Abstracts

Moduli of Differentials

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Let $\mu = (m_1, \ldots, m_n)$ be a collection of integers such that $\sum_{i=1}^n m_i = 2g-2$. Let $\mathbb{P}\mathcal{H}(\mu)$ be the projectivized stratum parameterizing differentials ω (up to scale) on genus g Riemann surfaces such that the associated divisor of ω is of zero and pole type μ, i.e. $(\omega) = \sum_{i=1}^n m_i p_i$ for distinct points $p_1, \ldots, p_n$. In this talk we study some aspects of $\mathbb{P}\mathcal{H}(\mu)$ from the viewpoint of algebraic geometry and compare them to the moduli space of pointed genus g curves $\mathcal{M}_{g,n}$.

First we consider complete subvarieties. Diaz ([7]) first proved that any complete subvariety in $\mathcal{M}_g$ has dimension bounded above by $g-2$. The bound is not known to be sharp in general. In particular we do not know whether $\mathcal{M}_4$ contains a complete surface. For the case of differentials, the author ([3]) proved that the strata of meromorphic differentials (i.e. some $m_i < 0$) do not contain any complete curve. It remains to be an open question whether the strata of holomorphic differentials can contain a complete curve.

Next we consider tautological rings. The tautological ring of $\mathcal{M}_{g,n}$ is generated by Mumford's κ_j classes and the ψ_i classes associated to each marked point. In general the tautological ring structure of $\mathcal{M}_{g,n}$ can be complicated. We define the tautological ring of the strata $\mathbb{P}\mathcal{H}(\mu)$ to be generated by the tautological classes pulled back from $\mathcal{M}_{g,n}$ together with the tautological line bundle class $\eta = c_1(\mathcal{O}(-1))$ of the projective Hodge bundle. The author ([4]) proved that the tautological ring of the strata of differentials without simple poles (i.e. all $m_i \neq -1$) is generated by η only. It remains to be an open question which powers of η vanish on the strata.

Finally we consider positivity of divisor classes. Cornalba and Harris ([6]) proved that $a\lambda - b\delta$ is ample on the Deligne-Mumford compactification $\overline{\mathcal{M}}_g$ if and only if $a > 11b > 0$, where λ is the first Chern class of the Hodge bundle and δ is the total boundary divisor class. The author ([5]) proved that $a(\kappa_1 + \psi) - b\eta$ is ample on the compactified stratum $\mathbb{P}\overline{\mathcal{H}}_{g,n}(\mu)$ (in the sense of [1]) if and only if $a > b > 0$, where $\psi = \sum_{i=1}^n \psi_i$ is the total ψ class. It remains open to study positivity of other divisor classes on the compactified strata.

We remark that the above results and questions can be generalized to the strata of k-differentials for all k ([2, 3, 4, 5]).

References

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