Abstract

This paper examines how a central bank’s choice of interest rate rule impacts the rate of mortgage default and welfare. I do this by constructing a quantitative equilibrium (QE) model that incorporates incomplete markets, aggregate uncertainty, overlapping generations, and realistic mortgage structure. Through a series of counterfactual simulations, I demonstrate five things: 1) nominal interest rate rules that exhibit cyclical behavior increase the average default rate and lower average welfare; 2) welfare can be substantially improved by adopting a modified Taylor rule that stabilizes house prices; 3) a decrease in the length of the interest rate cycle will tend to increase the average default rate; 4) if the business and housing cycles are not aligned, then aggressive inflation targeting will tend to increase the mortgage default rate; and 5) placing a legal cap on loan-to-value ratios will lower the average default rate and lessen the intensity of extreme events. In addition to these findings, my model also incorporates an important mechanism for default, which had not previously been included in the QE literature: default spikes happen when income falls and home equity is degraded at the same time. Overall, my results suggest that the univariate time series properties of interest rates (i.e. wavelength, persistence, and variance) may play a substantial role in generating mass mortgage-default events. If a central bank wishes to avoid such crises, they should either adopt a rule that generates interest rates with slow-moving cycles or use a modified Taylor rule that also targets house price growth.

JEL Classification: E50, E52, C63, C68

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1 Introduction

In July of 2000, the Federal Reserve initiated a series of rate cuts that lowered the effective federal funds rate (FFR) from a peak of 6.54% to 1% in late 2004, dropping it below 2% by the end of 2001. During this same period, house price growth jumped from an already high rate of 9.6% to 14.6%; and grew at an average of 6.8 percentage points per year faster than it did from 1988 to 2000. In late 2004 to mid-2007, interest rates shot up once again to 5.25%; and, with a lag, house price growth peaked at 15.9% and then dropped to -4.6% by mid-2007, falling further as the recession deepened. Not long after the house price drop, the default rate on first mortgages rose from a historically stable 1% to a peak of 5.39% in 2009.

After a casual glance at the data, we might conclude that the FFR cycle that spanned late 2000 to mid-2007 can safely be blamed for Great Recession—that is, it caused the twin housing and financial crises, bridged by a mortgage default spike. Indeed, when John Taylor addressed the Jackson Hole conference in 2007, that is exactly what he argued. In this paper, I will not attempt to find the cause of the “Great Recession”; however, I will try to elucidate how monetary policy can play a role in preventing such crises. In particular, I ask: how does the central bank’s choice of interest rate rule impact the frequency and intensity of mortgage default?

To give the reader a sketch of the mechanisms that connect mortgage default crises and monetary policy, consider what happens during a typical interest rate cycle. First, assume that the real, risk-free interest rate falls sharply and remains low; and that these changes are at least weakly transmitted to mortgage interest rates. This will increase demand for housing by reducing the cost to borrow. If the increase in demand also pushes up house prices, then household equity positions will improve, allowing homeowners to borrow more (i.e. withdraw additional equity), which could increase the amount of debt that homeowners hold—and possibly increase household leverage. Finally, a drop in the risk free rate could divert marginal bank depositors to instead invest in capital, increasing output (income).

Now, assume that the first part of this cycle is followed by a sharp increase in interest rates—just as the prolonged period of low interest rates from 2001 to 2004 was followed by a sharp rise in the FFR. Suddenly, the effects of the housing boom will be reversed. Demand will collapse, pushing house prices down, and leaving households who extracted equity or bought homes near the trough of the cycle with negative equity. This will prevent households from withdrawing equity to smooth consumption or to refinance into lower interest rate loans should they become available. High interest rates will also divert investment away from capital, lowering income, and will push up adjustable-rate mortgage payments with a lag. The combination of low incomes, high mortgage payments, and negative equity positions will push up default rates, which will deteriorate financial intermediary balance sheets, and may cause a credit crunch.

Prior to the Great Recession, few DSGE models contained many of the aforemen-

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1See Figure 1 in the appendix for a comparison of the Taylor Rule-implied interest rate and the effective federal funds rate leading up to the crisis.
tioned elements. Additionally, economists had only started to consider how to model multi-period mortgage structure realistically in general equilibrium.\(^2\) And many of the macro housing models that did exist lacked heterogeneity, eliminating the effect of a prolonged house price rise completely (Jeske 2005).

In this paper, I attempt to contribute to the now-vibrant housing literature that has drawn inspiration from Aiygari (1993), and Krusell and Smith (1998). In particular, I construct a quantitative equilibrium model that incorporates incomplete markets, aggregate uncertainty, overlapping generations, price stickiness, credit-scoring, optimal default, inter-sectoral productivity correlation, and realistic mortgage structure. I then calibrate the model to match the cross-sectional and time series dimensions in the data; and run a series of counterfactual simulations under different interest rate rules to determine how they impact default rates and welfare. In addition to considering popular classes of interest rate rules (e.g. the Taylor Rule), I also look at more fundamental components of interest rate behavior by testing rules that generate cycles, but do not endogenously respond to other macrovariables, such as autoregressive rules and sine wave rules.

2 Related Literature

This paper contributes to three subliteratures. The first consists of papers that attempt to explain the Great Recession and financial crises in general. The second consists of macro papers that have attempted to integrate housing. And the final looks at the optimality of the Taylor rule and other monetary policy rules. I will review each of these literatures in order below.

2.1 Great Recession Causes Literature

Mian and Sufi (2010) find that household leverage yields considerable predictive power for the 2007-09 recession. They show that the household debt-to-disposable income ratio accurately predicts movements in aggregate variables, such as unemployment, consumer default, and house prices. They suggest that measures of household leverage could provide a well-grounded empirical basis for explaining macroeconomic fluctuations. This finding is consistent with my theme, which explores interest rate rules as the cause of household balance sheet deteriorations, which lead to increased default rates and related macroeconomic fluctuations.

In a separate paper, Mian and Sufi (2009) consider the importance of the subprime loans and their securitization in the financial crisis. They perform their analysis at the county-level; and find three important results. First, they determine that counties that experienced aggressive growth in subprime lending tended to also experience the greatest default intensities. Second, they find that the credit expansion in 2002-2005 was negatively correlated with income growth—an unexpected reversal of normal credit expansion patterns. And third, they demonstrate that subprime loans tend to be

more common in areas with decreasing income. While my paper does not explicitly incorporate subprime borrowers, it does provide substantial heterogeneity and permits a changing, endogenous asset distribution (aggregate uncertainty), which allows us to consider whether borrower quality declines prior to crises.

Other plausible explanations for the Great Recession include Mayer (2009), who attempts to explain the crisis through a decline in underwriting standards and a large post-teaser rate jump on mortgage interest rates. Bucks and Pence (2008) argue that the large cluster of defaults was caused by a change in the composition of borrowers. That is, less financially literate individuals were induced to become homeowners. When they were hit with negative income shocks, they were less able to optimize budgeting correctly (often ignoring shocks entirely), which lead to a sharp rise in defaults.

In line with the theme of this paper, Leamer (2007) argues that recessions in the U.S. have largely been driven (or preceded) by downturns in housing investment and consumer durables. He claims that monetary policy should explicitly incorporate housing investment smoothing as an objective; and suggests that a modified Taylor Rule could incorporate such an objective. Similarly, Ahearne et. al (2005) draws the connection between monetary policy and house prices. In a study of 18 major industrial countries, they find that monetary easing typically precedes an increase in housing investment and prices.

Hott and Jokipii (2012) further cement the relationship between interest rate movements and housing investment. Using a multi-country dataset, they show that deviations in interest rates from the Taylor Rule account for 50% of the housing overvaluation that occurred prior to the Great Recession. Assesnmacher-Wesche and Gerlach (2008) find a similar relationship between interest rates and house prices using a 17-country VAR that spans 1986-2006. They find that a 25 basis point increase in short term rates pushes down GDP by 0.125% and housing prices by 0.375% with a lag. However, in contrast to Hott and Jokipii, they argue against using interest rates to smooth house prices.

Finally, Foote, Geraradi, Goette, and Willen (2008) provide an additional empirical nuance to the debate. They show that borrowers do not simply default if they have negative equity or if they receive a negative income shock, but rather, they default if both conditions are present. They call this the “double trigger” condition for mortgage default—a term that originated in finance.\footnote{See Abraham 1993.} To this author’s knowledge, this paper is the only rigorously-microfounded model of default in general equilibrium that replicates this empirical regularity.

### 2.2 Macro-Housing Literature

This paper also contributes to the macro-housing literature, which has become increasingly focused on heterogeneity, incomplete markets, and aggregate uncertainty. Early papers in the literature, such as Yang (2006), demonstrated the importance of lifecycle elements in housing; and devised mechanisms for modeling them correctly. Li and Yao
(2007) showed that the impact of house price changes depend on the degree of heterogeneity in the economy. Indeed, depending on how agents are modeled, price changes may have no effect on the macroeconomy, but can cause inter-group wealth transfers.

Ortalo-Magne and Rady (2005) contribute one of the foundational papers in this literature. They incorporate life-cycle elements, property ladders, and credit constraints into an equilibrium model with many heterogenous agents. While they greatly simplify the life-cycle elements (by including only four periods) and build a highly stylized model with no aggregate uncertainty, they still introduce a housing market prototype that is reused elsewhere in the literature. In particular, they assume that property comes in two varieties: “flats” and “houses”; and explicitly model the utility-based differences between the two. As for credit constraints, they assume that wealth cannot fall below some fraction of the value of the property. Finally, they assume that the supply of both flats and housing is fixed.

