Risk and Return & Capital Asset Pricing

RISK AND RETURN

1. Rates of return

- Percentage return = $\dfrac{\textit{capital gain} + \textit{dividend}}{\textit{initial share price}}$

Example: You bought the stock of Intel at the beginning of 1992 for $49 a share. By the end of the year the price had appreciated to $87. In addition, in 1992 Intel paid a dividend of $0.15 a share. What was your realized return if you sold your shares at the end of 1992?

$$r = \frac{(\$87 - \$49) + \$0.15}{\$49} = 0.779 = 77.9\%$$

2. Historical returns

- Stock-market indexes
  - Dow Jones Industrial Average
    - First computed in 1896 with only 20 stocks.
    - Now, consists of 30 large-firm "blue-chip" stocks.
    - The DJIA is a price-weighted index. That is, it assumes you hold one share in each of the 30 firms making up the index. When the DJIA began, it was computed by simply adding up the prices of the 20 stocks and dividing by 20. The divisor has now become very complicated because it has been adjusted for stock splits, firms being added or deleted from the index, etc.
  - Standard & Poor's Composite 500 Index
    - Comprised of 500 large-firm industrial stocks.
    - The S&P 500 is a market-value weighted index. That is, it assumes you hold shares in each of 500 major firms in proportion to the number of shares that have been issued. This gives you the average performance of investors in the 500 firms.

- Wilshire 5000
  - Broad market index comprised of all NYSE and Amex stocks plus actively traded OTC stocks.
  - Market-value weighted index.
- Dow Jones Banking Index
  - An example of an industry-specific index.
  - Comprised of 55 stocks in large commercial banks. (The number of banks is decreasing with mergers.)
- Nikkei Index (Tokyo exchange)
  - Comprised of 225 large-firm Japanese stocks.
  - Price-weighted index.

Note: Indices exclude dividends. Indices only capture the market value of the stocks in the portfolio. Thus, actual returns are understated.

- What is the historical relationship between return & risk (measured by the variability or standard deviation of returns)?

  Investors demand higher expected returns for taking on more risk.

  Expected return: $r = r_F + \text{risk premium}$

- Suppose we had a project with the same degree of risk as the market portfolio of common stocks. What is the appropriate opportunity cost of capital? Assume that the Treasury-bill interest rate is 4% and the normal risk premium is 8.6%.

  The expected market return is the interest rate on Treasury bills plus the normal risk premium.

  expected stock-market return = 4% + 8.6% = 12.6%

3. Asset risk

- Risk = degree of variability of investment returns
- Some measure of return variability will provide a reasonable measure of risk.
  - **variance (standard deviation)** measures the average degree of deviation from the average return.

Defining Expected Value

The expected value (or mean) of a discrete random variable *X* is given by

$$E[X] = \sum_{i=1}^{n} p_i X_i \quad \text{where} \quad \sum_{i=1}^{n} p_i = 1$$

What is the expected value of the coin flip when the stakes are $1?

- If the coin is fair then Pr(Heads) = p_1 = 0.5 and Pr(Tails) = p_2 = 0.5
- If the stakes are $1, then X_1 = $1 and X_2 = -$1
- Therefore, **E[X] = p_1 ($1) + p_2 (-$1) = .5 ($1) + .5 (-$1) = 0**

What is the expected value of the coin flip when the stakes are $1000?

What is the expected value of the coin flip when the stakes are $1000 and the Pr(Heads) = 0.55?

Defining Standard Deviation

The standard deviation of a discrete random variable *X* measures the dispersion of *X* around its expected value (or mean)

$$Var[X] \equiv \sigma^2(X) = \sum_{i=1}^{n} p_i (X_i - E[X])^2 \quad \text{where} \quad \sum_{i=1}^{n} p_i = 1$$

$$\text{Standard Deviation}(X) \equiv \sigma(X) = \sqrt{Var[X]} = \sqrt{\sum_{i=1}^{n} p_i (X_i - E[X])^2}$$

What is the standard deviation of the coin flip with $1 stakes?

$$\sigma(X) = \sqrt{p_1 \cdot (X_1 - E[X])^2 + p_2 \cdot (X_2 - E[X])^2}$$
$$\sigma(X) = \sqrt{.5 \cdot (1-0)^2 + .5 \cdot (-1-0)^2}$$
$$\sigma(X) = \sqrt{.5+.5} = 1$$

What is the standard deviation when the stakes are $1000?

$$\sigma(X) = \sqrt{p_1 \cdot (X_1 - E[X])^2 + p_2 \cdot (X_2 - E[X])^2}$$
$$\sigma(X) = \sqrt{.5 \cdot (1000-0)^2 + .5 \cdot (-1000-0)^2}$$
$$\sigma(X) = \sqrt{1{,}000{,}000} = 1{,}000$$

