Questions:
• Disney had the option to purchase 100% of NBC television from GE for $5 billion. Instead, Disney acquired ABC (and majority stake in ESPN, which now accounts for 15% of Disney’s profits) for $19 billion. How did they make this decision?
• Delta reduced capital spending by $500 million when it cut its order of Boeing 737-800s by 20 planes. Was this a good decision?
• Construction of tunnel under English Channel cost about $15 billion from 1986 to 1994. Was this a good investment?
• Should Intel retrofit an existing plant or build a new plant?
• Should the Red Sox replace Fenway Park?

CAPITAL BUDGETING

Objectives:
• Overview of Capital Budgeting
• Introduction to NPV
• NPV vs. IRR vs. Payback Rule
• Mutual Exclusive Projects
  – Equivalent Annual Annuities
  – Investment Timing and Replacement Decisions

Introduction to Capital Budgeting

Capital Investment Project
• “Any expenditure made in the hope of generating more cash later”

What is Capital Budgeting?
• Process firms use to determine which capital investment project(s) to undertake and how much to invest in each project.
• Involves measuring the incremental cash flows associated with a capital investment project and evaluating the attractiveness of those incremental cash flows to the project’s cost.
• Many corporate decisions are capital budgeting decisions
How Should Firms Make Capital Budgeting Decisions?

- As we’ll see, firms should use NPV to evaluate projects.
- The other decision rules that we consider can lead to erroneous decisions and typically are not any easier to implement.

Steps of Capital Budgeting Process

1. Forecast the projects’ cash flows
2. Estimate the opportunity cost of capital $=$
   \[= \text{expected rate of return given up by investing in the project} = \]
   \[= \text{required rate of return} \]
3. Evaluate the projects using one of the following methods
   1) Payback Period
   2) Net Present Value (NPV)
   3) Internal Rate of Return (IRR)
   4) Profitability Index (PI)
   5) Book (Accounting) Rate of Return
4. Select a project using one of these selection rules.

Introduction to Net Present Value (NPV)

You’re in the real estate business and considering construction of an office building. The land would cost $50,000 and construction would cost an additional $320,000. You predict that you’ll sell the office for $420,000 one year from now. Should you construct the office?

Assume that the $420,000 payoff is guaranteed and that the nominal rate of return on 1-year Treasury notes is 5%. What is the present value of receiving $420,000 one year from now?

\[
pV = \frac{CF_1}{(1 + r)^1} = \frac{420,000}{(1 + .05)^1} = 400,000
\]

Since the PV of office’s value next year exceeds the required investment of $370,000 \(\text{office building project is value increasing!}\)

This is the first example of a project with a positive NPV.

Net Present Value (NPV) is a decision criteria that takes into account all of the (incremental) costs and benefits associated with a project.

\[ NPV = PV(\text{payoffs}) - PV(\text{required investment}) \]
Present Value vs. Market Value

Once you’ve purchased the land and laid out the money for construction, what is the least you would sell office building for today?

\[
\text{Market Value (Office)} = \text{PV(Office)} = \frac{420,000}{1.05} = 400,000
\]

The market value of the office building today is the highest price that the buyer is willing to pay and the lowest price that you are willing to accept. (You could sell for less than the present value, but why would you choose to do so?)

*In this example, investing $370,000 today earns you an immediate return of $30,000. Is this realistic? What are we assuming?*

Previously, we assumed that we could definitely sell the office building for $420,000 next year. But what if the $420,000 sales price is uncertain? If the office building project is just as risky as an investment in the stock market (with a required rate of return of 12 percent), then you should use a discount rate that reflects this risk. Let’s recalculate the NPV using a discount rate of 12%.

\[
NPV = CF_0 + \frac{CF_1}{1 + r} = -370,000 + \frac{420,000}{1.12} = -370,000 + 375,000 = 5,000 > 0
\]

Because the discount rate of 12% is less than the expected 1-year return of 13.51%, the office building project still has a positive NPV. It creates value, but less value than when there was no risk.
Consumption Opportunities
Consider the choice between investing in the office building project and simply living off the $370,000.

The NPV Rule (Good)

**Rule:** Accept a project if it has a positive NPV
Reject a project if it has a negative NPV

\[
NPV = CF_0 + \sum_{n=1}^{N} \frac{CF_n}{(1 + r)^n}
\]
where the initial outlay \( CF_0 \) is typically negative

**Recipe:**
1. Estimate incremental after-tax cash flows associated with the project
2. Estimate the initial outlay (\( CF_0 \)) required to undertake the investment
3. Estimate the correct discount rates for those cash flows (based on the rates of returns offered by equivalent-risk investments in the capital market)

**Example of the NPV Rule**

The law firm of Dewey, Cheatem, and Howe must decide whether to purchase a computer system that will help them overcharge clients.