Chatterjee, Corbae, and Rios-Rull (2009) expand the literature further by incorporating default and credit scoring into a large scale macro context. They model agents who take out loans and repay them with an unknown probability. This leads to a system of credit-scoring, where lenders use a known credit-scoring function and pricing kernel to create loan terms. While this model provides a reasonable structure for incorporating default into a model with heterogenous agents, it can stay little about the timing and cause of default, since default probabilities are determined by an agent’s type (good or bad). In contrast, I allow agents to default optimally and use decision rules to compute the probability of default.

Other papers in this literature use the new class of rigorously-microfounded macro-housing models to run policy simulations. Jeske & Krueger (2005), for instance, find that mortgage interest rate subsidies tend to benefit and to increase homeownership rates among high-income and high-net worth individuals in general.

Another important part of modeling default choices is determining the structure of mortgages, since this may play a large role in determining default behavior. While there are a number of continuous time finance models that permit variation in payment schedules and default timing in a partial equilibrium context, this is not true in general for DSGE models, which frequently do not even model housing. In the wake of the Great Recession, however, a number of authors have attempted to create a serious role for mortgage structure in DSGE models. Most notably, Chambers, Garriga, and Schlagenhauf (2009) have constructed a general equilibrium model that permits individuals to choose between FRMs and ARMs—and even to obtain combo or piggyback loans. While I do not incorporate mortgage choice into my model, I do borrow modeling devices from the paper, and work to make them tractable in an environment with default.

Perhaps most closely related to this paper, Iacoviello and Pavan (2010) create a quantitative equilibrium model that accurately reproduces empirical co-movements in aggregate debt accumulation and housing investment. In their paper, they introduce (and borrow) several computational macro modeling devices that permit them to perform simulations in a realistic environment, but without making the computational
stage intractable. In particular, they use deterministic life-cycle productivity profiles to generate income heterogeneity, small pension payments after retirement to generate lifecycle asset accumulation motives, lump-sum taxes to simplify interactions with government, housing transaction costs that are proportional the to change in housing position (to make changes larger and infrequent), and a simple no-arbitrage condition to determine the price of housing. My paper borrows many of the modeling devices from Iacoviello and Pavan (2010), and adds optimal default, individual credit-scoring, inter-sectoral productivity correlation, and mortgage structure—and applies them to a different research question.

Goodhart, Osario, and Tsomocos (2009) attempt to advance the literature by creating a template for a new generation of macro-financial models. While they use only a handful of representative agents (unlike the other referenced housing papers), they model both default and complex linkages between the financial sector and the macroeconomy. They include corporate lending, interbank lending, and central bank lending into a model where most agents can “default.” However, since only a handful of representative heterogenous agents are used, individual agents do not default in a strict sense—rather, they choose a repayment rate. Banks respond to this by using rational expectations to predict repayment rates and to penalize default. Additionally, interbank and central bank lending is used to rescue banks in the event of mass defaults.

2.3 Optimal Monetary Policy

Finally, this paper contributes to the part of the optimal monetary policy literature that examines the welfare properties of interest rate rules. Woodford (2001), a landmark paper in the subliterature, considers whether the Taylor rule can be rationalized through a plausible central bank objective. He does this by constructing a simple model and testing its welfare properties. He finds that Taylor-style rules perform well, but suffer from two problems: 1) they rely on the output gap being measured correctly; and 2) they do not vary with the Wicksellian natural rate of interest, but instead assume a fixed natural interest rate. He suggests that future work should attempt to “analyze the consequences of inertial rules in the context of more detailed models.” This is one of the primary objectives of this paper.

Julliard et al (2006) test the welfare properties of the Taylor rule by constructing a DSGE model, using Bayesian methods to estimate its parameters, and performing counterfactual simulations under different parameter values. They find that the standard Taylor rule performs reasonably well. Ahrend (2010), on the other hand, performs an empirical investigation of the Taylor rule in practice; and finds that central bank departures from the rule can lead to substantial increases in asset prices. This paper explores both of these topics in a theoretical context.

Other papers, such as Giannoni (2012) find that simple Taylor-style rules may be inferior to Wicksellian rules, which permit some type of history-dependence. Giannoni (2012) claims that such rules are less prone to indeterminacy and are more robust to model mispecification; and argues that they are especially effective when coupled with
a “high degree of interest rate inertia.” Forlati and Lambertini (2011) also consider interest rate inertia, but do it in the context of richer model that includes housing. They find that inertial interest rate rules lead to larger contractions in output.

In summary—I expand on the macro-housing literature by adding optimal default, credit scoring, inter-sectoral productivity correlation, and mortgage structure to a quantitative equilibrium model. I add to the largely empirical Great Recession and financial crisis literature by performing counterfactual simulations using a theoretically consistent framework and a rigorously-microfounded model. And, finally, I add to the optimal monetary policy literature by evaluating a variety of different interest rate rules to determine their impact on default and welfare in a detailed model.

3 The Model

In order to answer the questions above, I start by constructing a macro-housing model in which agents default optimally. That is, default is not forced, arbitrary, or determined by type, but emerges from optimization. In building this model, my primary goal is to attack the problem as simply as possible, but with enough detail to capture the important features of default and housing choices. For this reason, I construct a rigorously-microfounded model in the style of Aiyagari (1993), and Krusell and Smith (1998).

The initial formulation will focus on a real model with capital and endogenously-determined house prices. However, I will later modify the model by removing capital and the endogenous component of house prices. I will also explain how sticky and flexible prices are added to the model.

In the model, there are infinitely many heterogeneous households of measure 1, who consume non-durables, invest in capital, purchase housing, and choose whether or not to default on mortgage debt. Firms produce a non-durable good with Cobb-Douglas technology. There is a representative financial intermediary, which accepts deposits from households, and then uses those deposits to issue mortgages to households. The government collects lump-sum taxes, pays pension benefits, insures deposits at financial intermediaries, and collects housing from deceased agents. Finally, a central bank implicitly determines the risk-free return on deposits by setting the interbank lending rate.

3.1 Firms

The firm side of the economy consists of 1) a consumption good producer who rents capital and labor services; and 2) a technology that permits all households to transform the consumption good into housing units.

3.1.1 Consumption Goods

The consumption goods are produced using Cobb-Douglas technology:
\[ Y_t = e^{A_t} K_t^\alpha N_t^{1-\alpha}, \]  

where \( A_t = c_A + \rho A_{t-1} + \epsilon_A, \epsilon_A \sim N(0, \sigma_A), \) and \( \mu_A = \frac{c_A}{1-\rho}. \) Firms maximize profits, yielding the familiar first order conditions:

\[ w_t = (1 - \alpha) e^{A_t} \left( \frac{K_t}{N_t} \right)^\alpha \]  

\[ R_t = \alpha e^{A_t} \left( \frac{N_t}{K_t} \right)^{1-\alpha} - \delta K, \]

Note that capital is assumed to depreciate at a constant rate, \( \delta K. \) Furthermore, \( N_t \) is the mass of employed workers. The mass (or fraction) of workers employed in any given period is derived from the conditional Markov process for employment and the assumption that households supply labor inelastically. For simplicity, I have assumed that this process does not depend on housing investment.

For computational purposes, I discretize \( A_t \) using the Rouwenhorst (1995) method,\(^4\) which Kopecky and Suen (2009) argue is more accurate than other approximation algorithms for highly persistent AR(1) processes. In particular, this method allows me to generate a discrete approximation of an AR(1) process by setting four parameters and the desired number of states: \( \rho_A, q_A, \sigma_A, \) and \( \mu_A, \) where \( \rho_A \) is the probability of the highest state, \( q_A \) is the probability of the lowest state, \( \sigma_A \) is the desired standard deviation of the process, and \( \mu_A \) is the desired mean of the process. The algorithm generates the Markov chain and the associated transition probability matrix.

To pin down the mass of employed workers, I use a conditional Markov process that depends on the technology shock. That is, roughly speaking, when the technology shock is high, the probability of transitioning into employment will be high (and vice versa). More specifically, I will calibrate the conditional Markov process to generate unemployment statistics that match U.S. business cycle data.

I use \( Pr(\epsilon_E' | \epsilon_E, \epsilon_A) \) to denote the probability of transitioning to employment state \( \epsilon_E', \) given last period’s employment state, \( \epsilon_E, \) and this period’s technology shock state, \( \epsilon_A. \) Note that the Markov chain will remain the same in all periods; however, the transition matrix will change, depending on the state of the technology shock, which means that we will have a different transition matrix for each state of \( \epsilon_A. \)

### 3.1.2 Housing Investment

The housing investment specification is similar to Glover, Heathcote, Krueger, and Rios-Rull (2011). In particular, I assume that households have access to a linear technology that transforms the consumption good into housing. That is, if the household

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\(^4\)See Appendix 1 for an explanation of the Rouwenhorst method.
builds with $c^h_t$ units of the consumption good, it will yield $h^h_t$ new units of housing:

$$h^h_t = \delta(IH_t)c^h_t e^{U_t},$$  

(4)

where $U_t = u_H + \rho_H + \epsilon_H$, where $\epsilon_H \sim N(0, \sigma_H)$, $IH_t$ is aggregate housing investment, $\delta(IH_t)$ is an analogy to capacity utilization,\(^5\) and $\delta'(IH_t) < 0$.