What is the standard deviation when the stakes are $1000, the probability of heads is 0.55, and you win with heads?

$$\sigma(X) = \sqrt{p_1 \cdot (X_1 - E[X])^2 + p_2 \cdot (X_2 - E[X])^2}$$
$$\sigma(X) = \sqrt{.55 \cdot (1000-100)^2 + .45 \cdot (-1000-100)^2}$$
$$\sigma(X) = \sqrt{990{,}000} = 994.99$$

Practice Calculating *E[X]* and *σ(X)*

Example: You are one of ten people chosen to compete next session on the Apprentice. The runner-up receives $100,000 in endorsements, the apprentice receives $1,000,000 and one-year position as Trump's personal stylist, and everyone else wins a trip to the boardroom. What are *E[X]* and *σ(X)?* What assumptions do you need to make to calculate these statistics?

Solution: To answer this question you need to decide on the probability of each outcome. Assuming that Pr($1 million) = Pr(Runner-up) = 0.10 and Pr(boardroom) = 0.80, we first estimate *E[X]* and then *σ(X)*.

$$E[X] = \sum_{i=1}^{n} p_i X_i = 0.1(1000000) + 0.1(100000) + .8(0) = \$110{,}000$$
$$\begin{aligned}\sigma(X) &= \sqrt{p_1(X_1 - E[X])^2 + p_2(X_2 - E[X])^2 + p_3(X_3 - E[X])^2} \\ &= \sqrt{0.1(1000000-110000)^2 + 0.1(100000-110000)^2 + 0.8(0-110000)^2} \\ &= \$298{,}161\end{aligned}$$

Estimating *E[r]* and *σ(r)* for a Stock

Now consider firm ABC for which we observe T years of historical stock return data. To estimate the expected return, we take the average of the historical returns *assuming each return is equally likely*:

$$\hat{E}[r] = \bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t \quad \text{where } T \text{ is the number of past returns}$$

And we can estimate the standard deviation of the return similarly:

$$\hat{\sigma}(r) = \sqrt{\hat{\sigma}^2(r)} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

When outcomes are equally likely, $p_i = 1 / T$. But here I'm using 1 / T-1 instead of 1 / T. What gives? The short answer is that we divide by T whenever we observe all possible outcomes and we divide by T-1 whenever we observe a random sample of outcomes.

Practice Estimating *E[r]* and *σ(r)*

Consider a stock that has traded at the following prices for each of the last six years and has paid an annual dividend of $1 beginning in year 1:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Price (P_t)	**$10.00**	**$10.00**	**$12.25**	**$11.50**	**$13.80**	**$12.90**

Estimate the expected return and standard deviation of this stock…

Solution:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Return (r_t)	--	10.00%	32.50%	2.04%	28.70%	0.72%

where $r_1 = \dfrac{10.00 - 10.00 + 1.00}{10.00} = 10.00\%,\ r_2 = \dfrac{12.25 - 10.00 + 1.00}{10.00} = 32.50\%, \ldots$

and so on…

Solution: The five annual returns, we given below

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Return (r_t)	--	10.00%	32.50%	2.04%	28.70%	0.72%

Now, we can estimate both the stock's expected return and the standard deviation of its return.

Expected Return:

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t = \frac{1}{5}(10.00 + 32.50 + 2.04 + 28.70 + 0.72) = 14.79\%$$

Standard Deviation:

$$\hat{\sigma}(r) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2} = \sqrt{222.61} = 14.92\%$$

- let's compute average return and variance using the following data:

Year	% Rate of Return	% Deviation from Average Return	Squared Deviation
1	16.8	16.8 - 16.7 = 0.1	$0.1^2 = 0.0$
2	31.5	31.5-16.7 = 14.8	$14.8^2 = 219.0$
3	-3.2	-3.2-16.7 = -19.9	$-19.9^2 = 396.0$
4	30.6	30.6-16.7 = 13.9	$13.9^2 = 193.2$
5	7.7	7.7-16.7 = -9.0	$-9.0^2 = 81.0$
	83.4		889.2

  - Average rate of return = 83.4% / 5 = 16.7%
  - Variance = σ^2 = average of squared deviations = 889.2 / 5 = 177.8
  - However, the variance is in terms of $\%^2$. We want in terms of %.
  - Standard deviation = σ = square root of variance = $\sqrt{177.8}$ =13.3%

- We can also compute the average returns and standard deviations using <u>expected</u> returns:

State	Probability	Auto Stock Return	Gold Stock Return
Recession	1/3	-8%	+20%
Normal	1/3	+5%	+3%
Boom	1/3	+18%	-20%

- Expected return = what is the average return you can expect to earn on your investment.
- Expected returns:
  - $E(R_A)$ = 1/3 (-0.08) + 1/3 (0.05) + 1/3 (0.18) = .05 or 5%

