The following decision tree applies.

Calculations

\[
\begin{align*}
EV (\text{node 6}) & = 0.34(60) + 0.32(60) + 0.34(50) = 56.6 \\
EV (\text{node 7}) & = 0.34(80) + 0.32(80) + 0.34(30) = 63.0 \\
EV (\text{node 8}) & = 0.34(100) + 0.32(70) + 0.34(10) = 59.8 \\
EV (\text{node 9}) & = 0.20(60) + 0.26(60) + 0.54(50) = 54.6 \\
EV (\text{node 10}) & = 0.20(80) + 0.26(80) + 0.54(30) = 53.0 \\
EV (\text{node 11}) & = 0.20(100) + 0.26(70) + 0.54(10) = 43.6 \\
EV (\text{node 12}) & = 0.30(60) + 0.30(60) + 0.40(50) = 56.0 \\
EV (\text{node 13}) & = 0.30(80) + 0.30(80) + 0.40(30) = 60.0 \\
EV (\text{node 14}) & = 0.30(100) + 0.30(70) + 0.40(10) = 55.0 \\
EV (\text{node 3}) & = \text{Max}(56.6,63.0,59.8) = 63.0 \quad 2 \text{ lots} \\
EV (\text{node 4}) & = \text{Max}(54.6,53.0,43.6) = 54.6 \quad 1 \text{ lot}
\end{align*}
\]
EV (node 5) = \text{Max}(56.0,60.0,55.0) = 60.0 \quad 2 \text{ lots}

EV (node 2) = 0.70(63.0) + 0.30(54.6) = 60.5
EV (node 1) = \text{Max}(60.5,60.0) = 60.5 \quad \text{Prediction}

Optimal Strategy:
If prediction is excellent, 2 lots
If prediction is very good, 1 lot

c. \text{EVwPI} = 0.3(100) + 0.3(80) + 0.4(50) = 74
\text{EVPI} = 74 - 60 = 14
\text{EVSI} = 60.5 - 60 = 0.5

\text{Efficiency} = \frac{\text{EVSI}}{\text{EVPI}} = \frac{0.5}{14} \times 100 = 3.6\%

The V.P.’s recommendation is only valued at EVSI = $500. The low efficiency of 3.6\% indicates other information is probably worthwhile. The ability of the consultant to forecast market conditions should be considered.

22.

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>(P(s_j))</th>
<th>(P(I \mid s_j))</th>
<th>(P(I \cap s_j))</th>
<th>(P(s_j \mid I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1)</td>
<td>0.2</td>
<td>0.10</td>
<td>0.020</td>
<td>0.1905</td>
</tr>
<tr>
<td>(s_2)</td>
<td>0.5</td>
<td>0.05</td>
<td>0.025</td>
<td>0.2381</td>
</tr>
<tr>
<td>(s_3)</td>
<td>0.3</td>
<td>0.20</td>
<td>0.060</td>
<td>0.5714</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>(P(I) =)</td>
<td></td>
<td>0.105</td>
</tr>
</tbody>
</table>

\text{P}(I) = 0.31 1.0000

23. a. \text{EV} (d_1) = 0.8(15) + 0.2(10) = 14.0
\text{EV} (d_2) = 0.8(10) + 0.2(12) = 10.4
\text{EV} (d_3) = 0.8(8) + 0.2(20) = 10.4

Decision \(d_1\) \quad \text{Expected Value 14}

b. \text{EVwPI} = 0.8(15) + 0.2(20) = 16
\text{EVPI} = 16 - 14 = 2

c. \text{Indicator I}

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>Prior Probabilities</th>
<th>Conditional Probabilities</th>
<th>Joint Probabilities</th>
<th>Posterior Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>State s1</td>
<td>0.8</td>
<td>0.20</td>
<td>0.16</td>
<td>0.52</td>
</tr>
<tr>
<td>State s2</td>
<td>0.2</td>
<td>0.75</td>
<td>0.15</td>
<td>0.48</td>
</tr>
</tbody>
</table>

\text{P}(I) = 0.31 1.00

\text{EV} (d_1) = 0.5161(15) + 0.4839(10) = 12.6
\text{EV} (d_2) = 0.5161(10) + 0.4839(12) = 11.0
\text{EV} (d_3) = 0.5161(8) + 0.4839(20) = 13.8

If indicator I occurs, decision \(d_3\) is recommended.
24. The revised probabilities are shown on the branches of the decision tree.

\[
\begin{align*}
\text{EV (node 7)} &= 30 \\
\text{EV (node 8)} &= 0.98(25) + 0.02(45) = 25.4 \\
\text{EV (node 9)} &= 30 \\
\text{EV (node 10)} &= 0.79(25) + 0.21(45) = 29.2 \\
\text{EV (node 11)} &= 30 \\
\text{EV (node 12)} &= 0.00(25) + 1.00(45) = 45.0 \\
\text{EV (node 13)} &= 30 \\
\text{EV (node 14)} &= 0.85(25) + 0.15(45) = 28.0 \\
\text{EV (node 3)} &= \text{Min}(30, 25.4) = 25.4 \text{ Expressway} \\
\text{EV (node 4)} &= \text{Min}(30, 29.2) = 29.2 \text{ Expressway} \\
\text{EV (node 5)} &= \text{Min}(30, 45) = 30.0 \text{ Queen City} \\
\text{EV (node 6)} &= \text{Min}(30, 28) = 28.0 \text{ Expressway} \\
\text{EV (node 2)} &= 0.695(25.4) + 0.215(29.2) + 0.09(30.0) = 26.6 \\
\text{EV (node 1)} &= \text{Min}(26.6, 28) = 26.6 \text{ Weather}
\end{align*}
\]
Strategy:

Check the weather, take the expressway unless there is rain. If rain, take Queen City Avenue.

Expected time: 26.6 minutes.

25. a. \( d_1 = \) Manufacture component  \( s_1 = \) Low demand
\( d_2 = \) Purchase component  \( s_2 = \) Medium demand
\( s_3 = \) High demand

![Decision Tree Diagram]

\[
\begin{align*}
EV(\text{node 2}) & = (0.35)(-20) + (0.35)(40) + (0.30)(100) = 37 \\
EV(\text{node 3}) & = (0.35)(10) + (0.35)(45) + (0.30)(70) = 40.25 \\
\text{Recommended decision: } d_2 \ (\text{purchase component})
\end{align*}
\]

b. Optimal decision strategy with perfect information:

- If \( s_1 \) then \( d_2 \)
- If \( s_2 \) then \( d_2 \)
- If \( s_3 \) then \( d_1 \)

Expected value of this strategy is \( 0.35(10) + 0.35(45) + 0.30(100) = 49.25 \)

\[ EVPI = 49.25 - 40.25 = 9 \text{ or } \$9,000 \]

c. If \( F \) - Favorable

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>( P(s_j) )</th>
<th>( P(F \mid s_j) )</th>
<th>( P(F \cap s_j) )</th>
<th>( P(s_j \mid F) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0.35</td>
<td>0.10</td>
<td>0.035</td>
<td>0.0986</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0.35</td>
<td>0.40</td>
<td>0.140</td>
<td>0.3944</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>0.30</td>
<td>0.60</td>
<td><strong>0.180</strong></td>
<td>0.5070</td>
</tr>
</tbody>
</table>

\[ P(F) = 0.355 \]
If $U$ - Unfavorable

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>$P(s_j)$</th>
<th>$P(U \mid s_j)$</th>
<th>$P(U \cap s_j)$</th>
<th>$P(s_j \mid U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>0.35</td>
<td>0.90</td>
<td>0.315</td>
<td>0.4884</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.35</td>
<td>0.60</td>
<td>0.210</td>
<td>0.3256</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.30</td>
<td>0.40</td>
<td>0.120</td>
<td>0.1860</td>
</tr>
</tbody>
</table>

$P(U) = 0.645$

The probability the report will be favorable is $P(F) = 0.355$

d. Assuming the test market study is used, a portion of the decision tree is shown below.