- Initial Outlay = $30,000
- Incremental cash flows = $15,000 per year for 3 years
- Discount rate = 10% in each year (i.e. term structure is flat)
Should Dewey, Cheatem, and Howe invest in the new computer system?

\[ NPV = CF_0 + \sum_{n=1}^{N} \frac{CF_n}{(1 + r)^n} = -30,000 + \frac{15,000}{1.1} + \frac{15,000}{1.1^2} + \frac{15,000}{1.1^3} \]
\[ = 30,000 + 13,636 + 12,397 + 11,270 \]
\[ = 7,302 > 0 \Rightarrow \text{Purchase new computers} \]

**The IRR Rule (Potentially Bad)**

**Rule:** Accept a project if it has an internal rate of return (IRR) greater than the required return on investments ("hurdle rate") and reject a project if it has an IRR less than the required return on investments.

**IRR:** The internal rate of return is the single discount rate that equates the discounted value of a project’s cash flows with the initial outlay. To calculate IRR, you either need to play the Price is Right High / Low Game or use a spreadsheet or financial calculator.

\[ CF_0 + \frac{CF_1}{(1 + IRR)^1} + \frac{CF_2}{(1 + IRR)^2} + \frac{CF_3}{(1 + IRR)^3} = 0 \]

**Note:** IRR is the discount rate at which the NPV equals zero; it is unrelated to the opportunity cost of capital.

**Recipe:**

1. Estimate incremental after-tax cash flows
2. Estimate the initial outlay \((CF_0)\) required to undertake the investment
3. Find the IRR which discounts the cash flows back to the initial outlay
4. Find “Hurdle Rate” to which you can compare the IRR

**Example of the IRR Rule**

Should the law firm of Dewey, Cheatem, and Howe purchase a new computer system that will help them overcharge clients?

Cost = $30,000
Incremental cash flows = $15,000 per year for 3 years
Hurdle rate = 10% (constant over 3 years)

\[-CF_0 = \frac{CF_1}{(1 + IRR)^1} + \frac{CF_2}{(1 + IRR)^2} + \frac{CF_3}{(1 + IRR)^3} \]
\[30,000 = \frac{15,000}{(1 + .2337)^1} + \frac{15,000}{(1 + .2337)^2} + \frac{15,000}{(1 + .2337)^3} \]

\[ IRR = 23.37\% \]
Because the IRR of 23.37% is higher than our hurdle rate of 10.00%, the IRR rule tells us to accept this project.

**Why is the IRR Rule Potentially Bad?**

**Problems:**
1. Relative to NPV, IRR is difficult to calculate
2. When there are multiple negative cash flows, the IRR is not uniquely defined
3. IRR does not always exist
   - *For example, try CF₁=100, CF₂=-200, CF₃=150*
4. IRR implicitly assumes the term structure is flat. When term structure is not flat, IRR can lead to bad decisions.
5. IRR does not help you choose between mutually exclusive projects that involve significantly different initial outlays.

**The Payback Rule (Ugly!)**

**Rule:** Accept any project that has cash inflows which exceed the initial outlay within a specified number of years

**Example:** A two year payback rule would say that an investment of $5000 would have to have cash flows totaling $5000 in the first two years to be accepted.

**Back to Dewey, Cheatem, and Howe…**

1. What if the company has a 1 year payback rule? Will they buy the computer system?
2. What if the company has a 3 year payback rule? Will they buy the computer system?
3. What if the price of the computer system increases to $40,000? Will they buy the computer system (using a 3 year payback rule)?

**Why is the Payback Rule Ugly?**

**Problems:**
1) Choice of payback period is arbitrary. Rule ignores any cash flows beyond the length of the payback rule.
   - *For example, a project with CF₀ = -100, CF₁=49, CF₂=49, CF₃=101, CF₄=101 would be rejected using a 1-year or 2-year payback rule*
2) Payback rule does not discount cash flows
   - *For example, a project with CF₀ = -100, CF₁=50, and CF₂=51 would be accepted then though the present value of the payments is surely less than $100; same is true of a project with CF₀ = -100, CF₁=0, and CF₂=100*

**Example:**
“A new high-efficiency washing machine costs $400. Since it’ll save you $2 per week on laundry, the new washing machine will pay for itself in less than 4 years!
More details of Problem: Rising Term Structure

**Problems:** IRR implicitly assumes the term structure is flat. When term structure is not flat, IRR can lead to bad decisions.