- $E(R_G) = 1/3\ (0.20) + 1/3\ (0.03) + 1/3\ (-0.20) = .01$ or 1%
- Variances:
  - $\sigma^2(R_A) = 1/3(-0.08 - .05)^2 + 1/3(.05 - .05)^2 + 1/3(0.18 - .05)^2 = 0.01127$
  - $\sigma^2(R_G) = 1/3(0.20 - .01)^2 + 1/3(.03 - .01)^2 + 1/3(-0.20 - .01)^2 = 0.02687$
- Standard deviations
  - $\sigma(R_A) = \sqrt{0.0117} = .106$ or 10.6%
  - $\sigma(R_G) = \sqrt{0.02687} = .164$ or 16.4%

- **We can also compute the average returns and standard deviations using <u>expected</u> values**

- Example:

State of the Economy	Cash Flow (X_i)	P_i	X_iP_i
Recession	20,000	.2	4,000
Normal	25,000	.6	15,000
Boom	30,000	.2	6,000
		Expected Cash Flow =====	25,000 = X

$$\sigma_X = \sqrt{(20{,}000-25{,}000)^2(.2) + (25{,}000-25{,}000)^2(.6) + (30{,}000-25{,}000)^2(.2)}$$

$$\sigma_X = 3162.28$$

$$C_V = \frac{3162.28}{25000} = .1264$$

- Alternatively there is another mutually exclusive project with the following information:

$X_B = 14{,}500 \qquad \sigma_{XB} = 2260$

$$C_V = \frac{2260}{14500} = .1558$$

- Example:

State of the Economy	Prob. of State Occurring	Rate of Return: Allen Paper	Rate of Return: Collins Glass
Boom	.2	40%	70%
Normal	.6	20%	20%
Recession	.2	0%	-30%

Allen Paper

i	State	k_i	k	k_i-k	$(k_i-k)^2$	Pr_i	$(k_i-k)^2 \times Pr_i$
1	Boom	40%	20%	20	400	0.20	80
2	Normal	20%	20%	0	0	0.60	0
3	Recession	0%	20%	-20	400	0.20	80

$$\sum_{i=1}^{3}(k_i-k)^2 \times Pr_i = 160$$

$$\sigma_{\text{Allen Paper}} = \sqrt{\sum_{i=1}^{3}(k_i-k)^2 \times Pr_i} = \sqrt{160} \approx 12.65\%$$

Collins Glass

i	State	k_i	k	k_i-k	$(k_i-k)^2$	Pr_i	$(k_i-k)^2 \times Pr_i$
1	Boom	70%	20%	50	2500	0.20	500
2	Normal	20%	20%	0	0	0.60	0
3	Recession	-30%	20%	-50	2500	0.20	500

$$\sum_{i=1}^{3}(k_i-k)^2 \times Pr_i = 1000$$

$$\sigma_{\text{Collins Glass}} = \sqrt{\sum_{i=1}^{3}(k_i-k)^2 \times Pr_i} = \sqrt{1000} \approx 31.62\%$$

- Example:

Stocks A, B and C have the following probability distributions of expected returns

i	A		B		C	
	Pr_i	k_i	Pr_i	k_i	Pr_i	k_i
1	.1	40%	.1	40%	.4	35
2	.2	20%	.2	10%	.3	10
3	.4	10%	.4	0%	.3	-20
4	.2	0%	.2	-5%		
5	.1	-20%	.1	-10%		

(a) $k_A = (40\%)(.1) + (20\%)(.2) + (10\%)(.4) + (0\%)(.2) + (-20\%)(.1) = \underline{\underline{10\%}}$

$k_B = (40\%)(.1) + (10\%)(.2) + (0\%)(.4) + (-5\%)(.2) + (-10\%)(.1) = \underline{\underline{4\%}}$

$k_C = (35\%)(.4) + (10\%)(.3) + (-20\%)(.3) = \underline{\underline{11\%}}$

At an 11 percent expected rate of return, stock C appears to be the most profitable investment.

(b) $\sigma_A = \sqrt{(40\text{-}10)^2 (.1) + (20\text{-}10)^2 (.2) + (10\text{-}10)^2 (.4) + (0\text{-}10)^2 (.2) + (-20\text{-}10)^2 (.1)}$

$$= \underline{\underline{14.83\%}}$$

$$CV_A = \frac{\sigma_A}{k_A} = \frac{14.83\%}{10\%} = \underline{\underline{1.483}}$$

$$\sigma_B = \sqrt{(40\text{-}4)^2\,(.1) + (10\text{-}4)^2\,(.2) + (0\text{-}4)^2\,(.4) + (-5\text{-}4)^2\,(.2) + (-10\text{-}4)^2(.1)}$$

$$= \underline{\underline{13.38\%}}$$

$$CV_B = \frac{13.38\%}{4\%} = \underline{\underline{3.345}}$$

$$\sigma_C = \sqrt{(35\text{-}11)^2\,(.4) + (10\text{-}11)^2\,(.3) + (-20\text{-}11)^2(.3)}$$

$$= \underline{\underline{22.78\%}}$$

$$CV_C = \frac{22.78\%}{11\%} = \underline{\underline{2.071}}$$

(c) In terms of absolute variation, stock C, with the highest standard deviation at 22.78 percent, is the riskiest. Relative to its expected return, however, stock B is the riskiest with the highest coefficient of variation at 3.345.

