Errata for *An Introduction To Analysis, Second Edition*


This list was last updated on November 10, 2018, and is maintained by G. E. Keough, keough@bc.edu, who would greatly appreciate any reports of inaccuracies in the text.

First Printing.

*Items are listed in order by page number.*

7  Definition 1.2.1 is missing an important second condition, that for every \( a \in A \), there exists \( b \in B \) for which \( (a, b) \in f \). Surprisingly, this error was present in the First Edition, but it was never reported.

14  Definition 1.3.2(b) should end with *see Theorem 1.5.16* (not 1.5.14).

50  Exercise 8(c) was intended to be the sequence \( \{(1 + 1/n)^n\} \). This will make the hint more meaningful!

A student may ask “what is the limit?” At this point, while we do not want to prove anything more than existence, we expect to add the suggestion “For a discussion on this limit, see Exercise 11 of Exercise Set 5.4.”

72  Line 5 of Paragraph 2: The indicated limit should be \( \lim_{x \to 0} \sin x/x = 1 \) (not the limit as \( n \to \infty \)).

72  The displayed equation 10 lines from the bottom of the page is missing an equal sign and should read

\[
|x - 3| < \delta = \varepsilon/5 \text{ implies } |f(x) - L| = 5|x - 3| < 5 \cdot \frac{\varepsilon}{5} = \varepsilon.
\]

93  Exercise 6 has a “0” where “\( f(0) \)” should appear. It should end with “For each, construct a sequence \( \{x_n\} \) such that \( x_n \to 0 \) and \( f(x_n) \neq f(0) \).” Note. Many different sequences \( \{x_n\} \) can be constructed as a solution, and it is not required that \( \{f(x_n)\} \) actually converge.

110  The heading “Sequences in \( \mathbb{R}^2 \)” should be left-aligned.

111  Definition 3.7.5 contains a one-character typesetting error and an important omission at the very end, and should be corrected to read: “... the limit of \( f \) as \( p \) goes to \( p_0 \) ... if, for any \( \varepsilon > 0 \), there is a \( \delta > 0 \) such that \( |f(p) - L| < \varepsilon \) for all \( p \in D \) satisfying \( 0 < \|p - p_0\| < \delta \).

Note. Definition 3.7.5 does not require that \( f \) be defined in a deleted neighborhood of \( p \), although such a requirement is enforced in the one-variable case of Definition 3.1.1 on page 71.

116  Exercise 4: So that the student might correctly discover that Cauchy had no sons (as indicated on page 319), the following computation should be added as a sixth part of the Exercise (*preferably as the first part*):

\[
\lim_{(x,y) \to (0,0)} \frac{x^2y^2}{x^4 + y^4}
\]

126  Exercises 13(b) and 13(c) require that the product and chain rules learned in Calculus be assumed in advance of Section 4.2. Although
consistent with the introduction to the Chapter which stated that the reader “is probably already quite familiar with the results of this Chapter,” the reader may have the impression that all derivatives in this exercise set must be computed from definition, a task none of the authors had intended a student execute.

140 Exercise 24(b) should end with “. . . that \( f\left(\frac{1}{(2n)}\right) > f\left(\frac{1}{(2n - 1)}\right)\).”

140 The parenthetical definition of a strict local minimum in Exercise 26(a) should read “\( f(x) > f(c) \) for \( x \) in a deleted neighborhood of \( c \).”

159 Exercise 2(a) as stated contains minor typesetting errors in (ii) and (iii), and may also lead to some confusion. The suggested rewriting of part (a), followed by a relabelling of parts (b) and (c) is as follows:

(a) Show that \( \nabla f(0, 0) = 0 \).

(b) Compute \( \lim_{\lambda \to 0^+} \frac{f((a, b) + \lambda u)}{\lambda} \) in each of the following cases:

(i) when \( u = (u_1, u_2) \) has at least one of \( u_1 \leq 0 \) or \( u_2 \leq 0 \).