Notice that the housing specification implies a relative price limit. No one will pay more than $\frac{1}{\delta(IH_t)e^{U_t}}$ for a unit of housing, since it is possible to generate one using that many units of the numeraire good. Additionally, I assume that investment is reversible,\(^6\) so no one will sell housing for less than that price. This implies that house prices will rise when the economy is hit by a negative housing sector productivity shock or when housing investment demand increases.

Note that total housing investment can be written as follows:

$$IH_t = \sum_{i \in I} \delta(IH_t)c^h_i \mu_i e^{U_t} = \delta(IH_t)C^h_t e^{U_t},$$  

(5)

where $\mu_i$ is agent i’s mass and where $IH_t$ and $C^h_t$ are used to denote aggregates. Additionally, the housing stock evolves as follows:

$$H_{t+1} = H_t + IH_t - \delta_H H_t,$$  

(6)

where $\delta_H$ is housing stock depreciation.

### 3.2 Households

Households are born at $a=1$ and work until $a=T$. After retirement, households receive a pension, $x_t$, for $T^R$ periods, and then perish with certainty at age $T+T^R$. Note that this generates a hump-shaped paper asset profile for households, since they must accumulate assets in order to smooth post-retirement consumption.

At any point in time, heterogeneity across households is driven by two mechanisms: 1) employment status ($\epsilon^E_{it} = 1$ or 0); and 2) age-specific productivity, $\eta_a$. Following Heer and Maussner (2008), I assume that the former is generated by a conditional Markov process that depends on non-durables shocks (as given in the firms section). For the latter, I follow Iacoviello and Pavan (2010) in adopting a single, deterministic profile for age-specific productivity, which is computed using CPS data. These two mechanisms for heterogeneity will drive differences in asset holdings and default decisions.

Unemployed agents receive per period unemployment benefits, $x^U_t$, for the dura-

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\(^5\)That is, when housing investment is high, the capacity of the sector to produce an additional unit is increasingly strained, making it more costly.

\(^6\)While this is not a particularly realistic assumption, it should not have a qualitative impact on welfare and default results, since house prices in the model fall when investment is reversed.
tion of their jobless spells. Similarly, retired agents will receive pension benefits of $x_t^R$. Employed household $i$, which is age $a$ at time $t$ receives a wage, $w_t \eta_a$, where $\sum_{a=1}^{T} \eta_a \mu_a = 1$, where $\mu_a$ is the density of age cohort $a$.

In the baseline model, households consume non-durable goods, $c_{it}$, and housing service flows, which are assumed to be directly proportional to their housing stock, $h_{it}$. They choose how much to save in bank deposits, $d_{it}$, how much capital to hold, $k_{it}$, how much collateralized debt to borrow in the form of mortgages, $b_{it}$, and whether or not to default on the mortgage they hold.

Investment in housing is lumpy; that is, households tend to make large and infrequent changes in housing size (i.e. by moving), rather than changing housing size frequently and in small increments. Here, I follow the standard assumption that lumpiness is generated by housing stock adjustment costs, $\phi(h_{it}, h_{it-1})$, which depend on the size of the new and old housing stock.

Furthermore, I adopt Iacoviello and Pavan’s (2010) assumption that houses have a minimum size, $h$. While they principally use this assumption to match the empirical fact that younger households tend to be renters (and that, in fact, households cannot purchase very small houses), I use this device largely to generate household leverage. When young households enter the model, they must take out a large mortgage in order to purchase a house. The high degree of leverage will translate into interest rate risk exposure—and, thus, a higher probability of default.

Additionally, I assume that all households have access to a small, fixed amount of non-housing shelter. This includes both defaulters and young households who have not yet purchased a home. This non-housing shelter can be interpreted as living with friends or relatives—or staying in a low-quality, but free apartment.

For simplicity, households are assumed to supply labor inelastically. Additionally, we may write the household’s instantaneous utility from non-durables consumption and housing service flows as follows:

$$u(c_{it}, h_{it}) = \gamma \frac{c_{it}^{1-\sigma_c}}{1-\sigma_c} + (1-\gamma) \frac{h_{it}^{1-\sigma_h}}{1-\sigma_h}$$  \hspace{1cm} (7)

This specification is compatible with Jeske’s (2005) finding that the ratio of housing to consumption tends to be hump-shaped over the life-cycle. Following Chambers, Garriga, and Schlafenhauf (2009), we set the curvature parameters for the utility function to $\sigma_c = 1$ (log utility) and $\sigma_h = 3$, which will give us a hump-shaped profile for $\frac{h}{c}$ over the lifecycle that matches the data.

Individuals have two sources of income: wages, net of taxes ($\Gamma_{a,t}$), from working at the

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7In the baseline specification, unemployed agents will receive benefits and will not be taxed. In the original set of simulations, I considered a version of the model without unemployment benefits. My findings did not differ qualitatively for the two versions of the model.

8The model is later extended to incorporate apartment rentals.

9For a discussion of the impact that non-housing shelter has on the model, see the appendix.
consumption goods firm and pensions (or unemployment benefits):

\[
y_{it} = \begin{cases} 
  w_{it}n_{it} - \Gamma_{a.t} & \text{if employed} \\
  x_{it} & \text{if unemployed or retired}
\end{cases}
\]  
(8)

Household i faces the following budget constraint:

\[
c_{it} + \phi(h_{it}, h_{it-1}) + d_{it} + p_{it}^h h_{it} + m_{it}^h + k_{it} = y_{it} + (1 + r_{t-1})d_{it-1} + p_t \\
  h_{it-1}(1 - \delta_H) + (1 + R_t)k_{it-1} + b_{it}
\]  
(9)

Note that \( p_{it}^h \) is the relative price of housing, \( m_{it} \) is the mortgage payment, \( b_{it} \) is the unpaid balance on the mortgage, \( R_t \) is the return to capital, and \( k_{it} \) denotes i’s capital holdings.

I depart from the current literature in applying a novel constraint that makes holders of one-period mortgages behave as if they held long term debt instruments:

\[
b_{it}^H \leq \begin{cases} 
  \lambda p_{it}^h h_{it} & \text{if } h_{it} - h_{it-1} > 0 \\
  \lambda p_{it}^h h_{it} & \text{if } b_{it-1} < \lambda p_t h_{it} & h_{it} = h_{it-1} \\
  b_{it-1} & \text{otherwise},
\end{cases}
\]  
(10)

where \( \lambda \in (0, 1) \) denotes the collateral constraint–that is, the maximum loan-to-value ratio.

According to the equation above, if homeowners move (adjust their housing stock), then they face a collateral constraint in that period, since they must obtain a new mortgage. Similarly, if they do not move, but carry forward less debt from the previous period than would be allowed by the collateral constraint in this period, then they have the option to borrow up to the constraint. Finally, if they did not move and have exceeded the collateral constraint, then they cannot borrow more, but do not have to reduce the size of their mortgage. The intent of these constraints is to achieve the following with one-period mortgages:

1. Avoid “forcing” default. In many endogenous default models, default is ultimately forced by a collateral constraint that is repeatedly applied to one-period mortgages. Empirically, this constraint only applies at origination–and not to existing loans. If it is forcefully applied to existing loans (i.e. by modeling them as repeated one-period loans), households will default whenever they are unable to borrow enough to repay last period’s debt (i.e. if house prices fall steeply). This generates a spurious channel for default (i.e. negative equity immediately triggers default), which is simply an artifact of one-period loan financing.
2. Allow mortgage equity withdrawal (MEW). Since we are interested in understanding how the path of interest rates impacts default, allowing for MEW may provide a critical channel for default. If households can borrow against the values of their homes, this may cause certain interest rate rules to generate a higher default rate.\footnote{Certain classes of interest rate rules may make it favorable to borrow against the value of your home immediately prior to a hike in interest rates, which will lead to a high degree of interest rate risk.}

3. Allow negative equity. In one-period loan models with collateral constraints, households typically cannot have negative equity, since it will violate the collateral constraint. However, in reality, if the price of housing falls, but individuals choose to remain in their homes, there is no constraint that forces them to maintain positive equity. Since negative equity is an important part of most default crises, this constraint is maintained to generate more realistic household balance sheets.

Overall, this constraint will make it possible to maintain the simplicity of a one-period loan framework, while simultaneously generating household balance sheets and default behavior that more closely approximate what we would observe if 30-period debt instruments were available.

In addition to this, I borrow a constraint from Iacoviello and Pavan (2010) that limits borrowing to a fraction, $\gamma$, of discounted, remaining lifetime earnings:

$$b_{it}^I = \gamma E_t \sum_{j=t}^{T-a+j} \beta^{T-a+t} y_{ij} \quad (11)$$

The purpose behind this constraint is to impose a feasibility condition on repayment. If households cannot reasonably be expected to repay a mortgage with their remaining income flows, then a financial intermediary will not be willing to originate it. Empirically, this is similar to the income and debt-servicing ratios that banks require borrowers to meet; however, it is tied to expected, discounted future income, rather than just current-period income and assets.\footnote{An alternative–and arguably more realistic–specification of this constraint might incorporate current asset holdings. The impact of including this modification would be to permit more borrowing later in the life cycle, since there is no within-cohort heterogeneity. For the purposes of this paper, I do not consider this constraint explicitly, but may add it as a future extension.}

This constraint, coupled with the previous one, yields the final borrowing constraint:

$$b_{it} \leq \min\{b_{it}^H, b_{it}^I\} \quad (12)$$

That is, the maximum amount a household can borrow is the minimum implied by the two borrowing constraints. Furthermore, mortgage interest rates are adjustable and are given as follows:

$$r_{it}^* = \left(\frac{r_t}{1 - q_t} + \xi_t\right) \quad (13)$$
Note that \( r_t \) is the rate earned on deposits, \( q_{it} \) is household \( i \)'s probability of default (computed from household decision rules),\(^{12}\) and \( \xi_t \) is the mortgage premium.