1. Consider the following scenario analysis:

		Rate of Return	
Scenario	Probability	Stocks	Bonds
Recession	.20	-5%	14%
Normal economy	.60	15%	8%
Boom	.20	25%	4%

- calculate the expected rate of returns

$E(R_S) = (.20)(-5\%) + (.60)(15\%) + (.20)(25\%) = -1 + 9 + 5 = 13\%$

$E(R_B) = (.20)(14\%) + (.60)(8\%) + (.20)(4\%) = 2.8 + 4.8 + 0.8 = 8.4\%$

- calculate the standard deviations
  - $\sigma^2(R_S) = (.20)(-5-13)^2 + (.60)(15-13)^2 + (.20)(25-13)^2 = 96$

  $\sigma(R_S) = \sqrt{96} = 9.8\%$
  - $\sigma^2(R_B) = (.20)(14-8.4)^2 + (.60)(8-8.4)^2 + (.20)(4-8.4)^2 = 10.24$

  $\sigma(R_B) = \sqrt{10.24} = 3.2\%$
- which investment would you prefer?

2. consider a portfolio with .60 in stocks and .40 in bonds (use numbers from the previous example).

- calculate the expected return and standard deviation on this portfolio

		Rate of return		
Scenario	Prob.	stocks	bonds	Portfolio
Recession	.20	-5%	14%	(.60)(-5%) + (.40)(14%) = 2.6%
Normal	.60	15%	8%	(.60)(15%) + (.40)(8%) = 12.2%
Boom	.20	25%	4%	(.60)(25%) + (.40)(4%) = 16.6%
Expected return		13%	8.4%	(.2)(2.6%) + (.6)(12.2) + (.2)(16.6) = 11.16%
Variance		96	10.24	$(.2)(2.6-11.16)^2+(.6)(12.2-11.16)^2+(.2)(16.6-11.16)^2= 21.2$
St. dev.		9.8%	3.2%	$\sqrt{21.2} = 4.6\%$

Mathematics of Risk

Consider two assets: A & B

--w_A and w_B are the fractions of total funds invested in each asset

--σ_A and σ_B are the standard deviations of returns of each asset

--ρ_{AB} is the correlation between the two assets' returns

$$\rho_{AB} = \frac{Cov(r_A, r_B)}{\sigma_A \cdot \sigma_B} \Leftrightarrow Cov(r_A, r_B) = \sigma_A \cdot \sigma_B \cdot \rho_{AB}$$

where we estimate $Cov(r_A, r_B)$ as

$$C\hat{o}v(r_A, r_B) = \sum_{t=1}^{T} \frac{1}{T-1}(r_{A_t} - \bar{r}_A)(r_{B_t} - \bar{r}_B)$$

Armed with these assumptions and definitions, we can derive the standard deviation of a portfolio consisting of two assets…

Let's start by estimating the expected return of the portfolio:

$$E[r_p] = w_A E[r_A] + w_B E[r_B] \Leftrightarrow \bar{r}_p = w_A \bar{r}_A + w_B \bar{r}_B$$

Now let's estimate the variance of the portfolio:

$$\begin{aligned}\hat{\sigma}_p^2 &= \sum_{t=1}^{T} \frac{1}{T-1} (r_{p_t} - \bar{r}_p)^2 \\ &= w_A^2 \hat{\sigma}_A^2 + w_B^2 \hat{\sigma}_B^2 + 2 w_A w_B (\hat{\sigma}_A \cdot \hat{\sigma}_B \cdot \hat{\rho}_{AB})\end{aligned}$$

Variance Return of Asset A

Variance Return of Asset B

Correlation Returns of Assets A

Now let's see where the formula comes from…

$$\sigma_P^2 = \sum_{t=1}^{T} \frac{1}{T-1} (r_{p_t} - \bar{r}_p)^2$$

$$= \sum_{t=1}^{T} \frac{1}{T-1} (w_A r_{A_t} + w_B r_{B_t} - w_A \bar{r}_A - w_B \bar{r}_B)^2$$

$$= \sum_{t=1}^{T} \frac{1}{T-1} (w_A r_{A_t} - w_A \bar{r}_A + w_B r_{B_t} - w_B \bar{r}_B)^2$$

$$= \sum_{t=1}^{T} \frac{1}{T-1} (w_A^2 (r_{A_t} - \bar{r}_A)^2 + w_B^2 (r_{B_t} - \bar{r}_B)^2 + 2 w_A w_B (r_{A_t} - \bar{r}_A)(r_{B_t} - \bar{r}_B))$$

$$= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B Cov(R_A, R_B)$$

$$= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B (\sigma_A \cdot \sigma_B \cdot \rho_{AB})$$