Summary of Calculations

<table>
<thead>
<tr>
<th>Node</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>64.51</td>
</tr>
<tr>
<td>5</td>
<td>54.23</td>
</tr>
<tr>
<td>6</td>
<td>21.86</td>
</tr>
<tr>
<td>7</td>
<td>32.56</td>
</tr>
</tbody>
</table>
Decision strategy:

If $F$ then $d_1$ since $\text{EV(node 4)} > \text{EV(node 5)}$

If $U$ then $d_2$ since $\text{EV(node 7)} > \text{EV(node 6)}$

$\text{EV(node 1)} = 0.355(64.51) + 0.645(32.56) = 43.90$

e. With no information:

$\text{EV}(d_1) = 0.35(-20) + 0.35(40) + 0.30(100) = 37$

$\text{EV}(d_2) = 0.35(10) + 0.35(45) + 0.30(70) = 40.25$

Recommended decision: $d_2$

f. Optimal decision strategy with perfect information:

If $s_1$ then $d_2$
If $s_2$ then $d_2$
If $s_3$ then $d_1$

Expected value of this strategy is $0.35(10) + 0.35(45) + 0.30(100) = 49.25$

$\text{EVPI} = 49.25 - 40.25 = 9$ or $9,000$

Efficiency $= (3650 / 9000)100 = 40.6\%$

26. Risk avoider, at $20$ payoff $p = 0.70$

$\therefore \text{EV(Lottery)} = 0.70(100) + 0.30(-100) = 40$

$\therefore$ Will Pay $40 - 20 = 20$

Risk taker B, at $20$ payoff $p = 0.45$

$\therefore \text{EV(Lottery)} = 0.45(100) + 0.55(-100) = -10$

$\therefore$ Will Pay $20 - (-10) = 30$

27. Risk Avoider

$\text{EU}(d_1) = 0.25(7.0) + 0.50(9.0) + 0.25(5.0) = 7.5$

$\text{EU}(d_2) = 0.25(9.5) + 0.50(10.0) + 0.25(0.0) = 7.375$

Risk Taker

$\text{EU}(d_1) = 0.25(4.5) + 0.50(6.0) + 0.25(2.5) = 4.75$

$\text{EU}(d_2) = 0.25(7.0) + 0.50(10.0) + 0.25(0.0) = 6.75$
28. a. 
\[
\begin{align*}
\text{EV}(d_1) &= 0.40(100) + 0.30(25) + 0.30(0) = 47.5 \\
\text{EV}(d_2) &= 0.40(75) + 0.30(50) + 0.30(25) = 52.5 \\
\text{EV}(d_3) &= 0.40(50) + 0.30(50) + 0.30(50) = 50.0
\end{align*}
\]

b. Using Utilities

<table>
<thead>
<tr>
<th>Decision Maker A</th>
<th>Decision Maker B</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU($d_1$) = 4.9</td>
<td>EU($d_1$) = 4.45</td>
</tr>
<tr>
<td>EU($d_2$) = 5.9</td>
<td>EU($d_2$) = 3.75</td>
</tr>
<tr>
<td>EU($d_3$) = 6.0</td>
<td>EU($d_3$) = 3.00</td>
</tr>
</tbody>
</table>

c. Difference in attitude toward risk. Decision maker A tends to avoid risk, while decision maker B tends to take a risk for the opportunity of a large payoff.

29. a. 
P(Win) = \frac{1}{250,000} \quad P(Lose) = \frac{249,999}{250,000}

\[
\begin{align*}
\text{EV}(d_1) &= \frac{1}{250,000}(300,000) + \frac{249,999}{250,000}(-2) = -0.80 \\
\text{EV}(d_2) &= 0
\end{align*}
\]

\[\therefore \quad d_2 - \text{Do not purchase lottery ticket.}\]

b. 

<table>
<thead>
<tr>
<th>Purchase</th>
<th>$d_1$</th>
<th>$s_1$ \quad \text{Win}</th>
<th>$s_2$ \quad \text{Lose}</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{EU}(d_1) &= \frac{1}{250,000}(10) + \frac{249,999}{250,000}(0) = 0.00004 \\
\text{EU}(d_2) &= 0.00001
\end{align*}
\]

\[\therefore \quad d_1 - \text{purchase lottery ticket.}\]

30. a. \text{EV}(d_1) = 10,000

\[
\text{EV}(d_2) = 0.96(0) + 0.03(100,000) + 0.01(200,000) = 5,000
\]

Using EV approach \( \rightarrow \) No Insurance \( (d_2) \)

b. Lottery:

\[p = \text{probability of a$0 Cost} \quad 1 - p = \text{probability of a$200,000 Cost}\]
c. 

<table>
<thead>
<tr>
<th></th>
<th>$s_1$ None</th>
<th>$s_2$ Minor</th>
<th>$s_3$ Major</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance</td>
<td>$d_1$</td>
<td>9.9</td>
<td>9.9</td>
</tr>
<tr>
<td>No Insurance</td>
<td>$d_2$</td>
<td>10.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

$EU(d_1) = 9.9$

$EU(d_2) = 0.96(10.0) + 0.03(6.0) + 0.01(0.0) = 9.78$

$\therefore$ Using EU approach $\rightarrow$ Insurance ($d_1$)

d. Use expected utility approach.

31. a. $EV(d_1) = 0.60(1000) + 0.40(-1000) = $200

$EV(d_2) = 0$

$\therefore d_1 \rightarrow$ Bet

b. Lottery: $p$ of winning $1,000

(1 - $p$) of losing $1,000

Most students, if realistic, should require a high value for $p$. While students will differ, let us use $p = 0.90$ as an example.

c. $EU(d_1) = 0.60(10.0) + 0.40(0.0) = 6.0$

$EU(d_2) = 0.60(9.0) + 0.40(9.0) = 9.0$

$\therefore d_2 \rightarrow$ Do Not Bet (Risk Avoider)

d. No, different decision makers have different attitudes toward risk, therefore different utilities.

32. a. $EV(A) = 0.80(60) + 0.20(70) = 62$

$EV(B) = 0.70(45) + 0.30(90) = 58.5$

b. Lottery:

$p$ = probability of a 45 minute travel time

(1 - $p$) = probability of a 90 minute travel time

c. 