Default, \( \psi_{it}^d \), is captured by a binary variable. Defaulters are not able to re-enter the mortgage market for a period of time, \( f_p \). A separate binary variable, \( \psi_{it}^c \), denotes whether an individual has defaulted recently enough to be excluded from the mortgage market.\(^{13}\)

In contrast to Chatterjee (2009) and other papers in this literature, I assume that the housing market exclusion period is fixed, rather than random, for the sake of tractability. Furthermore, when a household defaults, the model requires that \( h_{it} = 0 \) and \( b_{it} = 0 \). That is, the housing stock (which serves as collateral) is transferred to the financial intermediary and the mortgage debt is eliminated.\(^{14}\)

Finally, note that some households in the model will perish with housing remaining. In the literature, it is common for the dying generation to either bequest the housing to the incoming generation or turn it over to the government. For simplicity, we assume that the government takes the housing when the outgoing cohort perishes.\(^{15}\)

Now that all of the pieces of the model in place, we may collect the individual-level state variables, \( z_{it} = \{ d_{it-1}, \psi_{it-1}^d, \psi_{it-1}^c, h_{it-1}, b_{it-1}, k_{it-1}, \epsilon_{it}, a \} \), the aggregate-level state variables, \( Z_t = \{ K_t, A_{t-1}, U_{t-1}, IH_t, \epsilon_{At}, \epsilon_{Ut}, \epsilon_c \} \), and the parameters \( \Omega = \{ \alpha, \sigma_c, \sigma_h, \gamma, \lambda, \rho_A, \rho_U, \sigma_U, \sigma_A, \delta_K, \delta_H, c_A, c_U, \sigma_{A,U} \} \) to simplify notation. The dynamic programming problem (DPP) for the household may now be written as follows:

\[
V_{it}(z_{it}, Z_t; \Omega) = \max \{ c_{it}, d_{it}, k_{it}, b_{it}, \psi_{it}^d \} u(c_{it}, h_{it}) + \beta \sum_{A', U', \epsilon' \in \{1,0\}} \Pr(A') \Pr(U') \Pr(\epsilon'|\epsilon) V_{it+1}(z_{it+1}, Z_{t+1}; \Omega)
\]

s.t.

\[
c_{it} + \phi(h_{it}, h_{it-1}) + d_{it} + p_{it}^b h_{it} + m_{it}^b + k_{it} = y_{it} + (1 + r_{it-1}) d_{it-1} + p_{it}^b h_{it-1}(1 - \delta_H) + (1 + R_t) k_{it-1} + b_{it}
\]

\[
b_{it} \leq \min\{b_{it}^H, b_{it}^J\}
\]

If \( \psi_{it}^c > 0 \), then \( b_{it}, h_{it} = 0 \).

\[
m_{it} = \begin{cases} r_{it-1}^{-1} b_{it-1} & \text{if } \psi_{it}^d = 0 \\ 0 & \text{if } \psi_{it}^d = 1 \end{cases}
\]

---

\(^{12}\)See the appendix for a shortcut for computing default probabilities without using the decision rules.\(^{13}\)Note that defaulters have access to non-housing shelter, which means that they will not receive negative infinity utility from defaulting.\(^{14}\)It is important to note that 1) the foreclosure process is costly—and, thus, the amount recovered will be less than the value of the house prior to the foreclosure; and 2) the value of housing at default will not exceed the size of the mortgage. If it did, the household would simply sell it, rather than defaulting.\(^{15}\)Note that this will have a fairly insignificant impact on government’s budget constraint, since each perishing cohort accounts for 1/60th of the population and will tend to draw down its housing position near the end of the lifecycle.
This problem is solved using a custom approximate dynamic programming (ADP) algorithm, which is described in the appendix.

### 3.3 The Financial Intermediary

I adopt a largely novel specification for the financial intermediary that generates a number of desirable results related to mortgage pricing and solvency. In particular, I place structure on the financial intermediary’s objective in order to obtain simple decision rules without solving a dynamic programming problem. I assume the following:

1. Deposits made at financial intermediaries yield the risk free rate, \( r_{t-1} \).
2. Households may obtain competitively-priced, one-period mortgages from financial intermediaries, which are subject to the constraints given in the housing section.
3. Financial intermediaries are risk-neutral.
4. There are infinitely many financial intermediaries, which are represented by a single financial intermediary with zero net cashflows.
5. Financial intermediaries use rational expectations (i.e. a household’s decision rules) to determine a household’s probability of defaulting, \( q_{it} \).
6. Financial intermediaries add a state-contingent premium, \( \xi_t \), to the mortgage interest rate in order to generate zero net cash flows.
7. The foreclosure process is costly, leaving financial intermediaries with only a fraction, \( \Lambda \) of the housing, which they liquidate in the same period at the market rate, \( p_H \).

Using these assumptions, the intermediary sets the mortgage payment for household \( i \), who obtained a loan in period \( t \) as follows:

\[
m_{it+1} = (1 + r_{it}^*) b_{it}, \tag{19}
\]

where \( b_{it} \) is the size of the mortgage. Notice that the interest rate on the mortgage contains two components: 1) \( (1 + r_{it}^*) \), which is specific to the individual and accounts for idiosyncratic default risk; and 2) a spread component, \( \xi_t \), which is identical for all borrowers and clears the market.

Furthermore, note that the intermediary will receive \( m_{it} \mu_i \) when a household repays a loan originated at time \( t-1 \) and \( \Lambda p_H h_{it-1} \), when it does not. Thus, in order to obtain zero net cashflows from period \( t \) loans and deposits, it must set \( \xi_t \) to solve the following equation:

\[
(1 + r_t) \sum_i d_{it-1} + \sum_i b_{it} = \sum_i 1(\psi^d_{it} = 0)m_{it} + p_H \sum_i 1(\psi^d_{it} = 1)h_{it-1}(1 - \delta_H) + \sum_i d_{it}
\]

\[
\text{Outflows} \quad \text{Inflows} \tag{20}
\]
Notice that $d_{it-1}$, $q_{it-1}$, $h_{it-1}$ are all predetermined at time $t$. Thus, the intermediary sets $\xi_t$ to reduce or increase mortgage volume until net cashflows are zero.\(^{16}\)

### 3.4 The Government

The government has one function in the model: to make transfer payments to retired and unemployed individuals using taxes collected. For simplicity, the government is assumed to use a constant replacement ratio, $\zeta$. That is, it transfers $\zeta w_t$ to retired and unemployed individuals.

#### 3.4.1 Transfers

In order to cover payments to the unemployed and retired, the government must allocate the following amount to outgoing transfer payments:

$$
\tau_t = \zeta w_t \left( \sum_{a=T^R+1}^{T^R+T} \mu_a + \sum_{a=1}^{T^R+T} (1 - \epsilon_{at}^E) \mu_a \right)
$$

(21)

Note that $\sum_{a=T^R+1}^{T^R+T} \mu_a$, the mass of retired individuals, is constant in this model, so it may be rewritten as $\mu^R$. The mass of unemployed, $\sum_{a=1}^{T^R+T} (1 - \epsilon_{at}^E) \mu_a$, changes over time, so it is denoted by $\mu^U_t$. This yields:

$$
\tau_t = \zeta w_t (\mu^R + \mu^U_t)
$$

(22)

That is, the government must collect enough in taxes to pay the mass of retired, $\mu_R$, and the mass of unemployed, $\mu^U_t$, $\zeta w_t$ in transfers.

#### 3.4.2 Revenue

For simplicity, I assume the following about taxes: rates scale with productivity and unemployed agents do not pay taxes. With these assumptions, the tax for employed households in cohort $a$ can be written as follows:

$$
\Gamma_{at} = \zeta w_t \eta_a \epsilon_{at}^E \mu_R + \mu^U_t \frac{h^D_t}{(1 - \mu^U_t) - \mu^U_t^+}
$$

(23)

Note that $h^D_t$ denotes the housing stock that households turn over to the government in period $t$ after perishing. Additionally, notice that this specification for taxes will

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\(^{16}\)There are two important things to note. First, in practice, we use a state-contingent function to set $\xi_t$, rather than setting it in all periods. This adds tractability; and is discussed more in the appendix. Second, $\xi_t$ will also have an impact on the default rate and deposits at time $t$, but the effects will be substantially smaller than those on mortgage volume.
require the government to maintain a balanced budget at all times. That is, aggregate incoming transfer payments are equal to aggregate outgoing transfer payments:

\[ \Gamma_t = \sum_{a=1}^{T+R} \left( \zeta w_t \epsilon_t^E \mu_R + \mu_t^U \right) \frac{h_t^D}{\left(1 - \mu_t^U \right)} - \frac{h_t^D}{\left(1 - \mu_t^U \right)} \eta_a \mu_a + h_t^D = \zeta w_t (\mu_R + \mu_t^U) = \tau_t \] (24)

To see why this is the case, recall that \( \sum_{a=1}^{T} \eta_a \mu_a = 1 \). Since \( \eta_a = 0 \) and \( \epsilon_t^E = 0 \) when \( a > T \) (i.e. individuals are retired), it will also be the case that \( \sum_{a=1}^{T+R} \eta_a \mu_a = 1 \). Thus, \( \sum_{a=1}^{T+R} \eta_a \epsilon_t^E \mu_a = (1 - \mu_t^U) \), which gives us the above equation.