Practice Calculating σ_P

Consider a portfolio consisting of the following two assets

- Return of asset A has σ_A = 20% and w_A = 2/3
- Return of asset B has σ_B = 40% and w_B = 1/3

The weighted average standard deviation is 26.67%

But recall that the standard deviation of the portfolio depends upon the correlation between the returns of asset A and asset B (ρ_{AB}).

Case 1. Assume $\rho_{AB} = 1$

$$\begin{aligned}\sigma_P &= \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B(\sigma_A \cdot \sigma_B \cdot \rho_{AB})} \\ &= \sqrt{\left(\tfrac{2}{3}\right)^2(.2)^2 + \left(\tfrac{1}{3}\right)^2(.4)^2 + 2\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)(.2)(.4)(1)} \\ &= 26.67\%\end{aligned}$$

Case 2. Assume $\rho_{AB} = 0$

$$\begin{aligned}\sigma_P &= \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B(\sigma_A \cdot \sigma_B \cdot \rho_{AB})} \\ &= \sqrt{\left(\tfrac{2}{3}\right)^2(.2)^2 + \left(\tfrac{1}{3}\right)^2(.4)^2 + 2\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)(.2)(.4)(0)} \\ &= 18.86\%\end{aligned}$$

Case 3. Assume $\rho_{AB} = -1$

$$\begin{aligned}\sigma_P &= \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B(\sigma_A \cdot \sigma_B \cdot \rho_{AB})} \\ &= \sqrt{\left(\tfrac{2}{3}\right)^2(.2)^2 + \left(\tfrac{1}{3}\right)^2(.4)^2 + 2\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)(.2)(.4)(-1)} \\ &= 0\%\end{aligned}$$

CAPITAL ASSET PRICING MODEL (CAPM)

Different Types of Risk

- The standard deviation of a stock's return (as presented in the previous slides) measures the overall risk of investing in that stock.
- Likewise, the standard deviation of a portfolio's return measures the overall risk of investing in that portfolio of stock. For a diversified portfolio of stocks with the same expected return, the portfolio has a lower standard deviation $\Rightarrow$ less total risk.
- Note, however, that we can break risk down into two categories:

  -Systematic risk
  -Unsystematic risk
- This distinction between systematic risk and unsystematic risk is crucial when considering the relation between risk and return…

Systematic Risk

- Also known as nondiversifiable risk or market risk
- This is the part of risk that cannot be eliminated through diversification. Involves events that "systematically" effect all stocks
- Examples:

  --Interest rates
  --State of the economy
  --World wars
  --Oil shocks
- Do financial markets reward you for bearing systematic risk?

Unsystematic Risk

- Also known as diversifiable risk or company-specific risk
- This is risk that gets washed away by opposite events occurring to companies inside your portfolio because the events are uncorrelated
- Examples:

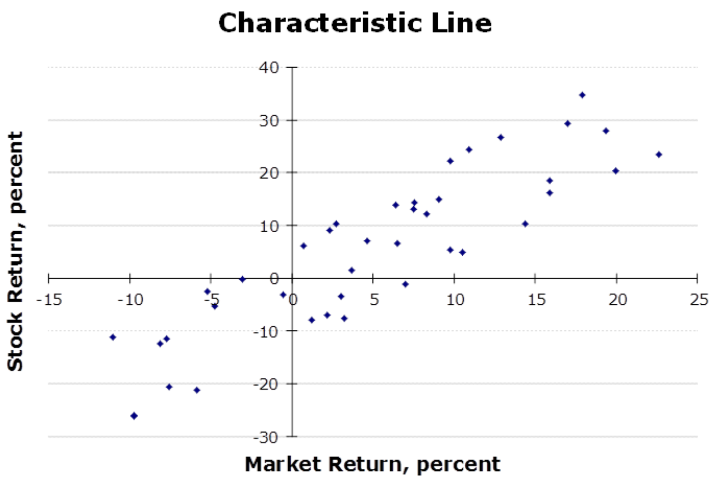
  --BP oil spill → bad news for BP (and the environment)
  --Fire at company headquarters (e.g., Office Space)
  --Albertson's decision to install sushi vending machines
  --Death of firm's CEO (or a competitor's CEO)
  --SEC investigation into a firm's accounting practices (or the accounting practices of its competitors)
- Do financial markets reward you for bearing unsystematic risk?

Measuring Systematic Risk

Question: What is the largest portfolio that we can invest in?