<table>
<thead>
<tr>
<th></th>
<th>Route Open</th>
<th>Route Delays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route A</td>
<td>$d_1$</td>
<td>8.0</td>
</tr>
<tr>
<td>Route B</td>
<td>$d_2$</td>
<td>10.0</td>
</tr>
</tbody>
</table>

$EU(A) = 0.80(8.0) + 0.20(6.0) = 7.6$

$EU(B) = 0.70(10.0) + 0.30(0.0) = 7.0$
33. a. EV = 0.10(150,000) + 0.25(100,000) + 0.20(50,000) + 0.15(0) + 0.20(-50,000) + 0.10(-100,000)
   = $30,000

   Market the new product.

   b. Lottery

     \[ p \] = probability of $150,000
     \( (1 - p) \) = probability of -$100,000

   c. Risk Avoider.

   d. EU(market) = 0.10(10.0) + 0.25(9.5) + 0.20(7.0) + 0.15(5.0) + 0.20(2.5) + 0.10(0.0) = 6.025
     EU(don't market) = EU($0) = 5.0

   Market the new product.

   e. Yes - Both EV and EU recommend marketing the product.

34. a.

<table>
<thead>
<tr>
<th></th>
<th>s₁</th>
<th>s₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bet</td>
<td>d₁</td>
<td>350</td>
</tr>
<tr>
<td>Do Not Bet</td>
<td>d₂</td>
<td>0</td>
</tr>
</tbody>
</table>

b. \[ EV(d₁) = \frac{1}{38}(350) + \frac{37}{38}(-10) = -$0.53 \]
   \[ EV(d₂) = 0 \]

   ∴ \( d₂ \mathrm{ \rightarrow Do Not Bet } \)

c. Risk takers, because risk neutral and risk avoiders would not bet.

d. \[ EU(d₁) \geq EU(d₂) \] for decision maker to prefer Bet decision.

   \[ \frac{1}{38}(10.0) + \frac{37}{38}(0.0) \geq EU(d₂) \]
   \[ 0.26 \geq EU(d₂) \]

   ∴ Utility of $0 payoff must be between 0 and 0.26.
Chapter 15
Multicriteria Decision Problems

Learning Objectives

1. Understand the concept of multicriteria decision making and how it differs from situations and procedures involving a single criterion.

2. Be able to develop a goal programming model of a multiple criteria problem.

3. Know how to use the goal programming graphical solution procedure to solve goal programming problems involving two decision variables.

4. Understand how the relative importance of the goals can be reflected by altering the weights or coefficients for the decision variables in the objective function.

5. Know how to develop a solution to a goal programming model by solving a sequence of linear programming models using a general purpose linear programming package.

6. Know what a scoring model is and how to use it to solve a multicriteria decision problem.

7. Understand how a scoring model uses weights to identify the relative importance of each criterion.

8. Know how to apply the analytic hierarchy process (AHP) to solve a problem involving multiple criteria.

9. Understand how AHP utilizes pairwise comparisons to establish priority measures for both the criteria and decision alternatives.

10. Understand the following terms:

    - multicriteria decision problem
    - analytic hierarchy process (AHP)
    - goal programming
    - hierarchy
    - deviation variables
    - pairwise comparison matrix
    - priority levels
    - synthesisization
    - goal equation
    - consistency
    - preemptive priorities
    - consistency ratio
    - scoring model
Solutions:

1. a. 

<table>
<thead>
<tr>
<th>Raw Material</th>
<th>Amount Needed to Achieve Both $P_1$ Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{2}{5} (30) + \frac{1}{2} (15) = 12 + 7.5 = 19.5$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{5} (15) = 3$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{3}{5} (30) + \frac{3}{10} (15) = 18 + 4.5 = 22.5$</td>
</tr>
</tbody>
</table>

Since there are only 21 tons of Material 3 available, it is not possible to achieve both goals.

b. Let 

- $x_1 =$ the number of tons of fuel additive produced 
- $x_2 =$ the number of tons of solvent base produced 
- $d^+_1 =$ the amount by which the number of tons of fuel additive produced exceeds the target value of 30 tons 
- $d^-_1 =$ the amount by which the number of tons of fuel additive produced is less than the target of 30 tons 
- $d^+_2 =$ the amount by which the number of tons of solvent base produced exceeds the target value of 15 tons 
- $d^-_2 =$ the amount by which the number of tons of solvent base is less than the target value of 15 tons

Min $d^-_1 + d^-_2$

s.t.

$\frac{2}{5} x_1 + \frac{1}{2} x_2 \leq 20$ Material 1 
$\frac{1}{5} x_2 \leq 5$ Material 2 
$\frac{3}{5} x_1 + \frac{3}{10} x_2 \leq 21$ Material 3 
$x_1 - d^+_1 + d^-_1 = 30$ Goal 1 
$x_2 - d^+_2 + d^-_2 = 15$ Goal 2 
$x_1, x_2, d^+_1, d^-_1, d^+_2, d^-_2 \geq 0$

c. In the graphical solution, point A minimizes the sum of the deviations from the goals and thus provides the optimal product mix.
d. In the graphical solution shown above, point B minimizes $2d_1^- + d_2^-$ and thus provides the optimal product mix.

2. a. Let

$x_1 =$ number of shares of AGA Products purchased

$x_2 =$ number of shares of Key Oil purchased

To obtain an annual return of exactly 9%

$$0.06(50)x_1 + 0.10(100)x_2 = 0.09(50,000)$$

$$3x_1 + 10x_2 = 4500$$

To have exactly 60% of the total investment in Key Oil

$$100x_2 = 0.60(50,000)$$

$$x_2 = 300$$

Therefore, we can write the goal programming model as follows:

$$\text{Min } P_1(d_1^-) + P_2(d_2^-)$$

s.t.

$$50x_1 + 100x_2 \leq 50,000 \text{ Funds Available}$$

$$3x_1 + 10x_2 - d_1^+ + d_1^- = 4,500 \text{ } P_1 \text{ Goal}$$

$$x_2 - d_2^+ + d_2^- = 300 \text{ } P_2 \text{ Goal}$$

$$x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$$
b. In the graphical solution shown below, \( x_1 = 250 \) and \( x_2 = 375 \).