Note that three components of the tax collected from households vary: 1) the wage; 2) the age-specific productivity component; and 3) the unemployment rate, \( \mu_t^U \). When an individual’s productivity component is higher (i.e. the individual is earning more), she will pay more in taxes. This is also true if the wage is higher, which results in a generally progressive tax. However, an increase in unemployment will still tend to increase taxes, since the tax base will decline. However, if the wage also declines, this effect may be limited.

In order to test the magnitude of the these effects, I used data from the autoregressive rule simulation with capital and endogenous house prices, which is described later in the paper. I found that the average tax rate was positively correlated with both the consumption goods sector shock and aggregate income, which suggests that any countercyclical tax behavior generated by a shrinking tax base (as described above) is dominated by the magnitude of wage changes. That is, in the model, aggregate tax revenue tends to fall when output falls and rise when output rises.

### 3.5 The Central Bank

I assume that the central bank sets the interbank lending rate. Since each financial intermediary is indifferent between borrowing from other banks and from households, the interbank lending rate will also determine the risk-free rate earned on deposits, \( r_t \).

Since the model employs a representative financial intermediary, net interbank lending is zero and is excluded. In the first set of exogenous interest rate rule simulations, I will assume that the central bank adopts an autoregressive interest rate rule. I will then test the properties of this rule by varying the autoregressive coefficient to determine how interest rate persistence impacts default and welfare.

In addition to autoregressive rules, the central bank will also employ other exogenous interest rate rules, such as a sine wave rule, in different simulations. The purpose of this exercise will be to capture the impact of certain interest rate behaviors on default.

The central bank will also employ a number of different rules that endogenously

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17This might happen if the consumer goods sector is hit with negative productivity shock, since this will have a negative effect on wage through the productivity decline, but a positive effect through the increase in unemployment.
respond to the economy’s dynamics. In different simulations, these rules will incorpo-
rate house price level targeting, inflation targeting, house price inflation targeting, and
output gap targeting. The Taylor rule and modifications of it will also be tested to
determine the optimal coefficients for reducing default and maximizing welfare.

Finally, it is important to note that inflation will have real effects in my model,
even in the absence of sticky prices. Since agents are heterogenous and the model
incorporates realistic mortgage structure, an increase in inflation will reduce the real
value of mortgage debt. Furthermore, inflation will tend to increase the market value
of homes relative to the size of the mortgage contracts that were used to purchase
those homes. This will improve homeowner balance sheets by increasing home equity
positions. Ultimately, these effects (and others) make it possible to perform all of the
analysis in a flexible price–rather than sticky price–framework. However, I will defer
further discussion of sticky prices to later sections and the appendix.

## 3.6 Aggregate Consistency Conditions

In addition to satisfying individual-level constraints, the economy is also subject to
aggregate consistency conditions. Each constraint requires an aggregate-level variable
to be equal to the weighted sum of the individual-level variables:

\[
K_t = \sum_{a=1}^{T^{R+T}} k_{at} \mu_a \tag{25}
\]

\[
N_t = \sum_{a=1}^{T^{R+T}} \epsilon_{at} \mu_a \tag{26}
\]

\[
C_t = \sum_{a=1}^{T^{R+T}} c_{at} \mu_a \tag{27}
\]

\[
\Gamma_t = \sum_{a=1}^{T^{R+T}} \Gamma_{at} \mu_a \tag{28}
\]

\[
B_t = \sum_{a=1}^{T^{R+T}} b_{at} \mu_a \tag{29}
\]

\[
D_t = \sum_{a=1}^{T^{R+T}} d_{at} \mu_a \tag{30}
\]

\[
H_t = \sum_{a=1}^{T^{R+T}} h_{at} \mu_a \tag{31}
\]

\[
C_t^h = \sum_{a=1}^{T^{R+T}} c_{at}^h \mu_a \tag{32}
\]
\[ \Phi_t = \sum_{a=1}^{T_{R+T}} \phi(h_{at}, h_{at-1}) \mu_a \] (33)

\[ Y_t = C_t + IH_t + K_t - (1 - \delta_K)K_{t-1} + \Phi_t \] (34)

3.7 Price Stickiness

The model was originally solved and simulated with sticky prices. After further evaluation, it became clear that sticky prices were not an essential feature of my model—and, thus, were removed to highlight the importance of other mechanisms. Below, I briefly describe the version of sticky prices that were originally incorporated into the model. It is important to note that their inclusion in the model did not result in qualitatively different results. Other than that, I will restrict further discussion of the sticky price case to the appendix.

In order to generate sticky prices, I adopt an approach similar to the one taken in Chari, Kehoe, and McGrattan (2000) and Taylor (1979, 1980), but in a cashless economy. That is, there is a final goods producer who assembles intermediates with the following production function:

\[ Y_t = \left[ \int Y_t(j) \, di \right]^{\frac{1}{q}}, \quad 0 < q \leq 1 \] (35)

Additionally, there is a continuum of intermediate goods firms who use Cobb-Douglas technology to produce individual varieties:

\[ Y_t(j) = e^{A_t} K_t(j)^\alpha N_t(j)^{1-\alpha}, \] (36)

Intermediate goods firm j will choose \( P(j), K(j), \) and \( N(j) \) to maximize profits:

\[ \pi_t(j) = P_t(j)Y_t(j) - RK_t(j) - W_tN_t(j) \] (37)

I assume that only half of intermediate goods firms are able to set prices in each period.\(^{18}\) For instance, in period t, all j, such that \( j \in [0, 0.5) \) will set prices for periods t and t+1; and in period t+1, all j, such that \( j \in [0.5, 1] \) will set prices for periods t+1 and t+2. The price set by intermediate goods firms in period t is denoted by \( \bar{P}_t \). This yields the following optimal price for intermediate goods firms:

\[ \bar{P}_t = E_t \left[ P_t^\theta V_t Y_t + (1 + r_t)^{-1} P_{t+1}^\theta V_{t+1} Y_{t+1} \right], \] (38)

where \( \theta = \frac{2 - q}{1 - q} \) and \( V_t \) is the minimized unit cost of production. Finally, the zero profit condition yields the following equation for the price index:

\[ P_t = \left[ \frac{1}{2} \bar{P}_{t-1}^\frac{q}{1-q} + \frac{1}{2} \bar{P}_t^\frac{q}{1-q} \right]^{\frac{1}{q}} \] (39)

\(^{18}\)In the flexible price specification, all firms can set prices in all periods.
In each period, I solve for $P_t$ using an approximation scheme, which is outlined in the appendix.

4 Model Properties

Now that all of the pieces of the model are in place, we may take a more careful look at the economy, starting with the characteristics of the individuals who default. Figure 2 shows cumulative density function (CDF) plots of the age, income, home equity, and capital holding for simulated households. The plots show defaulters in red and non-defaulters in blue.

There are four useful things to take away from these plots. First, all defaulting households have negative equity positions, but not all non-defaulters have positive home equity positions. As outlined earlier, negative equity is a necessary—but not sufficient—condition for default. Second, the income of defaulters is low relative to non-defaulters. In fact, 47.86% of defaulters are unemployed; whereas, only 5.7% of non-defaulters are unemployed. This should not be surprising, since income shocks are the second trigger. Third, defaulters tend to be substantially younger. In fact, all defaulters in this particular simulation were under age 50. Individuals over age 50 typically have sufficient home equity and capital holdings to endure shocks to income and equity. Finally, the capital stocks of non-defaulters tend to be considerably higher. There are two reasons for this: 1) having more capital helps agents to smooth shocks that affect wages and house prices, which makes them less likely to default; and 2) the types of agents who are unlikely to default (i.e. older agents with substantial home equity) have had more time to accumulate capital to smooth consumption during retirement.

In addition to looking at default, I also consider other properties of individual-level variables in the model. The household shown in Figure 3 is drawn from the baseline model's simulation and is used to demonstrate the degree of heterogeneity in the model. Note that the household shown experiences multiple unemployment spells, defaults at an early age, holds a mortgage until age 60, and never manages to accumulate a substantial amount of assets. As a result, they experience unusually low consumption in retirement. Figure 4 shows the average household lifecycle profile for comparison. Note that Figure 4 replicates the stylized facts for U.S. homeowners according to Jeske (2005). That is, they accumulate housing from age 25 until age 50-59, but then stay in the same house or move into a smaller one thereafter. Additionally, non-housing consumption increases until about age 40, but then declines thereafter. Net paper assets are initially negative, but then become positive and grow as the individual pays off debt and accumulates savings for retirement.