Consider a project that costs $30,000 today and generates cash flows of $2000 in year 1, $2000 in year 2, and $40,000 in year 3; further assume that $r_1 = 5\%$, $r_2 = 10\%$, and $r_3 = 15\%$.

What is the NPV of this project?

$$NPV = -30,000 + \frac{2,000}{1.05} + \frac{2,000}{(1.10)^2} + \frac{40,000}{(1.15)^3} = -141.70 < 0$$

What is the IRR of this project?

$$30,000 = \frac{2,000}{(1 + IRR)} + \frac{2,000}{(1 + IRR)^2} + \frac{40,000}{(1 + IRR)^3} \Rightarrow IRR = 14.391\%$$

*If hurdle rate is less than 14.391\%, you undertake a negative NPV project*

**Problem: Size Disparity**

You are going to build a new bridge across the Columbia River and have the choice of using concrete or wood. If you build a concrete bridge, trucks will be able to use the bridge and pay lots of tolls. If you build a wooden bridge, only cars will be able to use the bridge, resulting in much lower tolls. Your opportunity cost of capital is 5 percent.

- Concrete bridge costs $1,000 to build and provides cash flows of $250 for each of the next five years.
- Wooden bridge costs $50 to build and provides cash flows of $15 for each of the next five years.

<table>
<thead>
<tr>
<th>Bridge</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>-1000</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Wooden</td>
<td>-50</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

*What are the NPVs and IRRs of the two bridges?*
What are the NPVs of the (C)oncrete and (W)ooden bridges?

\[
NPV(C) = -1000 + 250 \left[ \frac{1}{.05} - \frac{1}{.05 \cdot (1.05)^5} \right] = -1000 + 250 \cdot [4.32948] = 82.37
\]

\[
NPV(W) = 50 \cdot 15 \left[ \frac{1}{.05} - \frac{1}{.05 \cdot (1.05)^5} \right] = -50 + 15 \cdot [4.32948] = 14.94
\]

What are the IRRs of the (C)oncrete and (W)ooden bridges?

\[
IRR(C) : -1000 + 250 \left[ \frac{1}{.093} - \frac{1}{.093 \cdot (1.093)^5} \right] = 0
\]

\[
IRR(C) = 7.93\%
\]

\[
IRR(W) : -50 + 15 \left[ \frac{1}{.1524} - \frac{1}{.1524 \cdot (1.1524)^5} \right] = 0
\]

\[
IRR(W) = 15.24\%
\]

Therefore the concrete bridge has a higher NPV than the wooden bridge ($80.37 > $14.94) but a lower IRR (7.93% < 15.24%). The wooden bridge provides the higher return, but on a much smaller investment.

Which bridge should you build?

- You should build the concrete bridge because it increases your wealth (or the value of your firm) by $82.37 versus $14.94.
- Since the sizes of the initial outlays of the projects are so different, IRR is misleading. This is the size disparity problem.

Now, imagine that you can build one concrete bridge for $1000 or twenty wooden bridges for $1000, where each wooden bridge produces cash flows of $15 per year for 5 years. What are the IRR and NPV of building the twenty wooden bridges?

- IRR still 15.24% because CFs simply scaled by factor of 20
- However NPV increases to $298.84 (= $14.94 x 20)
- Since the initial outlays of the concrete and wooden bridges are now comparable, you can go with the project with the higher IRR
DECISION RULES: choosing between two projects (mutually exclusive or independent)

Basic Data

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1000</td>
<td>-1000</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>600</td>
</tr>
</tbody>
</table>

1. Payback Period

Payback Period = time until cash flows recover the initial investment of the project.
Payback A = 2 1/3 years
Payback B = 4 years

- The payback selection rule: accept the project if the payback period < cutoff
- Weaknesses of Payback:
  - Ignores the time value of money.
  - Ignores cash flows occurring after the payback period.
  - Preference of short-level projects
2. Net Present Value (NPV) Method

The net present value (NPV) of a project is the present value of its cash flows discounted at the opportunity cost of capital minus initial investment.

\[
NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - Inv_0
\]

\[
NPV_A = \frac{500}{(1.1)} + \frac{400}{(1.1)^2} + \frac{300}{(1.1)^3} + \frac{400}{(1.1)^4} + \frac{10}{(1.1)^5} + \frac{10}{(1.1)^6} - 1000
\]

\[= 455 + 331 + 225 + 68 + 6 + 6 - 1000 = 91\]

\[
NPV_B = \frac{100}{(1.1)} + \frac{200}{(1.1)^2} + \frac{300}{(1.1)^3} + \frac{400}{(1.1)^4} + \frac{500}{(1.1)^5} + \frac{600}{(1.1)^6} - 1000
\]

\[= 91 + 165 + 225 + 273 + 310 + 339 - 1000 = 403\]

- The NPV selection rule: accept the project if the NPV ≥ 0
- Therefore, if the projects are independent, accept both.
- If the projects are mutually exclusive, accept Project B since NPV_B > NPV_A.