3. a. Let

\[
\begin{align*}
    x_1 &= \text{number of units of product 1 produced} \\
    x_2 &= \text{number of units of product 2 produced}
\end{align*}
\]

Min 

\[
\begin{align*}
    P_1(d_1^+) &+ P_1(d_1^-) + P_2(d_2^+) + P_2(d_2^-) + P_3(d_3^-) \\
\end{align*}
\]

s.t.

\[
\begin{align*}
    1x_1 + 1x_2 - d_1^+ + d_1^- &= 350 \quad \text{Goal 1} \\
    2x_1 + 5x_2 - d_2^+ + d_2^- &= 1000 \quad \text{Goal 2} \\
    4x_1 + 2x_2 - d_3^+ + d_3^- &= 1300 \quad \text{Goal 3} \\
    x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^- \geq 0
\end{align*}
\]

b. In the graphical solution, point A provides the optimal solution. Note that with \( x_1 = 250 \) and \( x_2 = 100 \), this solution achieves goals 1 and 2, but underachieves goal 3 (profit) by $100 since \( 4(250) + 2(100) = 1200 \).
c.  
\[
\text{Max } 4x_1 + 2x_2 \\
\text{s.t.} \\
\begin{align*}
1x_1 + 1x_2 & \leq 350 \quad \text{Dept. A} \\
2x_1 + 5x_2 & \leq 1000 \quad \text{Dept. B} \\
x_1, x_2 & \geq 0 
\end{align*}
\]

The graphical solution indicates that there are four extreme points. The profit corresponding to each extreme point is as follows:

<table>
<thead>
<tr>
<th>Extreme Point</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4(0) + 2(0) = 0$</td>
</tr>
<tr>
<td>2</td>
<td>$4(350) + 2(0) = 1400$</td>
</tr>
<tr>
<td>3</td>
<td>$4(250) + 2(100) = 1200$</td>
</tr>
<tr>
<td>4</td>
<td>$4(0) + 2(200) = 400$</td>
</tr>
</tbody>
</table>

Thus, the optimal product mix is $x_1 = 350$ and $x_2 = 0$ with a profit of $1400$. 


d. The solution to part (a) achieves both labor goals, whereas the solution to part (b) results in using only \(2(350) + 5(0) = 700\) hours of labor in department B. Although (c) results in a $100 increase in profit, the problems associated with underachieving the original department labor goal by 300 hours may be more significant in terms of long-term considerations.

e. Refer to the graphical solution in part (b). The solution to the revised problem is point B, with \(x_1 = 281.25\) and \(x_2 = 87.5\). Although this solution achieves the original department B labor goal and the profit goal, this solution uses \(1(281.25) + 1(87.5) = 368.75\) hours of labor in department A, which is 18.75 hours more than the original goal.

4. a. Let
\[
\begin{align*}
  x_1 &= \text{number of gallons of IC-100 produced} \\
  x_2 &= \text{number of gallons of IC-200 produced}
\end{align*}
\]

Minimize
\[
P_1(d_1^-) + P_1(d_2^+) + P_2(d_3^-) + P_2(d_4^-) + P_3(d_5^-)
\]
subject to
\[
\begin{align*}
20x_1 + 30x_2 - d_1^- &+ d_1^+ = 4800 \quad \text{Goal 1} \\
20x_1 + 30x_2 - d_2^- &+ d_2^+ = 6000 \quad \text{Goal 2} \\
x_1 - d_3^- &+ d_3^+ = 100 \quad \text{Goal 3} \\
x_2 - d_4^- &+ d_4^+ = 120 \quad \text{Goal 4} \\
x_1 + x_2 - d_5^- &+ d_5^+ = 300 \quad \text{Goal 5}
\end{align*}
\]

\(x_1, x_2, \text{all deviation variables} \geq 0\)

b. In the graphical solution, the point \(x_1 = 120\) and \(x_2 = 120\) is optimal.
5. a. 
- May: \( x_1 - s_1 = 200 \)
- June: \( s_1 + x_2 - s_2 = 600 \)
- July: \( s_2 + x_3 - s_3 = 600 \)
- August: \( s_3 + x_4 = 600 \) (no need for ending inventory)

b. 
- May to June: \( x_2 - x_1 - d_1^+ + d_1^- = 0 \)
- June to July: \( x_3 - x_2 - d_2^+ + d_2^- = 0 \)
- July to August: \( x_4 - x_3 - d_3^+ + d_3^- = 0 \)

c. No. For instance, there must be at least 200 pumps in inventory at the end of May to meet the June requirement of shipping 600 pumps.

The inventory variables are constrained to be nonnegative so we only need to be concerned with positive deviations.

- June: \( s_1 - d_1^+ = 0 \)
- July: \( s_2 - d_2^+ = 0 \)
- August: \( s_3 - d_3^+ = 0 \)

d. Production capacity constraints are needed for each month.

- May: \( x_1 \leq 500 \)
- June: \( x_2 \leq 400 \)
- July: \( x_3 \leq 800 \)
- August: \( x_4 \leq 500 \)
e. Min $d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^- + d_4^+ + d_4^- + d_5^+ + d_6^+$

s.t.

3 Goal equations in (b)
3 Goal equations in (c)
4 Demand constraints in (a)
4 Capacity constraints in (d)

$x_1, x_2, x_3, x_4, s_1, s_2, s_3, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, d_4^+, d_4^- \geq 0$

Optimal Solution: $x_1 = 400, x_2 = 400, x_3 = 700, x_4 = 500, s_1 = 200, s_2 = 0, s_3 = 100,$
$d_1^+ = 300, d_1^- = 200, d_2^+ = 200, d_2^- = 100$

f. Yes. Note in part (c) that the inventory deviation variables are equal to the ending inventory variables. So, we could eliminate those goal equations and substitute $s_1, s_2,$ and $s_3$ for $d_1^+, d_2^-$ and $d_6^+$ in the objective function. In this case the inventory variables themselves represent the deviations from the goal of zero.

6. a. Note that getting at least 10,000 customers from group 1 is equivalent to $x_1 = 40,000$ (25% of 40,000 = 10,000) and getting 5,000 customers is equivalent to $x_2 = 50,000$ (10% of 50,000 = 5,000). Thus, to satisfy both goals, 40,000 + 50,000 = 90,000 letters would have to be mailed at a cost of 90,000($1) = $90,000.

Let

$x_1 =$ number of letters mailed to group 1 customers
$x_2 =$ number of letters mailed to group 2 customers
$d_1^+ =$ number of letters mailed to group 1 customers over the desired 40,000
$d_1^- =$ number of letters mailed to group 1 customers under the desired 40,000
$d_2^+ =$ number of letters mailed to group 2 customers over the desired 50,000
$d_2^- =$ number of letters mailed to group 2 customers under the desired 50,000
$d_3^+ =$ the amount by which the expenses exceeds the target value of $70,000
$d_3^- =$ the amount by which the expenses falls short of the target value of $70,000

Min $P_1(d_1^+) + P_1(d_2^+) + P_2(d_3^+)$

s.t.

$x_1 - d_1^+ + d_1^- = 40,000$ Goal 1
$x_2 - 1d_2^+ + 1d_2^- = 50,000$ Goal 2
$1x_1 + 1x_2 - d_3^+ + d_3^- = 70,000$ Goal 3

$x_1, x_2, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^- \geq 0$

b. Optimal Solution: $x_1 = 40,000, x_2 = 50,000$

c. Objective function becomes

$$\min P_1(d_1^+) + P_1(2d_2^+) + P_2(d_3^+)$$
Optimal solution does not change since it is possible to achieve both goals 1 and 2 in the original problem.