Additionally, note that the age-income profile, as shown in Figure 3 and 4 is calibrated to match Consumer Population Survey (CPS) data. That is, income increases until the individual reaches her early 40s. It then levels off and begins to decline. Furthermore, the individual in Figure 3 receives a random unemployment shock at age 62, which lasts a year. And, at age 65, the individual retires and accepts transfers from
the government at the replacement ratio, \( \zeta \).

Some of the basic features of the aggregate economy are illustrated in Figure 5, which shows 100 periods of simulated data. One thing to note is that aggregate housing is substantially more volatile than consumption, as is also true in the historical data. Additionally, the simulated economy exhibits leverage cycles, as measured by the debt-to-income ratio. Finally, over the course of the 100 years of simulated data, we see significant changes in default rates, which range from 0% to 7%. This is roughly consistent with the U.S. housing market over the last 50 years.

As far as the calibration of the unemployment rate is concerned–the simulated rate has a mean of 6.49% and a standard deviation of 1.47% over the full simulation period (not just the 50 years shown in Figure 6). In the actual data for the U.S. from 1957 to 2012, the mean unemployment rate was 6.03% and the standard deviation was 1.58%. Additionally, it is important to note that idiosyncratic unemployment shocks are based off of shocks to consumption good production, which makes the unemployment rate move with the business cycle.

In addition to examining default in the model, I also consider how macrovariables in the model respond to shocks by constructing impulse responses. I assume that the central bank uses the following standard version of the Taylor rule with \( \alpha_1 = 1.5 \) and \( \alpha_2 = .5 \):

\[
\begin{align*}
    r_t &= r^* + \alpha_1 (\pi_t - \pi_t^*) + \alpha_2 (\log(Y_t) - \log(Y_t^*)) + \epsilon_t
\end{align*}
\]  

Note that the impulse responses do not come from a linearized version of the model, but instead are constructed by solving the nonlinear version, simulating without shocks initially, and then introducing a single, one-standard deviation shock. All responses shown are from the full-employment steady state, rather than the ergodic rate of unemployment steady state. Finally, notice that the non-smoothness and the fluctuations in the impulse responses arise from two things: 1) non-convexities in the model; and 2) the non-linearity of the solution method. Occasionally-binding constraints, house-size minimums, mortgage structure, and post-default housing market exclusion result in discrete adjustments of individual-level (and aggregate) variables in response to shocks. Fluctuations may arise from both non-convexities and second (or higher) order effects, which are captured by the nonlinear solution method.

Figure 7 shows the impulse responses for a positive, one-standard deviation technology shock. The graphs for output, consumption, capital investment and the housing stock show percentage deviations from steady state. The graphs for the interest rate and the debt-to-GDP ratio are shown in deviations from their initial values. A one-

\[\text{In the model, } \pi_t^* = 0 \text{ and } Y_t^* \text{ is equal to its ergodic mean in the full employment steady state.}\]

\[\text{For technical reasons, the ergodic rate of unemployment steady state presents several problems. Most importantly, however, even if the unemployment rate is fixed at its ergodic mean, macrovariables will continue to fluctuate slightly, since individuals with different characteristics will become unemployed in different periods.}\]

\[\text{Note that there is very little housing investment in the steady state, since depreciation is zero. The only investment in the steady state is generated by replacing housing from agents who perish. For this reason, impulse responses are given for the housing stock, rather than housing investment.}\]
standard deviation technology shock increases the productivity of workers and capital, leading to an initial 4.1% increase in output, which decays over the course of 13 years. Consumption also rises by 3.5%, but does so more slowly and remains higher for 30 years. The interest rate increases by 200 basis points to respond to the jump in output; and remains high for 15 years. Investment in physical capital and the size of the housing stock increases in response to the shock—and remains above their respective ergodic means longer than output.

Figure 8 shows the impulse responses for a positive, one-standard deviation shock to housing productivity. Note that this shock reduces the price of houses by 6.3%, which fuels housing investment, resulting in an increase in the stock of housing by about 2.5% after 7 years. Physical capital investment, however, drops by 13% over 7 years as investment is diverted to housing. The drop in physical capital depresses output by 0.55% over 7 years; and reduces consumption with a similar magnitude, but lower and over a longer period of time.

Figure 9 shows the impulse responses for a positive, 100 basis point shock to the interest rate. First, notice that the shock increases the return to saving, diverting investment away from capital to bank deposits, which increase by 8% over the course of 20 years. The reduction in capital investment depresses output by .5% and also puts downward pressure on consumption. Note, however, that the reduction in consumption (3%) is much larger than the drop in output; and is most likely caused by the increase in interest rates, which pushes up mortgage payments, constraining households to consume less. The increase in interest rates puts downward pressure on housing investment, reducing the housing stock by 3% after 10 years. Average household leverage rises by 6 percentage points over the course of 20 years, suggesting that households may be withdrawing equity from their homes, even if they are not buying more housing.

Table 1 shows business cycle moments for 1000 periods of simulated data. This is compared to the to the actual business cycle moments for the U.S. using annual data over the 1957-2011 period. In general, the magnitudes of the relative standard deviations and the signs of the covariances accurately represent the actual data; however, simulated investment tends to be less volatile and more procyclical than its empirical counterpart.

Table 2 shows the parameter values used in the autoregressive rule simulations. Note that some parameter values and all of the simulated business cycle moments will vary from simulation to simulation. For this reason, Tables 1 and 2 should not be used to interpret all of the simulated results.

Figure 10 compares the Lorenz Curve for the simulated economy (shown in blue) with an empirical Lorenz Curve for the United States (show in black). The straight, red line represents perfect equality. In general, the simulated economy reproduces the empirical income distribution well with two minor departures: 1) wealth is more evenly distributed within the top 50% of earners in the simulated data; and 2) the bottom 50% is relatively poorer in the simulated data.

Note that the starting year is 1957 because it is the first year that I have observations for the housing investment data.
5 Simulation Results

In the subsections below, I detail the simulation results, starting with what I will refer to as endogenous interest rate rules. This is intended to refer to any rule that specifies how interest rates should respond to other macrovariables. Alternatively, we might refer to them as policy rules. Later on, we will look at exogenous interest rate rules (or rules that do not specify comovement with other macrovariables) to get a better sense of how certain types of univariate time series properties of interest rates affect default and welfare. We will also use these rules to examine shock amplification channels in the model.

Note that the results given in the subsections below are robust to several model specification changes. In particular, the results do not change qualitatively if we make any of the following modifications: 1) add sticky prices; 2) remove unemployment benefits; or 3) remove non-housing shelter. Sticky prices were included in the original set of simulations, but were removed because they played only a small role in the model and obscured important mechanisms. The other two items tended to increase the intensity of welfare results, but did not qualitatively change any findings. All of these modifications will be discussed briefly in the appendix; however, unless I state otherwise, all results shown below are for the flexible price case.

5.1 Endogenous Rules

In this section, I will consider endogenous (or policy) rules for setting interest rates. I will do this by examining the properties of inertial Taylor rules, Taylor rules with different coefficients, price-level targeting rules, output-targeting rules, and Taylor rules with a house price targeting component.

5.1.1 Inertial Taylor Rules

I will start this section by considering the class of inertial Taylor rules. This consists of standard Taylor rules that are augmented by autoregressive components. In the specification given below, \( r_t \) is the interest rate set by the central bank, \( \alpha_\rho \) is a constant term, \( \rho_R \) is the autoregressive parameter, \( \pi_t \) is the rate of inflation, \( \pi_t^* \) is the target inflation rate, \( Y_t \) is aggregate demand, and \( Y_t^* \) is an “efficient” or target level of aggregate demand. I use the standard Taylor rule coefficients of \( \alpha_1 = 1.5 \) and \( \alpha_2 = .5 \).

\[
    r_t = \alpha_\rho + \rho_r r_{t-1} + \alpha_1 (\pi_t - \pi_t^*) + \alpha_2 (\log(Y_t) - \log(Y_t^*)) + \epsilon_t \tag{41}
\]

Note that everything in the above equation will be mean zero other than the autoregressive component and intercept. Thus, the mean of the interest rate process will be determined by setting the intercept term. In order to be consistent with later simulations, I set \( \alpha_\rho = (1 - \rho_r) \bar{r} \), which will generate an interest rate process with a mean equal to the ideal rate, \( \bar{r} \).

In order to determine the impact of a higher degree of inertia, I solve and simulate the model 150 times and at 10 different parameter values. I then fit a curve to the
results. Figure 11 and Figure 12 in the appendix show the curves for default and average normalized utility respectively. Note that each of these values is plotted against the value of $\alpha_\rho$ used in the simulation.

The results suggest that a high degree of interest rate inertia will increase the default rate and reduce normalized average utility. In particular, increasing the autoregressive parameter from 0 (a standard Taylor rule with no inertia) to .9 (a highly persistent Taylor rule) will increase the default rate from .55% to 2%. Furthermore, the increase in the default rate does not seem to be offset by utility-increasing benefits that might come from a slow-moving interest rate. Rather, an individual living in the economy with no interest rate inertia who is consuming the average amount would need to be given a 32% increase in consumption in order to be willing to live in the economy with $\alpha_\rho = .9$—a non-trivial difference in living standards.

These results appear to confirm the findings in papers like Forlati and Lambertini (2011), which suggest that inertial interest rate rules lead to deeper output contractions—and, thus, may harm welfare. However, as we’ll see later, “inertia” itself may not be driving all of the reductions in welfare we see in this model. Rather, the way in which we generate inertia (i.e. by using an autoregressive process) may create other undesirable univariate time series properties in interest rates—and these may be responsible for a substantial part of the default rate increase and welfare decline.