Note: NPV declines as r increases and NPV rises as r decreases.
3. **Internal Rate of return (IRR)**

The **internal rate of return (IRR)** is the discount rate at which project NPV=0.

\[
IRR := \sum_{t=1}^{n} \frac{CF_t}{(1 + IRR)^t} - INV_0 = 0.
\]

\[
IRR_A = \frac{500}{1+R} + \frac{400}{(1+R)^2} + \frac{300}{(1+R)^3} + \frac{100}{(1+R)^4} + \frac{10}{(1+R)^5} + \frac{10}{(1+R)^6} - 1000 = 0
\]

\[
IRR_A \approx 15\%
\]

\[
IRR_B = \frac{100}{1+R} + \frac{200}{(1+R)^2} + \frac{300}{(1+R)^3} + \frac{400}{(1+R)^4} + \frac{500}{(1+R)^5} + \frac{600}{(1+R)^6} - 1000 = 0
\]

\[
IRR_B \approx 20\%
\]

- The **IRR selection rule**: accept the project if the IRR ≥ cost of capital.
- Therefore, if the projects are independent, accept both (both IRR > 10%).
- If the projects are mutually exclusive, accept Project B since IRR_B > IRR_A.

**Note**: IRR is independent of the cost of capital.
**Profitability Index**

**Profitability Index** = PV(Cash flows) / Initial investment.

\[ PI = \frac{\sum_{t=1}^{n} CF_t \cdot (1+k)^t}{Co} \]

\[ PI_A = \frac{1091}{1000} = 1.091 \]

\[ PI_B = \frac{1403}{1000} = 1.403 \]

- The **Profitability Index selection rule**: accept the project if the PI \( \geq 1 \).
- Therefore, if the projects are independent, accept both (both PIs > 1).
- If the projects are mutually exclusive, accept Project B since PI\(_B\) > PI\(_A\).

**Note**: PI declines as r increases and PI rises as r decreases.

**Review of Capital Budgeting Rules**

**NPV Rule**
Good for every situation, although you must use NPV carefully or in conjunction with EAA when projects are mutually exclusive and have different lives.

**IRR Rule**
Widely used but can be misleading when choosing between mutually exclusive projects (different project lives & size disparity problem) or when there are multiple negative cash flows.

**Payback Rule**
Widely used in practice but can lead to serious errors. The main drawbacks are the ad hoc choice of a payback period, zero weight placed on cash flows after the payback period, and failure to acknowledge the time value of money.
DECISION RULES: ADDITIONAL EXAMPLES

Basic Data

<table>
<thead>
<tr>
<th>Year</th>
<th>Project L</th>
<th>Project S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>($100)</td>
<td>($100)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>20</td>
</tr>
</tbody>
</table>

1. **Payback Period**

   **Expected Net Cash Flow**

<table>
<thead>
<tr>
<th>Year</th>
<th>Project L</th>
<th>Project S</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Annual</td>
<td>Cumulative</td>
</tr>
<tr>
<td>0</td>
<td>($100)</td>
<td>($100)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>(90)</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>(30)</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>50</td>
</tr>
</tbody>
</table>

   Payback\(_L\) = 2 + 30/80 = 2.40 years

   Payback\(_S\) = 1 + 30/50 = 1.6 years.

2. **Net Present Value (NPV) Method**
\[ NPV = \sum_{t=1}^{n} \frac{CF_t}{(1 + r)^t} - Inv_0 \]

**Project L:**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IRR</strong></td>
<td>-100.00</td>
<td>10</td>
<td>60</td>
<td>80</td>
</tr>
</tbody>
</table>

\[ 10 / 1.1 = 9.09 \]
\[ 60 / (1.1)^2 = 49.59 \]
\[ 80 / (1.1)^3 = 60.11 \]

\[ NPV_L = $18.79 \]

Project S:

\[ NPV_S = 70 / 1.1 + 50 / (1.1)^2 + 20 / (1.1)^3 - 100 = $19.98. \]

- The **NPV selection rule**: accept the project if the NPV ≥ 0
- Therefore, if the projects are independent, accept both.
- If the projects are mutually exclusive, accept Project S since NPV_S > NPV_L.