7. a. Let
   \[ x_1 = \text{number of TV advertisements} \]
   \[ x_2 = \text{number of radio advertisements} \]
   \[ x_3 = \text{number of newspaper advertisements} \]

   Min \[ P_1(d_1^+) + P_2(d_2^+) + P_3(d_3^+) + P_4(d_4^+) \]

   s.t.
   \[
   \begin{align*}
   x_1 & \leq 10 \text{ TV} \\
   x_2 & \leq 15 \text{ Radio} \\
   x_3 & \leq 20 \text{ Newspaper} \\
   20x_1 + 5x_2 + 10x_3 - d_1^- + d_1^+ & = 400 \text{ Goal 1} \\
   0.7x_1 - 0.3x_2 - 0.3x_3 - d_2^- + d_2^+ & = 0 \text{ Goal 2} \\
   -0.2x_1 + 0.8x_2 - 0.2x_3 - d_3^- + d_3^+ & = 0 \text{ Goal 3} \\
   25x_1 + 4x_2 + 5x_3 - d_4^- + d_4^+ & = 200 \text{ Goal 4} \\
   \end{align*}
\]

   \[ x_1, x_2, x_3, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, d_4^+, d_4^- \geq 0 \]

b. Optimal Solution: \( x_1 = 9.474, x_2 = 2.105, x_3 = 20 \)

   Rounding down leads to a recommendation of 9 TV advertisements, 2 radio advertisements, and 20 newspaper advertisements. Note, however, that rounding down results in not achieving goals 1 and 2.

8. Let \( x_1 = \text{first coordinate of the new machine location} \)
   \[ x_2 = \text{second coordinate of the new machine location} \]
   \[ d_i^+ = \text{amount by which } x_1 \text{ coordinate of new machine exceeds } x_1 \text{ coordinate of machine } i \]
   \[ d_i^- = \text{amount by which } x_1 \text{ coordinate of machine } i \text{ exceeds } x_1 \text{ coordinate of new machine} \]
   \[ e_i^+ = \text{amount by which } x_2 \text{ coordinate of new machine exceeds } x_2 \text{ coordinate of machine } i \]
   \[ e_i^- = \text{amount by which } x_2 \text{ coordinate of machine } i \text{ exceeds } x_2 \text{ coordinate of new machine} \]
The goal programming model is given below.

\[
\begin{align*}
\text{Min} & \quad d_1^- + d_1^+ + e_1^- + e_1^+ + d_2^- + d_2^+ + e_2^- + e_2^+ + d_3^- + d_3^+ + e_3^- + e_3^+ \\
\text{s.t.} & \quad x_1 + d_1^- - d_1^+ = 1 \\
& \quad x_2 + e_1^- - e_1^+ = 7 \\
& \quad x_1 + d_2^- - d_2^+ = 5 \\
& \quad x_2 + e_2^- - e_2^+ = 9 \\
& \quad x_1 + d_3^- - d_3^+ = 6 \\
& \quad x_2 + e_3^- - e_3^+ = 2 \\
& \quad x_1, x_2, d_1^-, d_1^+, e_1^-, e_1^+, d_2^-, d_2^+, e_2^-, e_2^+, d_3^-, d_3^+, e_3^-, e_3^+ \geq 0
\end{align*}
\]

b. The optimal solution is given by

\[
\begin{align*}
x_1 &= 5 \\
x_2 &= 7 \\
d_1^- &= 4 \\
e_2^- &= 2 \\
d_3^- &= 1 \\
e_3^+ &= 5
\end{align*}
\]

The value of the solution is 12.

9. Scoring Calculations

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Analyst</th>
<th>Accountant</th>
<th>Auditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career Advancement</td>
<td>Chicago 35</td>
<td>Denver 20</td>
<td>Houston 20</td>
</tr>
<tr>
<td>Location</td>
<td>10</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Management</td>
<td>30</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Salary</td>
<td>28</td>
<td>32</td>
<td>16</td>
</tr>
<tr>
<td>Prestige</td>
<td>32</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Job Security</td>
<td>8</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>Enjoy the Work</td>
<td>28</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Score</td>
<td>171</td>
<td>139</td>
<td>139</td>
</tr>
</tbody>
</table>

The analyst position in Chicago is recommended. The overall scores for the accountant position in Denver and the auditor position in Houston are the same. There is no clear second choice between the two positions.
10. Kenyon Manufacturing Plant Location

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
<th>Georgetown Kentucky</th>
<th>Marysville Ohio</th>
<th>Clarksville Tennessee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land Cost</td>
<td>4</td>
<td>7</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Labor Cost</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Labor Availability</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Construction Cost</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Transportation</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Access to Customers</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Long Range Goals</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scoring Calculations</th>
<th>Georgetown Kentucky</th>
<th>Marysville Ohio</th>
<th>Clarksville Tennessee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land Cost</td>
<td>28</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Labor Cost</td>
<td>18</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>Labor Availability</td>
<td>35</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Construction Cost</td>
<td>24</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>Transportation</td>
<td>15</td>
<td>21</td>
<td>12</td>
</tr>
<tr>
<td>Access to Customers</td>
<td>30</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>Long Range Goals</td>
<td>28</td>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

Score

178 184 151

Marysville, Ohio (184) is the leading candidate. However, Georgetown, Kentucky is a close second choice (178). Kenyon Management may want to review the relative advantages and disadvantages of these two locations one more time before making a final decision.

11. Myrtle Beach Smokey Branson

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Myrtle Beach South Carolina</th>
<th>Smokey Mountains</th>
<th>Branson Missouri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Distance</td>
<td>10</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Vacation Cost</td>
<td>25</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Entertainment Available</td>
<td>21</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Outdoor Activities</td>
<td>18</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Unique Experience</td>
<td>24</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>Family Fun</td>
<td>40</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

Score

138 131 127

Myrtle Beach is the recommended choice.

12. Midwestern State College Handover Techmec

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Midwestern University</th>
<th>State College at Newport</th>
<th>Handover College</th>
<th>Techmec State</th>
</tr>
</thead>
<tbody>
<tr>
<td>School Prestige</td>
<td>24</td>
<td>18</td>
<td>21</td>
<td>15</td>
</tr>
<tr>
<td>Number of Students</td>
<td>12</td>
<td>20</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>Average Class Size</td>
<td>20</td>
<td>25</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>Cost</td>
<td>25</td>
<td>40</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>Distance From Home</td>
<td>14</td>
<td>16</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Sports Program</td>
<td>36</td>
<td>20</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>Housing Desirability</td>
<td>24</td>
<td>20</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>Beauty of Campus</td>
<td>15</td>
<td>9</td>
<td>24</td>
<td>15</td>
</tr>
</tbody>
</table>

Score

170 168 190 183
Handover College is recommended. However Tecumseh State is the second choice and is less expensive than Handover. If cost becomes a constraint, Tecumseh State may be the most viable alternative.

### 13.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Park Shore</th>
<th>The Terrace</th>
<th>Gulf View</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>25</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Location</td>
<td>28</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>Appearance</td>
<td>35</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>Parking</td>
<td>10</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Floor Plan</td>
<td>32</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>Swimming Pool</td>
<td>7</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>View</td>
<td>15</td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>Kitchen</td>
<td>32</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>Closet Space</td>
<td>18</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>

**Score**  
- Park Shore: 202
- The Terrace: 176
- Gulf View: 192

Park Shore is the preferred condominium.