5.1.2 Standard Taylor Rules

Next, I’ll consider the set of standard Taylor rules. In the specification given below, $r_t$ is the interest rate set by the central bank, $\bar{r}$ is the ideal interest rate, $\pi_t$ is the rate of inflation, $\pi_t^*$ is the target inflation rate, $Y_t$ is aggregate demand, and $Y_t^*$ is an “efficient” or target level of aggregate demand.

$$r_t = \bar{r} + \alpha_1(\pi_t - \pi_t^*) + \alpha_2(log(Y_t) - log(Y_t^*)) + \epsilon_t \quad (42)$$

For the purposes of this first simulation exercise, I set $\pi_t^*=0$ and set $Y_t^*$ equal to the ergodic mean of $Y_t$. I then simulate over $\alpha_1 \in (0, 2)$ with $\alpha_2 = .5$. Additionally, for the first simulation, productivity shocks in the housing sector and consumption goods sector are negatively correlated, which makes the price level and housing prices positively correlated.

My results suggest that default is not minimized at the standard Taylor Rule coefficient of $\alpha_1 = 1.5$. In fact, the maximum default rate occurs at $\alpha_1 = 1.3$, with a local minimum at $\alpha_1 = 2$ and a global minimum at $\alpha_1 = 1$. Similarly, utility is maximized at $\alpha_1 = 1$.

Next, I consider the same simulation, but with positively correlated housing and consumption sector productivities (and negatively correlated prices). Here, the default

\[\text{In follow-up set of simulations, } \alpha_2 = .6 \text{ was found to be the default-minimizing and utility-maximizing value of the output gap parameter when } \alpha_1 = 1. \text{ While this isn’t necessarily a global maximum over the parameter space, it suggests that a Taylor rule with stronger output gap targeting, but weaker inflation targeting might be an improvement when housing and default are explicitly taken into consideration.}\]
rate is monotonically increasing in the coefficient on inflation. Furthermore, welfare is monotonically decreasing as $\alpha_1$ increases, suggesting that the welfare benefits of lower inflation may be outweighed by the higher default rate.

Why is there such an important qualitative difference when the sign of the correlation is changed? The answer lies in house prices. Recall that $p^h_t = \frac{1}{\delta_t \mu_t e^{\nu_t}}$. If productivity is positively correlated across sectors, then $A_t$ will then tend to be low when $U_t$ is also low. Since the unemployment rate will tend to be high when $A_t$ is low, output will also be low, which will depress the price level. This will prompt the central bank to respond by reducing interest rates, which will simulate housing investment. Since positive housing investment pushes up house prices, the central bank’s actions will actually tend to destabilize the housing market. That is, when house prices are high, then central bank will often make them higher. And when house prices are low, the central bank will tend to push them lower. Additionally, as the central bank pursues inflation more aggressively, it will become an increasingly destabilizing force in the housing market.

To summarize—the empirical relationship between housing and consumption sector productivities is a critical determinant of the efficacy of monetary policy because it partially determines the comovement of the price level and house prices. If housing and consumption sector productivities tend to move in opposite directions, then aggressive inflation targeting will destabilize house prices, leading to higher rates of default. Conversely, if they move in the same direction, then aggressive inflation targeting will tend to stabilize house prices, reducing default.

### 5.1.3 House Price Targeting

Next, I consider Taylor rules that incorporate house prices. The functional form for the rule is given below:

\[
r_t = \bar{r} + \alpha_1(\pi_t - \pi^*_t) + \alpha_2(\log(Y_t) - \log(Y^*_t)) + \alpha_3(\log(p^h_t) - \log(p^{h*}_t)) + \epsilon_t
\]  

(43)

Note that $p^h_t$ is the price of housing and $p^{h*}$ is the ergodic mean of house prices. For the purposes of this simulation exercise, I set $\alpha_1 = 1.5$ and $\alpha_2 = .5$; and then varied $\alpha_3$ from 0 to 1. The primary impact from this rule is to smooth house prices by affecting the housing investment component of price. That is, when investment increases or housing productivity is falls, $p^h_t$ will rise. If such a rule were in place, the central bank would respond by raising interest rates to push housing investment (and prices) down.

One important thing to note is that the central bank is still placing normal weights on inflation and the output gap; however, it has simply added a competing objective: house price smoothing. While this new term might occasionally lead to undesirable effects (i.e. high interest rates when output is low), the effects will be tempered by the

\[24\text{It is important to note that an adverse technology shock does not increase the price level in my model. While it will push up the unit costs of production, it will simultaneously increase the rate of unemployment, which will have a strong, countervailing impact.}\]
original components of the rule. Thus, a recession (if accompanied by fast house price growth) will be met with a smaller reduction in the interest rate than would otherwise happen.

In my simulations, this rule substantially outperforms the standard version of the Taylor rule with respect to default rate minimization and welfare maximization. When $\alpha_3 = 0$, this rule is equivalent to a Taylor rule with typical coefficients. As $\alpha_3$ increases, the house price targeting component becomes increasingly important. As can be seen in Figures 13 and 14, when $\alpha_3 = 1$, the default rate is 35% lower than it would be under a standard Taylor rule. Furthermore, in order to be indifferent between the standard Taylor rule and the same rule with a house price targeting component, a household who is consuming the average amount would need to receive a consumption increase of 23%.

This suggests that there could be substantial welfare gains to house price level targeting. However, it is important to emphasize two nuances here: 1) even when $\alpha_3 = 1$, the central bank is not targeting only house price level deviations. Rather, it is adding a house price targeting component to a typical Taylor rule. And 2) the ergodic mean of house prices is known in the model, but not in reality. That is, unlike price level targeting, there is no obvious or reasonable range of levels to target. Furthermore, unlike inflation targeting, it is not clear what the appropriate rate of growth of house prices is.

The second nuance raises an important question: if the central bank does not know what the correct house price level or rate target is, should it just ignore house prices entirely and use a standard Taylor rule, inflation-target, or price-level target instead? I would suggest that ignoring housing altogether is, in fact, taking a position on it; and is no safer than explicitly including housing in the rule with an imperfect target. One possible strategy for getting around this problem might be to only activate the house price component if house price inflation passes out of some safe band of growth:

$$r_t = \bar{r} + \pi_t + \alpha_1 (\pi_t - \pi_t^*) + \alpha_2 (\log(Y_t) - \log(Y_t^*)) + \alpha_3 f(\log(p^h_t) - \log(p^h_{t-1})) + \epsilon_t \quad (44)$$

$$f(\log(p^h_t) - \log(p^h_{t-1})) = \begin{cases} \log(p^h_t) - \log(p^h_{t-1}) & \text{if } (\log(p^h_t) - \log(p^h_{t-1})) > \nu^U \\ \log(p^h_t) - \log(p^h_{t-1}) & \text{if } (\log(p^h_t) - \log(p^h_{t-1})) < \nu^L \\ 0 & \text{otherwise,} \end{cases}$$

That is, if the growth rate of house prices is greater than some upper bound, $\nu^U$, then activate the housing component of the rule. Alternatively, if the growth rate of house prices falls below some lower bound, $\nu^L$, then activate the housing component. This will permit the central bank to conduct monetary policy as usual in normal times, but still respond to house price growth that is highly abnormal relative to its historical trend when it occurs.
5.1.4 House Price Inflation Targeting

For the sake of completeness, I perform simulations for house price inflation targeting in addition to house price level targeting. Here, instead of targeting the ergodic house price level, the central bank attempts to make house prices move slowly, whether they are returning to the ergodic house price level or departing from it:

\[ r_t = \bar{r} + \pi_t + \alpha_1(\pi_t - \pi_t^*) + \alpha_2(log(Y_t) - log(Y_t^*)) + \alpha_3(log(p^h_t) - log(p^h_{t-1})) + \epsilon_t \] (45)

Note that this approach has interesting implications for housing booms: if house prices start growing rapidly, but taper off as the boom reaches its height, then the central bank will respond by sharply raising the interest rate initially, but then lowering it as the boom continues. This is very different from a house price level targeting rule, which will tighten monetary policy as the boom continues; and will keep it tight even after the peak, when house price growth becomes negative.

Additionally, recall that there is no obvious “ideal” rate of house price growth in the actual economy. Rather, there is perhaps some rate of growth that is consistent with fundamentals and is unlikely to result in a large price growth reversal in the future; however, it is unknown and probably changes over time. In contrast, the model presented in this paper is stationary, which renders a positive or negative house price growth target meaningless. It is also not clear whether fast house price growth ever deteriorates welfare; although, based on the results in this paper, fast drops in house prices appear to reduce welfare because they outpace equity position improvements from mortgage payments—and result in negative equity, which is a necessary condition for default.

For all of the reasons above, this section will be considered for the sake of thoroughness, but will not be treated as a rule that could plausibly be used for policy-making purposes—at least not without a substantial amount of additional work and thought.

The results for house price inflation targeting suggest that default is minimized and welfare is maximized when \( \alpha_3 = 1.2 \)—that is, when the central bank changes the interest rate faster than the rate of house price growth. Note that there are two plausible explanations for this result: first, prolonged periods of house price increases are mitigated by aggressive interest rate hikes. And second (and perhaps more importantly), rapid house price declines are slowed by aggressive interest rate reductions.