3. **Internal Rate of return (IRR)**

\[ IRR: \quad NPV = \sum_{t=1}^{n} \frac{CF_t}{(1 + IRR)^t} - INV_0 = 0. \]

**Project L:**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IRR</strong></td>
<td>-100.00</td>
<td>10</td>
<td>60</td>
<td>80</td>
</tr>
</tbody>
</table>

\[ IRR_L = 18.1\% \]
\[ IRR_S = 23.6\% \]

- The **IRR selection rule**: accept the project if the IRR ≥ cost of capital.
• Therefore, if the projects are independent, accept both (both IRRs > 10%).
• If the projects are mutually exclusive, accept Project S since IRR_S > IRR_L.

**Note:** IRR is independent of the cost of capital.

### 4. Profitability Index

- **Profitability Index** = PV(Cash flows) / Initial investment.

  Assuming \( r = 10\% \)
  
  \[
  \text{PI}_L = \frac{18.79 + 100}{100} = 108.79/100 = 1.19  \\
  \text{PI}_S = \frac{19.98 + 100}{100} = 119.98/100 = 1.20
  \]

- The **Profitability Index selection rule**: accept the project if the \( \text{PI} \geq 1 \).
- Therefore, if the projects are independent, accept both (both PIs > 1).
- If the projects are mutually exclusive, accept Project S since \( \text{PI}_S > \text{PI}_L \).

**Note:** PI declines as \( r \) increases and PI rises as \( r \) decreases.

---

**Difference Between IRR + NPV**

When the two projects are mutually exclusive (we can select only one), the two methods may give different results.

**Example:**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-23616</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>B</td>
<td>-23616</td>
<td>0</td>
<td>5000</td>
<td>10000</td>
<td>32675</td>
</tr>
</tbody>
</table>

\( \text{IRR}_A = 25\% \)
\( \text{IRR}_B = 22\% \)
if \( k = 10\% \)
\( \text{NPV}_A = 8083 \)
\( \text{NPV}_B = 10347 \)
There is a conflict.
Conflict is due to different implicit compounding rate.

- **Pitfalls of the IRR rule**
  
  (1) Lending or Borrowing?

<table>
<thead>
<tr>
<th>Project</th>
<th>Cash Flows, $</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Project</td>
<td>C₀ (Inv₀)</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>-100</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>+100</td>
</tr>
</tbody>
</table>

(2) Multiple IRR's or no IRR

<table>
<thead>
<tr>
<th>Project</th>
<th>C₀</th>
<th>C₁</th>
<th>C₂</th>
<th>IRR</th>
<th>NPV at 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-4000</td>
<td>+25000</td>
<td>-25000</td>
<td>25% and 400%</td>
<td>-1934</td>
</tr>
</tbody>
</table>

To find the IRR, we set the NPV = 0:

\[
\text{NPV}_A = -4000 + 25000/(1 + IRR) - 25000/(1 + IRR)^2 = 0
\]

\[
= -4000(1 + IRR)^2 + 25000(1 + IRR) - 25000 = 0, \text{ divide by 1000,}
\]
\[
= -4(1 + IRR)^2 + 25(1 + IRR) - 25 = 0
\]
\[
= -4(IRR^3 + 2IRR + 1) + 25 + 25IRR - 25 = 0
\]
\[
= -4IRR^2 - 8IRR - 4 + 25 + 25IRR - 25 = 0
\]
\[
= 4IRR^2 + 17IRR - 4 = 0
\]
\[
= [(IRR - 4)(4IRR - 1)] = 0
\]

IRR = 4, or IRR=0.25
To find the IRR, we set the NPV=0:

$$NPV_B = +1000 - 3000/(1 + IRR) + 2500/(1 + IRR)^2 = 0$$

$$= +1000(1 + IRR)^2 - 3000(1 + IRR) + 2500 = 0$$ , divide by 1000,

$$= +1(1 + IRR)^2 - 3(1 + IRR) + 2.5 = 0$$

$$= ( IRR^2 + 2IRR + 1) - 3 - 3IRR + 2.5 = 0$$

$$= IRR^2 - IRR + 0.5 = 0$$

$$= 2IRR^2 - 2IRR + 1 = 0$$

No solution, we can not find an IRR

(3) Mutually Exclusive Projects

a) Different Project Scale

<table>
<thead>
<tr>
<th>Project</th>
<th>C₀</th>
<th>C₁</th>
<th>C₂</th>
<th>IRR</th>
<th>NPV at 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-10000</td>
<td>+20000</td>
<td>100</td>
<td></td>
<td>+8,182</td>
</tr>
<tr>
<td>D</td>
<td>-20000</td>
<td>+35000</td>
<td>75</td>
<td></td>
<td>+11,818</td>
</tr>
</tbody>
</table>