### 14. a.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>220 Bowrider</th>
<th>230 Overnight</th>
<th>240 Sundancer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>40</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Overnight Capability</td>
<td>6</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>Kitchen/Bath Facilities</td>
<td>2</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Appearance</td>
<td>35</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Engine/Speed</td>
<td>30</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Towing/Handling</td>
<td>32</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>Maintenance</td>
<td>28</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>Resale Value</td>
<td>21</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

**Score**  
- 220 Bowrider: 194
- 230 Overnight: 181
- 240 Sundancer: 144

Clark Anderson prefers the 220 Bowrider.

### 14. b.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>220 Bowrider</th>
<th>230 Overnight</th>
<th>240 Sundancer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>21</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>Overnight Capability</td>
<td>5</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>Kitchen/Bath Facilities</td>
<td>5</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>Appearance</td>
<td>20</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Engine/Speed</td>
<td>8</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Towing/Handling</td>
<td>16</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Maintenance</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Resale Value</td>
<td>10</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

**Score**  
- 220 Bowrider: 91
- 230 Overnight: 130
- 240 Sundancer: 144

Julie Anderson prefers the 240 Sundancer.

### 15. Synthesization

Step 1: Column totals are 8, 10/3, and 7/4

Step 2:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Accord</th>
<th>Saturn</th>
<th>Cavalier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord</td>
<td>1/8</td>
<td>1/10</td>
<td>1/7</td>
</tr>
<tr>
<td>Saturn</td>
<td>3/8</td>
<td>3/10</td>
<td>2/7</td>
</tr>
<tr>
<td>Cavalier</td>
<td>4/8</td>
<td>6/10</td>
<td>4/7</td>
</tr>
</tbody>
</table>
Step 3:

<table>
<thead>
<tr>
<th></th>
<th>Accord</th>
<th>Saturn</th>
<th>Cavalier</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord</td>
<td>0.125</td>
<td>0.100</td>
<td>0.143</td>
<td>0.123</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.375</td>
<td>0.300</td>
<td>0.286</td>
<td>0.320</td>
</tr>
<tr>
<td>Cavalier</td>
<td>0.500</td>
<td>0.600</td>
<td>0.571</td>
<td>0.557</td>
</tr>
</tbody>
</table>

Consistency Ratio

Step 1:

\[
\begin{bmatrix} 1 \\ 0.123 \\ 0.375 \\ 0.500 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0.300 \\ 0.600 \end{bmatrix} + \begin{bmatrix} 1/4 \\ 0.143 \\ 0.286 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0.557 \\ 0.571 \end{bmatrix}
\]

Step 2:

\[
\frac{0.369}{0.123} = 3.006
\]

\[
\frac{0.967}{0.320} = 3.019
\]

\[
\frac{1.688}{0.557} = 3.030
\]

Step 3:

\[
\lambda_{\text{max}} = \frac{3.006 + 3.019 + 3.030}{3} = 3.02
\]

Step 4:

\[
\text{CI} = \frac{3.02 - 3}{2} = 0.010
\]

Step 5:

\[
\text{CR} = \frac{0.010}{0.58} = 0.016
\]

Since CR = 0.016 is less than 0.10, the degree of consistency exhibited in the pairwise comparison matrix for price is acceptable.

16. Synthesization

Step 1: Column totals are 17/4, 31/21, and 12

Step 2:

<table>
<thead>
<tr>
<th>Style</th>
<th>Accord</th>
<th>Saturn</th>
<th>Cavalier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord</td>
<td>4/17</td>
<td>7/31</td>
<td>4/12</td>
</tr>
<tr>
<td>Saturn</td>
<td>12/17</td>
<td>21/31</td>
<td>7/12</td>
</tr>
<tr>
<td>Cavalier</td>
<td>1/17</td>
<td>3/31</td>
<td>1/12</td>
</tr>
</tbody>
</table>

Step 3:

<table>
<thead>
<tr>
<th></th>
<th>Accord</th>
<th>Saturn</th>
<th>Cavalier</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord</td>
<td>0.235</td>
<td>0.226</td>
<td>0.333</td>
<td>0.265</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.706</td>
<td>0.677</td>
<td>0.583</td>
<td>0.656</td>
</tr>
<tr>
<td>Cavalier</td>
<td>0.059</td>
<td>0.097</td>
<td>0.083</td>
<td>0.080</td>
</tr>
</tbody>
</table>

Consistency Ratio
Step 1:

\[
\begin{bmatrix}
1 & 1/3 & 4 \\
0.265 & 0.656 & +0.080 \\
0.795 & 0.320 & 0.802 \\
0.066 & 0.094 & 0.080 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1/4 & 1/7 & 1/1 \\
0.265 & 0.656 & +0.080 \\
0.795 & 0.320 & 0.802 \\
0.066 & 0.094 & 0.080 \\
\end{bmatrix}
\]

Step 2:

\[
\frac{0.802}{0.265} = 3.028 \\
\frac{2.007}{0.656} = 3.062 \\
\frac{0.239}{0.080} = 3.007 \\
\]

Step 3:

\[
\lambda_{\text{max}} = \frac{3.028 + 3.062 + 3.007}{3} = 3.032 \\
\]

Step 4:

\[
\text{CI} = \frac{3.032 - 3}{2} = 0.016 \\
\]

Step 5:

\[
\text{CR} = \frac{0.016}{0.58} = 0.028 \\
\]

Since CR = 0.028 is less than 0.10, the degree of consistency exhibited in the pairwise comparison matrix for style is acceptable.

17. a.

<table>
<thead>
<tr>
<th>Reputation</th>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>School B</td>
<td>1/6</td>
<td>1</td>
</tr>
</tbody>
</table>

b. Step 1: Column totals are 7/6 and 7

Step 2:

<table>
<thead>
<tr>
<th>Reputation</th>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>6/7</td>
<td>6/7</td>
</tr>
<tr>
<td>School B</td>
<td>1/7</td>
<td>1/7</td>
</tr>
</tbody>
</table>

Step 3:

<table>
<thead>
<tr>
<th>Reputation</th>
<th>School A</th>
<th>School B</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>0.857</td>
<td>0.857</td>
<td>0.857</td>
</tr>
<tr>
<td>School B</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
</tr>
</tbody>
</table>

18. a. Step 1: Column totals are 47/35, 19/3, 11

Step 2:

<table>
<thead>
<tr>
<th>Desirability</th>
<th>City 1</th>
<th>City 2</th>
<th>City 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>City 1</td>
<td>35/47</td>
<td>15/19</td>
<td>7/11</td>
</tr>
<tr>
<td>City 2</td>
<td>7/47</td>
<td>3/19</td>
<td>3/11</td>
</tr>
</tbody>
</table>
Step 3:

<table>
<thead>
<tr>
<th>City 3</th>
<th>5/47</th>
<th>1/19</th>
<th>1/11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desirability</td>
<td>City 1</td>
<td>City 2</td>
<td>City 3</td>
</tr>
<tr>
<td>City 1</td>
<td>0.745</td>
<td>0.789</td>
<td>0.636</td>
</tr>
<tr>
<td>City 2</td>
<td>0.149</td>
<td>0.158</td>
<td>0.273</td>
</tr>
<tr>
<td>City 3</td>
<td>0.106</td>
<td>0.053</td>
<td>0.091</td>
</tr>
</tbody>
</table>

b. Step 1:

\[
\begin{bmatrix}
1 & 5 & 7 \\
0.724 & 1/5 & + 0.193 & 1 & + 0.083 & 3 \\
1/7 & 1/3 & 1
\end{bmatrix}
\begin{bmatrix}
0.723 \\
0.145 \\
0.103
\end{bmatrix}
+ 0.965 + 0.193 + 0.249 = 2.273
\begin{bmatrix}
0.581 \\
0.083 \\
0.251
\end{bmatrix}
\]

Step 2: 2.273/0.724 = 3.141
0.588/0.193 = 3.043
0.251/0.083 = 3.014

Step 3: \(\lambda_{\text{max}} = (3.141 + 3.043 + 3.014)/3 = 3.066\)

Step 4: CI = (3.066 - 3)/2 = 0.033

Step 5: CR = 0.033/0.58 = 0.057

Since CR = 0.057 is less than 0.10, the degree of consistency exhibited in the pairwise comparison matrix is acceptable.