Answer: The Market Portfolio (often proxied by S&P 500 index)

- By definition, the Market Portfolio contains no unsystematic risk
- Therefore, the covariance of a security with the market portfolio provides a measure of the risk which cannot be diversified away
- *Characteristic Line* shows the relation between a securities' return and the market's return
- Slope of the Characteristic Line is known as beta (β)
- β measures the systematic risk of the security



Interpreting Beta

Greater exposure to systematic risk implies a greater beta

Common to sort stocks into three groups

- $\beta > 1$ ⎝ "aggressive stock"
- $\beta = 1$ ⎝ same systematic risk as market index
- $\beta < 1$ ⎝ "defensive stock"

If $\beta_{XYZ} = 3$, we expect XYZ's stock price to go up (down) 3% when the market goes up (down) 1%.

- $\beta_{rf} = 0$ (assuming U.S. government debt is risk free)

According to finance.yahoo.com on 10/23/13

- $\beta_{AAPL} = 0.58$ $\beta_F = 1.40$ $\beta_{TZOO} = 2.91$

Portfolio Betas are Easy to Calculate

- To determine the beta on a portfolio of stocks, you weight the beta on stock i by the fraction of your portfolio that is invested in stock
- Consider investing 20% in Apple, 60% in Ford, and 20% in Travel Zoo

$$\begin{aligned}\beta_{portfolio} &= \left(w_{AAPL} \cdot \beta_{AAPL}\right) + \left(w_F \cdot \beta_F\right) + \left(w_{TZOO} \cdot \beta_{TZOO}\right) \\ &= (0.2)(0.58) + (0.6)(1.40) + (0.2)(2.91) \\ &= 1.538\end{aligned}$$

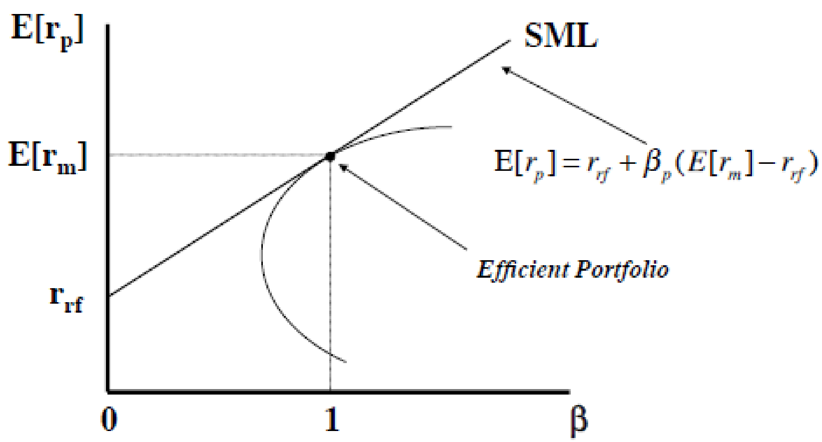
Note: A diversified portfolio with a beta of 1.538 will have less total risk than a stock with a beta of 1.538

Capital Asset Pricing Model

- The Capital Asset Pricing Model (CAPM) relates the expected return on a security to the expected return on the market portfolio
- "Two Fund Separation" ⌊ Every investors creates a portfolio from two assets. They invest some fraction of their wealth in the (efficient) market portfolio and the rest in a riskless asset (like Treasury Bills).
- The CAPM implies:
  $\mathrm{E}[r_p] = r_{rf} + \beta_p(E[r_m] - r_{rf})$
- In words, the expected return on an asset is equal to the risk-free rate of return, plus a risk factor (β_p) specific to the particular portfolio (p) times the risk premium.
- The risk premium is the difference between the expected returns on the market portfolio and the risk free asset.

Security Market Line

Provides the level of the expected return for portfolios that differ in the level of systematic risk, which we measure by β



Security Market Line in Action (1)

What is the expected return on the portfolio that invests 20% in Apple, 60% in Ford, and 20% in Travelzoo?

We need three pieces of information: the beta on the portfolio, the risk-free rate of return, and the expected return on the market.

- We've seen that this portfolio has a β of **1.538**.
- According to Treasury.gov, on 10/23/13 the yield on 1-year Treasury bills was **0.11%**.
- Finally, assume that analysts expect the market portfolio to return **6.00%** over the next year.

$$E[r_p] = r_{rf} + \beta_p(E[r_m] - r_{rf})$$
$$= 0.11\% + 1.538(6.00\% - 0.11\%)$$
$$= 9.16882\%$$

Security Market Line in Action (2)

Let's calculate the expected return on a different portfolio:

- Risk-free return is 5% and the expected return on the market is 15%
- You have calculated the β of your portfolio to be 1.8

$$E[r_p] = r_{rf} + \beta_p(E[r_m] - r_{rf})$$
$$= 5\% + 1.8(15\% - 5\%)$$
$$= 5\% + 18\% = 23\%$$

- What is the expected return on a portfolio with β of 1.0?

$$E[r_p] = r_{rf} + \beta_p(E[r_m] - r_{rf})$$
$$= 5\% + 1.0(15\% - 5\%)$$
$$= 5\% + 10\% = 15\%$$

Security Market Line in Action (3)

Assume that the risk-free rate is 3.6% and the risk premium is 8.4%.