$$IRR: \quad NPV = \sum_{t=1}^{n} \frac{C_t}{(1 + IRR)^t} - C_0 = \frac{C_1}{1 + IRR} - C_0 = $0.$$  =>  $$IRR = C_1/C_0 - 1$$

$$IRR_C = 20000/10000 - 1 = 1 = 100\%$$

$$IRR_D = 35000/20000 - 1 = 0.75 = 75\%$$

$$NPV_C = -10,000 + 20,000/(1.1) = $8,182$$

$$NPV_D = -20,000 + 35,000/(1.1) = $11,818$$
If we compare only IRRs....Project C is better (100 > 75) 
Based on the NPV,...Project D is better (11818 > 8182) 
Which project should we choose? 
When IRR & NPV give different solutions, we go with NPV; i.e., project D. 

b) Different Project Lives

<table>
<thead>
<tr>
<th>Project</th>
<th>C₀</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>Etc.</th>
<th>IRR (%)</th>
<th>NPV at 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>-9000</td>
<td>+6000</td>
<td>+5000</td>
<td>+4000</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>33</td>
<td>3592</td>
</tr>
<tr>
<td>I</td>
<td>-9000</td>
<td>+1800</td>
<td>+1800</td>
<td>+1800</td>
<td>+1800</td>
<td>0</td>
<td>...</td>
<td>20</td>
<td>9000</td>
</tr>
<tr>
<td>J</td>
<td>-6000</td>
<td>+1200</td>
<td>+1200</td>
<td>+1200</td>
<td>+1200</td>
<td>0</td>
<td>...</td>
<td>20</td>
<td>6000</td>
</tr>
</tbody>
</table>

Which project should we choose? 
Again reinforcing the intuition of a) choose project with the highest NPV...project I.

5. **Book Rate of Return (Accounting Rate of Return).**

\[ \text{BRR (ARR)} = \frac{\text{average income}}{\text{average book value over project life}} \]

- Problems:
  - does not account for the time value of money;
  - uses accounting data rather than cash flows and market values;
  - difficult to choose the decision rule.

**Problems**

1. You have two projects.

<table>
<thead>
<tr>
<th>Year</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$100</td>
<td>-$100</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>
If the opportunity cost of capital is 12%, which of these projects is worth pursuing?

\[
NPV_A = -100 + C \times \left( \frac{1}{r} - \frac{1}{r(1+r)^t} \right) = C \times PV \text{ Annuity Factor}(12\%, 4 \text{ yrs}) = 21.49
\]

\[
NPV_B = -100 + C \times \left( \frac{1}{r} - \frac{1}{r(1+r)^t} \right) = C \times PV \text{ Annuity Factor}(12\%, 3 \text{ yrs}) = 20.09
\]

Both projects are acceptable.

Suppose that the projects are mutually exclusive. Which project would you choose?
Choose the project A because \( NPV_A > NPV_B \).

Which project would you choose if the opportunity cost of capital were 16%?

\[
NPV_A = -100 + C \times \left( \frac{1}{r} - \frac{1}{r(1+r)^t} \right) = -100 + 40 \times PV \text{ Annuity Factor}(16\%, 4 \text{ yrs}) = 11.93
\]

\[
NPV_B = -100 + C \times \left( \frac{1}{r} - \frac{1}{r(1+r)^t} \right) = -100 + 50 \times PV \text{ Annuity Factor}(16\%, 3 \text{ yrs}) = 12.29
\]

Choose the project B because \( NPV_B > NPV_A \).

What are the internal rates of return on projects A and B?

\[
-100 + 40 \times PV \text{ Annuity Factor}(\text{IRR}, 4 \text{ yrs}) = 0
\]

\[
PV \text{ Annuity Factor}(\text{IRR}, 4 \text{ yrs}) = \frac{100}{40} = 2.5 \quad \Rightarrow \quad \text{IRR}_A = 21.86\%
\]

\[
-100 + 50 \times PV \text{ Annuity Factor}(\text{IRR}, 3 \text{ yrs}) = 0
\]

\[
PV \text{ Annuity Factor}(\text{IRR}, 3 \text{ yrs}) = \frac{100}{50} = 2.0 \quad \Rightarrow \quad \text{IRR}_B = 23.38\%
\]

Is there any reason to believe that the project with the higher IRR is the better project?
NO.