(ii) when \( u = (u_1, u_2) \) has both \( u_1 > 0 \) and \( u_2 > 0 \).

(c) Show that \( D_u f(0, 0) = \nabla f(0, 0) \cdot u \), for all unit vectors \( u \).

(d) Show that \( f \) is not differentiable at \( (0, 0) \).

161 Exercise 10(b) should more sensibly begin “Suppose that . . . ” rather than “Let . . . ”

175 The last displayed inequality is not incorrect, but should, in context, be written with strict inequalities on the outside, but a simple \( \leq \) is the center:

\[
\int_a^b f + \int_a^b g - \varepsilon < \int_a^b (f + g) \leq \int_a^b f + \int_a^b g + \varepsilon.
\]

The same is true for the preceding inequality, which follows from (5.2.2):

\[
\int_a^b (f + g) \leq U_P(f + g) \leq U_P(f) + U_P(g) < \int_a^b f + \int_a^b g + \varepsilon.
\]

176 The symbol \( m_J(f) \) appearing on the last line of the page should be defined as \( m_J(f) = \text{glb}_J f(x) \) (not \( \text{lub}_J f(x) \) as shown).

188 The statement of Exercise 2 should be “Give reasons, referring to theorems of this section, why each of the following functions is integrable on the indicated intervals.”

190 The paragraph at the top of the page (the continuation of Exercise 12 from the previous page) should be “outdented,” in the sense that it defines a rubric and notation that follows the completion of part (c), setting the stage prior to attempting part (d).

191 The reference on line –19 should be to “part (a) of Theorem 4.3.15” (not Theorem 4.4.4).

211 The domain for Exercise 9 is incorrect in the \( y \)-dimension. It should be \([1, 2] \times [0, 4]\).

211 The last displayed equation is missing a right parenthesis in the first integrand:

\[
= \left[ \int_c^u (f(x, y) - f(x_0, y)) \, dy \right] + \left[ \int_{u_0}^u f(x_0, y) \, dy \right]
\]
232 Exercise 19 should begin “Suppose that \( \{a_n\} \) is . . .”

233 Definition 6.3.1 should be understood to assert that each of the expressions \( \sum_{n=0}^{\infty} a_n(x-x_0)^n \) and \( \sum_{n=0}^{\infty} a_n x^n \) represents a notational convenience for writing \( a_0 + a_1(x-x_0) + \ldots \) and \( a_0 + a_1 x + \ldots \), respectively. There should be no confusion in what follows about the meaning of either \( \sum_{n=0}^{\infty} a_n(x-x_0)^n \) when \( x = x_0 \), or \( \sum_{n=0}^{\infty} a_n x^n \) when \( x = 0 \).

245 Exercise 8(e) may confuse the reader, given the language of the exercise. If \( \sum_{n=0}^{\infty} n^5 a_n 3^n \) converges, then it is easy to show that \( \sum_{n=0}^{\infty} a_n 3^n / n^5 \) converges. No conclusion can be drawn about the radius of convergence since the assertions are untenable.

245 Exercise 10 contains the assumption that \( 0 < R_1 < R_2 < \infty \). While not technically an error, this assumption will confuse the intention of the exercise which goes on to ask about the radius of convergence of \( \sum (a_n + b_n)x^n \), possibly in terms of \( \max\{R_1, R_2\} \) and \( \min\{R_1, R_2\} \) – when, in fact, these quantities are known to be \( R_2 \) and \( R_1 \), respectively. The intended exercise should assume simply that \( 0 < R_1, R_2 < \infty \), which then makes Exercise 10 both connected with the possible conclusions and consistent with the spirit of Exercise 11 that follows.

249 The third line from the bottom should read

\[
= - \frac{1}{m!} f^{(m)}(x_0) (x-x_0)^m
\]

270 The hint in Exercise 27 should reference Theorem 2.6.4, not Theorem 3.6.8.

315 The answer(s) for 11(b) should be \([-1, 1]\) and \((-\infty, \infty)\).