1. What is the β on a portfolio with an expected return of 12.0%?

$$E[r_p] = r_{rf} + \beta_p(E[r_m] - r_{rf})$$
$$12.0\% = 3.6\% + \beta_p(8.4\%)$$
$$\beta_p = \frac{12.0\% - 3.6\%}{8.4\%} = 1.0$$

2. What is the β on a portfolio with an expected return of 16.2%?

$$\beta_p = \frac{16.2\% - 3.6\%}{8.4\%} = 1.5$$

Risk and Return

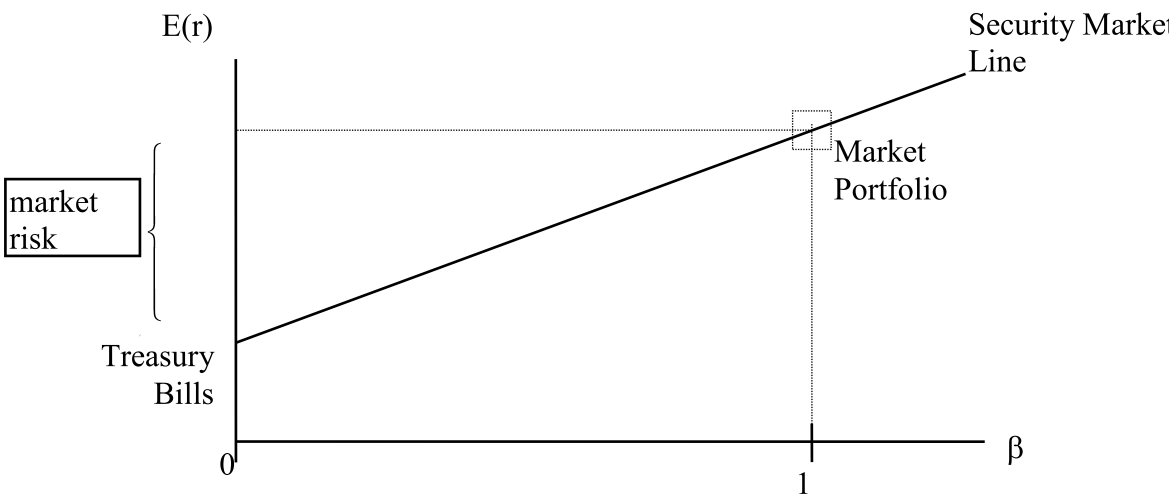
- What is the β of Treasury Bills?

The return on Treasury Bills is fixed. They are unaffected by what happens to the market. Therefore their β is zero.

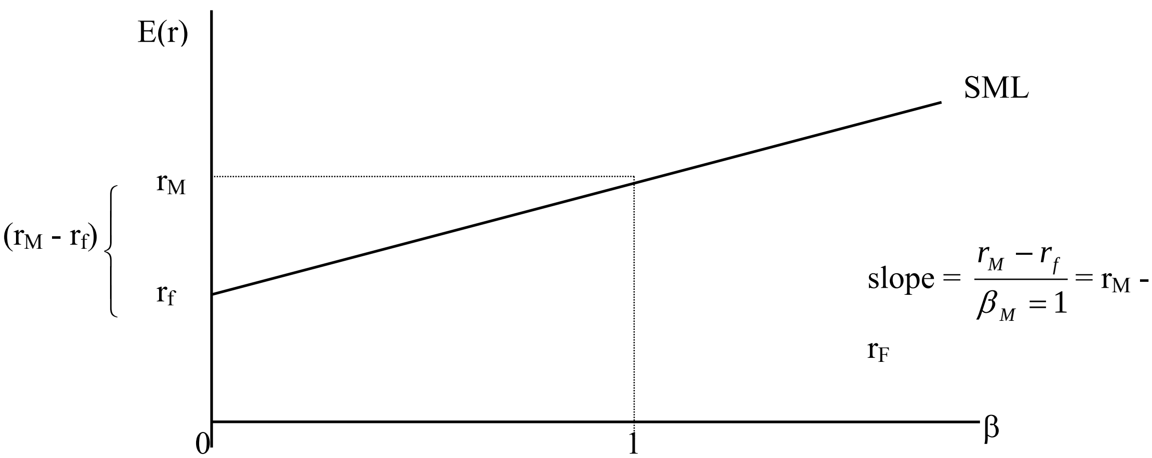
- What is the β of the market portfolio?