19. a. Step 1: Column totals are 4/3 and 4

Step 2:

\[
\begin{array}{c|c|c}
A & B \\
\hline
A & 3/4 & 3/4 \\
B & 1/4 & 1/4 \\
\end{array}
\]

Step 3:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>B</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

b. The individual's judgements could not be inconsistent since there are only two programs being compared.

20. a.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>1/3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>1/2</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>
b. Step 1: Column totals are 11/6, 21/5, and 8

Step 2:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6/11</td>
<td>15/21</td>
<td>2/8</td>
</tr>
<tr>
<td>B</td>
<td>2/11</td>
<td>5/21</td>
<td>5/8</td>
</tr>
<tr>
<td>C</td>
<td>3/11</td>
<td>1/21</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Step 3:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.545</td>
<td>0.714</td>
<td>0.250</td>
<td>0.503</td>
</tr>
<tr>
<td>B</td>
<td>0.182</td>
<td>0.238</td>
<td>0.625</td>
<td>0.348</td>
</tr>
<tr>
<td>C</td>
<td>0.273</td>
<td>0.048</td>
<td>0.125</td>
<td>0.148</td>
</tr>
</tbody>
</table>

c. Step 1:

\[
\begin{bmatrix}
0.503 \\
0.168 \\
0.252
\end{bmatrix}
\begin{bmatrix}
1/3 \\
3 \\
1/2
\end{bmatrix}
+
\begin{bmatrix}
0.348 \\
1 \\
1/5
\end{bmatrix}
\begin{bmatrix}
2 \\
5 \\
1
\end{bmatrix}
\]

Weighted Sum:

\[
\begin{bmatrix}
0.503 \\
0.168 \\
0.252
\end{bmatrix}
\begin{bmatrix}
1.044 \\
0.348 \\
0.070
\end{bmatrix}
+
\begin{bmatrix}
0.296 \\
0.740 \\
0.148
\end{bmatrix}
= 
\begin{bmatrix}
1.845 \\
1.258 \\
0.470
\end{bmatrix}
\]

Step 2: 

1.845/0.503 = 3.668  
1.258/0.348 = 3.615  
0.470/0.148 = 3.123

Step 3: 

\[\lambda_{\text{max}} = (3.668 + 3.615 + 3.123)/3 = 3.469\]

Step 4: 

CI = (3.469 - 3)/2 = 0.235

Step 5: 

CR = 0.235/0.58 = 0.415

Since CR = 0.415 is greater than 0.10, the individual's judgements are not consistent.

21. a.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1/2</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>1/5</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

b. Step 1: Column totals are 16/5, 17/10, and 11

Step 2:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5/16</td>
<td>5/17</td>
<td>5/11</td>
</tr>
<tr>
<td>B</td>
<td>10/16</td>
<td>10/17</td>
<td>5/11</td>
</tr>
</tbody>
</table>
Step 3:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.313</td>
<td>0.294</td>
<td>0.455</td>
<td>0.354</td>
</tr>
<tr>
<td>B</td>
<td>0.625</td>
<td>0.588</td>
<td>0.455</td>
<td>0.556</td>
</tr>
<tr>
<td>C</td>
<td>0.063</td>
<td>0.118</td>
<td>0.091</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Step 1:  

\[
\begin{bmatrix}
1 \\
0.354 \\
0.708 \\
0.071
\end{bmatrix} \times \begin{bmatrix}
1/2 \\
1 \\
1/5 \\
1/5
\end{bmatrix} = \begin{bmatrix}
5 \\
5 \\
1 \\
1
\end{bmatrix} 
\]

\[
\begin{bmatrix}
0.354 \\
0.708 \\
0.071
\end{bmatrix} + \begin{bmatrix}
0.278 \\
0.556 \\
0.111
\end{bmatrix} + \begin{bmatrix}
0.450 \\
0.450 \\
0.090
\end{bmatrix} = \begin{bmatrix}
1.083 \\
1.715 \\
0.272
\end{bmatrix} 
\]

Step 2:  

\[
\frac{1.083}{0.354} = 3.063 \\
\frac{1.715}{0.556} = 3.085 \\
\frac{0.272}{0.090} = 3.014
\]

Step 3:  

\[
\lambda_{\text{max}} = \frac{(3.063 + 3.085 + 3.014)}{3} = 3.054
\]

Step 4:  

\[
CI = \frac{(3.054 - 3)}{2} = 0.027
\]

Step 5:  

\[
CR = \frac{0.027}{0.58} = 0.046
\]

Since CR = 0.046 is less than 0.10, the individual's judgements are consistent.

22. a. Let  

<table>
<thead>
<tr>
<th>Location</th>
<th>D</th>
<th>S</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1</td>
<td>1/4</td>
<td>1/7</td>
</tr>
<tr>
<td>S</td>
<td>4</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

b. Step 1: Column totals are 12, 17/4, and 31/21

Step 2:  

<table>
<thead>
<tr>
<th>Location</th>
<th>D</th>
<th>S</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1/12</td>
<td>1/17</td>
<td>3/31</td>
</tr>
<tr>
<td>S</td>
<td>4/12</td>
<td>4/17</td>
<td>7/31</td>
</tr>
<tr>
<td>N</td>
<td>7/12</td>
<td>12/17</td>
<td>21/31</td>
</tr>
</tbody>
</table>
Step 3:

<table>
<thead>
<tr>
<th>Location</th>
<th>D</th>
<th>S</th>
<th>N</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.083</td>
<td>0.059</td>
<td>0.097</td>
<td>0.080</td>
</tr>
<tr>
<td>S</td>
<td>0.333</td>
<td>0.235</td>
<td>0.226</td>
<td>0.265</td>
</tr>
<tr>
<td>N</td>
<td>0.583</td>
<td>0.706</td>
<td>0.677</td>
<td>0.656</td>
</tr>
</tbody>
</table>

c. Step 1:

\[
\begin{bmatrix}
0.080 & 4 + 0.265 & 1 + 0.656 & 1/7 \\
0.320 & 3 + 0.265 & 0.094 & 2.007 \\
0.560 & 1/4 + 0.265 & 0.656 & 1/3 \\
\end{bmatrix}
\]

Step 2:

\[
\frac{0.239}{0.080} = 3.007 \\
\frac{0.802}{0.265} = 3.028 \\
\frac{2.007}{0.656} = 3.062
\]

Step 3:

\[
\lambda_{\text{max}} = \frac{(3.007 + 3.028 + 3.062)}{3} = 3.035
\]

Step 4:

CI = \frac{(3.035 - 3)}{2} = 0.017

Step 5:

CR = \frac{0.017}{0.58} = 0.028

Since CR = 0.028 is less than 0.10, the manager's judgements are consistent.