If the opportunity cost of capital is 12%, what is the profitability index for each project? Does the profitability index rank the projects correctly?
\( PI_A = \frac{(C_{0A} + NPV_A)}{C_{0A}} = \frac{121.49}{100} = 1.2149 \)
\( PI_B = \frac{(C_{0B} + NPV_B)}{C_{0B}} = \frac{120.09}{100} = 1.2009 \)

- What is the payback period of each project?
  \( PB_A = \frac{100}{40} = 2.5 \text{ years} \)
  \( PB_B = \frac{100}{50} = 2.0 \text{ years} \)

- Is there any reason to believe that the project with the lower payback period is the better project?
  NO

- Accountants have set up the following depreciation schedules for the two projects:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project A</td>
<td>$25</td>
<td>$25</td>
<td>$25</td>
<td>$25</td>
</tr>
<tr>
<td>Project B</td>
<td>33.33</td>
<td>33.33</td>
<td>33.33</td>
<td>33.33</td>
</tr>
</tbody>
</table>

Calculate each project’s average book rate of return. Do these figures rank the projects correctly?
  
  Average book values = \( \frac{100}{2} = 50 \)
  Income (A) = cash flow - depreciation = 40 - 25 = 15
  Book rate of return (A) = \( \frac{15}{50} = 30\% \)
  Income (B) = cash flow - depreciation = 50 - 33.33 = 16.67
  Book rate of return (B) = \( \frac{16.67}{50} = 33\% \)

2. Consider the following cash flows for the two investments:

<table>
<thead>
<tr>
<th>Year</th>
<th>Investment A</th>
<th>Investment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$100</td>
<td>-$100</td>
</tr>
<tr>
<td>1</td>
<td>52</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>63</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
<td>110</td>
</tr>
</tbody>
</table>

What are the paybacks on the two investments? If we require a two-year payback to take an investment, which of these two is acceptable? Is it necessarily the best investment?

- 100 - 52 = 48, \( \frac{48}{63} = 0.76 \) \( \Rightarrow \) payback(A) = 1.76 years
- 100 -41 - 55 = 4; \( \frac{4}{110} = 0.04 \) \( \Rightarrow \) payback(B) = 2.04 years

- Investment A would be accepted
• It is not necessarily the best because payback ignores cash flows that occur after the cutoff period, resulting in a bias in the short period.

3. If a project has conventional cash flows and a positive NPV, what do you know about its payback? Its profitability index? Its IRR?
   • payback < life of project
   • profitability index > 1
   • IRR > required rate of return

4. Project L has a cost of $65,000, and its expected net cash inflows are $15,000 per year for 8 years.
   • What is the project's payback period (to the closest year)?
     • \( \frac{65,000}{15,000} = 4.333 \) years
   • The cost of capital is 14%. What is the project's NPV?
     • \[ \text{NPV} = -65,000 + 15,000 \left\{ \frac{1}{0.14} - \frac{1}{(1 + 0.14)^8} \right\} = 4,583.50 \]
   • What is the project's IRR?
     \[ -65,000 + 15,000 \left\{ \frac{1}{\text{IRR}} - \frac{1}{\text{IRR}(1 + \text{IRR})^8} \right\} = 0 \]
     \[ \text{PVIFA}(8\text{yrs}, \text{IRR}) = \frac{65,000}{15,000} = 4.333 \]
     \[ \text{IRR} \approx 16\% \]

5. Parrish Engineering is considering including two pieces of equipment, a truck and an overhead pulley system, in this year's capital budget. The projects are not mutually exclusive. The cash outlay for the truck is $17,350, and that for the pulley system is $24,225. The firm's cost of capital is 15%. After-tax cash flows are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Truck</th>
<th>Pulley</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,300</td>
<td>$8,100</td>
</tr>
<tr>
<td>2</td>
<td>5,300</td>
<td>8,100</td>
</tr>
<tr>
<td>3</td>
<td>5,300</td>
<td>8,100</td>
</tr>
<tr>
<td>4</td>
<td>5,300</td>
<td>8,100</td>
</tr>
<tr>
<td>5</td>
<td>5,300</td>
<td>8,100</td>
</tr>
</tbody>
</table>
Calculate the IRR and NPV for each project, and indicate the correct accept/reject decision for each.