By definition, it is one.

- market risk premium = market return - risk-free rate (T-Bills)
- We can plot this information to get what is called the security market line (SML).



Or, purely in symbols:



- The expected rate of return for any asset lies on the SML.
- This leads us to the **Capital Asset Pricing Model (CAPM)**:

$$\mathbf{r = r_f + \beta(r_M - r_f)}$$

expected return = risk-free rate + risk premium

or, risk premium = $r - r_f = \beta(r_M - r_f)$

- To calculate expected return for particular stock, we need three things

r_f 1) *risk-free rate (e.g., T-bill rate)*

$(r_M - r_f)$ 2) *expected market risk premium (e.g., 8.6, historical estimate)*

β 3) *β (e.g., use historical data to estimate)*

- Intuitively, expected return demanded by investors for a particular asset depends on 2 things:

1. <u>The pure time value of money</u>

   Measured by risk-free rate, r_f. This is the **reward for** merely **waiting** for your money, without taking any risk.

2. <u>The reward for bearing risk = risk premium</u>

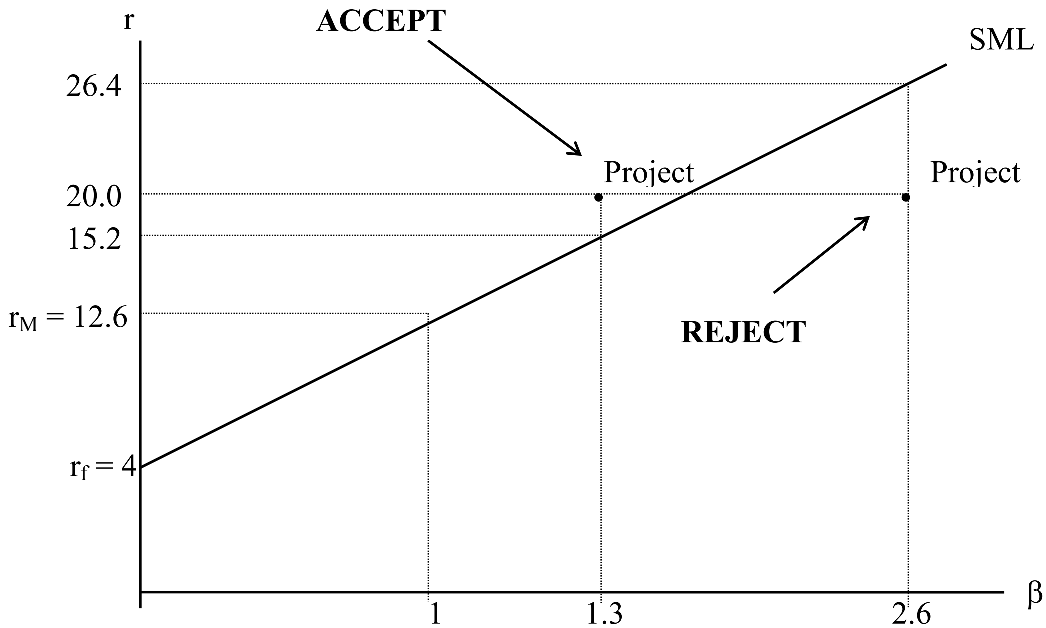
   Measured by b($r_M - r_f$.) This is the **reward for worrying**, i.e. for bearing β amount of systematic risk in addition to waiting for your money.

<u>Example</u>: Suppose r_f = 4%, $(r_M - r_f)$ = 8.6%, and β of stock = 1.3. Based on CAPM, what is the expected return of the stock?

- r = .04 + 1.3 (.086) = .04 + .112 = .152 or 15.2%
- What if the β doubled to 2.6?

  r = .04 + 2.6 (.086) = .04 + .224 = .264 or 26.4%

The security market line provides a standard for project acceptance:



Problem 10.

A share of stock with a β = 0.75 now sells for \$50. Investors expect the stock to pay a year-end dividend of \$3. The T-bill rate is 4%, and the market risk premium is perceived to be 8%. What is investors' expectation of the price of the stock at the end of the year?

- CAPM: $r = r_f + \beta(r_m - r_f) = 4 + (.75)\times 8 = 10\%$
- $r = \dfrac{Div + Capital\ gain}{Initial\ price} = \dfrac{Div + (P_1 - P_0)}{P_0} = \dfrac{3 + (P_1 - 50)}{50} = 0.10$

  P_1 = \$52

Problem 11.

Reconsider the stock problem 10. Suppose investors actually believe the stock will sell for \$54 at year-end. Is the stock a good or bad buy? What will investors do? At what point will the stock reach an "equilibrium" at which it again is perceived as fairly priced?

- fair current price, $P_0 = \dfrac{Div + P_1}{1 + r} = \dfrac{3 + 54}{1.10} = \$51.82$