23. a. Step 1: Column totals are 94/21, 33/4, 18, and 21/12

Step 2:

<table>
<thead>
<tr>
<th>Performance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21/94</td>
<td>12/33</td>
<td>7/18</td>
<td>4/21</td>
</tr>
<tr>
<td>2</td>
<td>7/94</td>
<td>4/33</td>
<td>4/18</td>
<td>3/21</td>
</tr>
<tr>
<td>3</td>
<td>3/94</td>
<td>1/33</td>
<td>1/18</td>
<td>2/21</td>
</tr>
<tr>
<td>4</td>
<td>63/94</td>
<td>16/33</td>
<td>6/18</td>
<td>12/21</td>
</tr>
</tbody>
</table>

Step 3:

<table>
<thead>
<tr>
<th>Performance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Row Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.223</td>
<td>0.364</td>
<td>0.389</td>
<td>0.190</td>
<td>0.292</td>
</tr>
<tr>
<td>2</td>
<td>0.074</td>
<td>0.121</td>
<td>0.222</td>
<td>0.143</td>
<td>0.140</td>
</tr>
<tr>
<td>3</td>
<td>0.032</td>
<td>0.030</td>
<td>0.056</td>
<td>0.095</td>
<td>0.053</td>
</tr>
<tr>
<td>4</td>
<td>0.670</td>
<td>0.485</td>
<td>0.333</td>
<td>0.571</td>
<td>0.515</td>
</tr>
</tbody>
</table>
b. Step 1:

\[
\begin{bmatrix}
0.292 \\
0.097 \\
0.042 \\
0.876 \\
\end{bmatrix}
+ 0.420 \\
0.140 \\
0.035 \\
0.560 \\
\begin{bmatrix}
0.371 \\
0.212 \\
0.053 \\
0.515 \\
\end{bmatrix}
+ 0.172 \\
0.129 \\
0.086 \\
0.318 \\
\begin{bmatrix}
1.257 \\
0.579 \\
0.216 \\
2.270 \\
\end{bmatrix}
\]

Step 2: 

\[
\frac{1.257}{0.292} = 4.305 \\
\frac{0.579}{0.140} = 4.136 \\
\frac{0.216}{0.053} = 4.075 \\
\frac{2.270}{0.515} = 4.408
\]

Step 3: 

\[\lambda_{\text{max}} = \frac{(4.305 + 4.136 + 4.075 + 4.408)}{4} = 4.231\]

Step 4: 

\[CI = \frac{(4.231 - 4)}{3} = 0.077\]

Step 5: 

\[CR = \frac{0.077}{0.90} = 0.083\]

Since CR = 0.083 is less than 0.10, the judgements are consistent.

24. a. Criteria: Yield and Risk

Step 1: 

Column totals are 1.5 and 3

Step 2:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Yield</th>
<th>Risk</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>0.667</td>
<td>0.667</td>
<td>0.667</td>
</tr>
<tr>
<td>Risk</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>

With only two criteria, CR = 0 and no computation of CR is made.

The same calculations for the Yield and the Risk pairwise comparison matrices provide the following:

<table>
<thead>
<tr>
<th>Stocks</th>
<th>Yield Priority</th>
<th>Risk Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCC</td>
<td>0.750</td>
<td>0.333</td>
</tr>
<tr>
<td>SRI</td>
<td>0.250</td>
<td>0.667</td>
</tr>
</tbody>
</table>

b. Overall Priorities:

\[
\text{CCC: } 0.667(0.750) + 0.333(0.333) = 0.611 \\
\text{SRI: } 0.667(0.250) + 0.333(0.667) = 0.389
\]
CCC is preferred.

25. a. Criteria: Leadership, Personal, Administrative

Step 1: Column Totals are 8, 11/6 and 13/4

Step 2:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Leader</th>
<th>Personal</th>
<th>Administrative</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leadership</td>
<td>0.125</td>
<td>0.182</td>
<td>0.077</td>
<td>0.128</td>
</tr>
<tr>
<td>Personal</td>
<td>0.375</td>
<td>0.545</td>
<td>0.615</td>
<td>0.512</td>
</tr>
<tr>
<td>Administrative</td>
<td>0.500</td>
<td>0.273</td>
<td>0.308</td>
<td>0.360</td>
</tr>
</tbody>
</table>

CR = 0.094 if computed.

The same calculations for the leadership, personal and administrative pairwise comparison matrices provide the following.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Leadership</th>
<th>Personal</th>
<th>Administrative</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacobs</td>
<td>0.800</td>
<td>0.250</td>
<td>0.667</td>
<td></td>
</tr>
<tr>
<td>Martin</td>
<td>0.200</td>
<td>0.750</td>
<td>0.333</td>
<td></td>
</tr>
</tbody>
</table>

b. Overall Priorities:

Jacobs 0.128(0.800) + 0.512(0.250) + 0.360(0.667) = 0.470
Martin 0.128(0.200) + 0.512(0.250) + 0.360(0.333) = 0.530

Martin is preferred.

26. a. Criteria: Price, Sound and Reception

Step 1: Column totals are 19/12, 13/3 and 8

Step 2:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Price</th>
<th>Sound</th>
<th>Reception</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0.632</td>
<td>0.692</td>
<td>0.500</td>
<td>0.608</td>
</tr>
<tr>
<td>Sound</td>
<td>0.211</td>
<td>0.231</td>
<td>0.375</td>
<td>0.272</td>
</tr>
<tr>
<td>Reception</td>
<td>0.158</td>
<td>0.077</td>
<td>0.125</td>
<td>0.120</td>
</tr>
</tbody>
</table>

CR = 0.064

The same calculations for the price, sound and reception pairwise comparison matrices provide the following:

<table>
<thead>
<tr>
<th>System</th>
<th>Price Priority</th>
<th>Sound Priority</th>
<th>Reception Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>System A</td>
<td>0.557</td>
<td>0.137</td>
<td>0.579</td>
</tr>
<tr>
<td>System B</td>
<td>0.123</td>
<td>0.239</td>
<td>0.187</td>
</tr>
<tr>
<td>System C</td>
<td>0.320</td>
<td>0.623</td>
<td>0.234</td>
</tr>
<tr>
<td>CR</td>
<td>0.016</td>
<td>0.016</td>
<td>0.046</td>
</tr>
</tbody>
</table>
b. Overall Priorities:

System A  \(0.608(0.557) + 0.272(0.137) + 0.120(0.579) = 0.446\)
System B  \(0.608(0.123) + 0.272(0.239) + 0.120(0.187) = 0.162\)
System C  \(0.608(0.320) + 0.272(0.623) + 0.120(0.046) = 0.392\)

System A is preferred.