**Truck:**

NPV(T) = -17,350 + 5,300 \left[ \frac{1}{0.15} - \frac{1}{(0.15)(1 + 0.15)^5} \right] =

= -17,350 + 17,766.42 = $416.42 \Rightarrow \text{ACCEPT}

NPV(T) = 0 \Rightarrow -17,350 + 5,300 \left[ \frac{1}{IRR} - \frac{1}{IRR(1 + IRR)^5} \right] = 0

\Rightarrow PVIFA (IRR, 5) = 3.2736

\Rightarrow IRR = 16\% \Rightarrow \text{ACCEPT (IRR > 15\%)}

**Pulley:**

NPV(P) = -24,225 + 8,100 \left[ \frac{1}{0.15} - \frac{1}{(0.15)(1 + 0.15)^5} \right] = $2,927.46

NPV(O) = -24,225 + 8,100 \left[ \frac{1}{IRR} - \frac{1}{IRR(1 + IRR)^5} \right] = 0

\Rightarrow PVIFA (IRR, 5) = 2.9907

\Rightarrow IRR = 20\% \Rightarrow \text{ACCEPT (IRR > 15\%)}

---

6. The Caviar Co. owns 140 acres of prime oceanfront property. It is considering several different development options. One option is a hotel and resort complex (Option A). Also under consideration is a more expensive motel/amusement park development (Option B). The cash flows (in millions of dollars) for the two options are projected to be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Option A</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$600</td>
<td>-$800</td>
</tr>
<tr>
<td>1</td>
<td>-40</td>
<td>-60</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>175</td>
</tr>
<tr>
<td>3</td>
<td>203</td>
<td>210</td>
</tr>
<tr>
<td>4</td>
<td>245</td>
<td>270</td>
</tr>
</tbody>
</table>
• What is the payback of Option A? For Option B?
   A: 4.33 years  
   B: 4.55 years

• Assuming a required return of 20%, what are the profitability indices for the projects? How do you interpret these?

   A: PV cash flows = \(-\frac{40}{1.2} + \frac{95}{1.2^2} + \frac{203}{1.2^3} + \frac{245}{1.2^4} + \frac{290}{1.2^5} + \frac{1,240}{1.2^6}\) = $800.09

   \[\text{PI}_A = \frac{800.09}{600} = 1.33\]

   B: PV cash flows = \(-\frac{60}{1.2} + \frac{175}{1.2^2} + \frac{210}{1.2^3} + \frac{270}{1.2^4} + \frac{375}{1.2^5} + \frac{1,510}{1.2^6}\) = $979.66

   \[\text{PI}_B = \frac{979.66}{800} = 1.22\]

   * since profitability indices are both greater than 1, then both projects have positive NPV

• Do the profitability index and NPV criteria always rank projects the same way? Why or why not? Which of these projects is preferable, again assuming a 20% discount rate?

   * They don’t rank projects the same way. PI does not take into account the total value a project adds.

   A: \(\text{NPV} = 800.09 - 600 = 200.09\)

   B: \(\text{NPV} = 979.66 - 800 = 179.66\)

   \[\Rightarrow \text{project A is better}\]

7. Besley Industries must choose between a gas-powered and an electric-powered forklift truck for moving materials in its factory. Since both forklifts perform the same function, the firm will choose only one. (They are mutually exclusive investments.) The electric-powered truck will cost more, but it will be less expensive to operate; it will cost $22,000, whereas the gas-powered truck will cost $17,600. The cost of capital that applies to both investments is 10%. The life for both types of truck is estimated to be 6 years, during which time the net cash flows for the electric-powered truck will be $6,600 per year and those for the gas-
powered truck will be $5,300 per year. Annual net cash flows include depreciation expenses. Calculate the NPV and IRR for each of truck, and decide which to recommend for purchase.

**electric:**

\[
\text{NPV} = -22,000 + 6,600 \left[ \frac{1}{0.10} - \frac{1}{(0.10)(1 + 0.10)^5} \right] = $6,745
\]

\[\text{IRR} = ? \quad \text{NPV} = -22,000 + 6,600 \left[ \frac{1}{IRR} - \frac{1}{IRR(1 + IRR)^5} \right] = 0\]

\[\Rightarrow \text{PVIFA (IRR, } 6) = \frac{22,000}{6,600} = 3.33\]

\[\Rightarrow \text{IRR} \approx 20\%\]

**gas:**

\[
\text{NPV} = -17,600 + 5,300 \left[ \frac{1}{0.10} - \frac{1}{(0.10)(1 + 0.10)^5} \right] = $5,483
\]

\[\text{IRR} = ? \quad \text{NPV} = 0; \quad \text{PVIFA (6, IRR)} = \frac{17,600}{5,300} = 3.32\]

\[\Rightarrow \text{IRR} \approx 20\%\]

NPV electric > NPV gas \[\Rightarrow\] company should purchase the electric