5.1.5 Price Level Targeting

Next, I consider the class of price-level targeting rules, using the following specification:

\[ r_t = \bar{r} + \alpha_p(log(P_t) - log(P^*)) + \epsilon_t \] (46)

This rule will increase the interest rate when the price level exceeds its ergodic mean, \( P^* \); and reduce it otherwise. I performed simulations for \( \alpha_p \in (0, 1) \) with positively correlated productivities, and found that default is minimized and welfare is
maximized at $\alpha_p = 0.5$. Beyond $\alpha_p = 0.5$, welfare declines and default increases. This should not be surprising, since strong price-level targeting can generate high interest rates when income and house prices are low. That is, if income is low and house prices are low, but the price level is high, then the central bank will respond by pushing up the interest rate. In the model, this would have particularly severe consequences for default. Conversely, under an inflation-targeting or output-gap targeting regime, this is much less likely to happen.

Similar to the inflation-targeting results, I find that price-level targeting performs poorly when productivities are positively correlated across sectors. In particular, default is minimized and welfare is maximized when $\alpha_p = 0$. That is—an interest rate that follows a white noise process around the ideal rate is preferable to price level targeting if house prices and the price level tend to move in opposite directions.

### 5.1.6 Output Smoothing Rules

Next, I’ll consider rules that smooth the output gap. In the specification given below, $r_t$ is the interest rate set by the central bank, $\bar{r}$ is the ideal interest rate, $Y_t$ is aggregate demand, and $Y^*_t$ is an “efficient” or target level of aggregate demand.

$$r_t = \bar{r} + \alpha_Y (\log(Y_t) - \log(Y^*_t)) + \epsilon_t \quad (47)$$

For the purposes of this simulation exercise, I set $Y^*_t$ equal to the ergodic mean of $Y_t$. I then simulate over $\alpha_Y \in (0, 1.5)$. I find the following three things: 1) increasing $\alpha_Y$ from 0 to 1.5 more than doubles the default rate; 2) positive correlation between sectoral productivities amplifies the increase in default caused by aggressive output smoothing; and 3) a moderate amount of output smoothing ($\alpha_Y = .5$) maximizes welfare, even if it increases the rate of default, as long as sectoral productivities are negatively correlated (i.e. house prices and the price level are positively correlated).

### 5.1.7 Regulating Mortgage Contracts

A final strategy that might be employed is to place legal limits on the availability of certain mortgage contracts.\footnote{Note that an alternative policy might involve taxing LTV ratios or placing stricter limitations on mortgages purchased by Freddie Mac.} If, for instance, banks were limited to offering fixed rate mortgages (FRMs) or low LTV ratio mortgages to individuals with poor credit histories or low incomes, this might help to eliminate one of the necessary conditions for default for many high-risk borrowers. Even if those individuals paid higher risk premia on FRMs or were excluded from the housing market temporarily (i.e. until they could make a sufficiently large downpayment), the impact might still be welfare-improving if it mitigated the aggregate risk that this group of borrowers imposes on the housing and mortgage markets.

Since mortgage choice is outside of the scope of this paper, I examine this problem by looking specifically at the LTV issue. In particular, I simulate the economy for an...
LTV ratio cap of .7 and of 1.1. I find that increasing the LTV ratio to 1.1 (i.e. allowing individuals to take out mortgages greater than the value of their homes) increases the average default rate by only 22.3%; however, it makes the economy more susceptible to catastrophes. When the cap is .7, the default rate never exceeds 10%. However, when the cap is 1.1, it exceeds 10% multiple times–and has a max default rate of 16.67%.

This suggests that regulating LTV maximums might be a way to create stability. Even though the impact of such a regulation will not be highly visible (i.e. it will not dramatically slash the default rate), it may prevent a crisis in which many households default at the same time; and house prices fall for a prolonged period of time.

### 5.1.8 Measures of Mispricing and Instability

In the years since the Great Recession, several papers have looked at the use of macroprudential policy and its role in creating financial stability without monetary intervention. Alichi, Ryoo, and Hong (2012), for instance, look at macroprudential policy in South Korea. In particular, they consider the taxation of financial intermediaries’ key financial ratios, such as the assets to non-core liabilities ratio. They also consider limiting or targeting other measures of leverage.

While I do not show results for macroprudential policy simulations, my model can be extended to incorporate some of these policies. For instance, the government in the model could generate some of its revenue by placing a tax on originations that is proportional to the LTV ratio. However, some macroprudential policies, such as the choice to tax non-core liabilities, as outlined in Shin’s (2010) memo, could not be evaluated in my framework without the introduction substantial changes.

Finally, Aydin and Volkan (2011) suggest that modified monetary policy rules can be used to promote stability and discourage mispricing. They focus on non-financial sector borrowing spreads, bank foreign exchange leverage, credit volume, and asset prices as potential targets. The set of interest rate rules they suggest can be represented by the following equation:

\[ r_t = \rho_m r_{t-1} + (1 - \rho_m) \{ \rho_{\pi} \tilde{\pi}_t + \rho_y \tilde{Y}_{d,t} + \rho_{\psi} \tilde{\psi}_t \} + \epsilon_{m,t} \]  

Note that \( r_{t-1} \) is the lagged interest rate, \( \tilde{\pi}_t \) is the inflation rate’s deviation from its target, \( \tilde{Y}_t \) is output’s deviation from its target, and \( \tilde{\psi}_t \) captures the target financial variable’s deviation from its target. In my model, this variable might be a key financial ratio, such as the debt-to-income ratio, the mortgage volume-to-income ratio, or the house price-to-GDP ratio–all of which could provide a measure of mispricing or instability. Alternatively, the average default risk premium from my model could be used to measure risk in the financial sector.

In short–macroprudential policies that either focus on taxation or are incorporated into interest rate rules may provide a reasonable alternative to house price level (or growth) targeting. Rather than picking an arbitrary threshold for “acceptable” growth, the central bank can make an attempt to prevent financial ratios from deviating from historically stable values. While this may sometimes lead to tighter policy during
contractions or looser policy during expansions, it will help to prevent the creation of financial vulnerabilities that have the possibility of greatly amplifying crises.

5.1.9 OER and House Prices

One final consideration is whether there is a need to target house prices when owner-equivalent rent (OER) is already incorporated into price indices like the CPI. That is, if the price index in my model were closer to the CPI, it might already capture house price growth; and there would be no need to modify the Taylor Rule, since it would already respond to house price increases.

In fact, OER is not a reasonable proxy for house prices—at least not in the years leading up to the Great Recession. As Cecchetti (2007) points out in a VOX editorial, targeting an inflation measure that incorporates OER is not equivalent to targeting an inflation measure that incorporates house prices. Between 2000 and 2007, OER increased by a mere 3.17% per year; whereas, house prices increased by 8.48% each year. Since OER accounts for roughly 30% of core CPI, changing this definition (or targeting house prices separately) would lead to dramatically different policy decisions.

Thus, relying on the OER component of CPI to capture house price growth may be misguided; and is unlikely to prevent future crises. For this reason, it may make sense to target house prices or house price growth separately. See the appendix for an extended discussion of this topic.

5.2 Exogenous Interest Rate Rules

In the next set of simulations, I consider exogenous interest rate rules that have different types of univariate time series properties. The purpose behind these simulations is to determine whether certain properties are more likely to increase the default rate and lower welfare. In addition to this, we will also look more closely at the model by removing several critical components; and then adding them back one at a time. We will start by looking at the class of autoregressive interest rate rules in a model without capital or endogenous house prices.

5.2.1 Autoregressive Interest Rate Rules

In order to get the simulated interest rate to approximate federal funds rate (FFR) movements, I estimated the univariate time series properties of the FFR. This entailed two steps: first, I performed an AR(1) regression on the FFR series from 1970-2010 to recover the autoregressive coefficient (.76). Next, I computed the standard deviation of the residual series (0.0156).

In the baseline autoregressive interest rate rule simulation, I used the aforementioned FFR properties to simulate the interest rate. For computational purposes, I discretized the AR(1) process using the Rouwenhorst method (1995), which is discussed further in the appendix. I also removed capital and the endogenous component
of house prices from the baseline simulation in order to examine the simplest version before adding further complexity.\textsuperscript{26}

For the purposes of this simulation, I assume that the central bank adopts a variety of different interest rate rules by varying the autoregressive parameter on the interest rate lag ($\rho_r$) between -1 and 1.\textsuperscript{27} In each case, I set the constant term, $c_r$ to $\mu_r(1 - \rho_r)$, where $\mu_r$ is the average interest rate target. This ensures that each simulation has the same average interest rate. In each case, I solve and simulate the model for a different $\rho_r$ and $c_r$ 150 times; and plot the main result in Figure 14. Note that the blue, solid line plots the simulated data, and the red, dotted line marks the estimated autoregressive coefficient for the annual FFR series.

Figure 15 shows a curve fitted to the default rate-autoregressive parameter pairs from the 150 simulations. Notice that the average default rate in the economy is minimized when the autoregressive coefficient is zero—that is, when the interest rate is a white noise process. Making the interest rate more persistent (by increasing the coefficient) or making it more volatile (by making the coefficient negative) generates a higher average rate of default. For example, adopting an interest rate that has the same univariate time series properties as the FFR will generate a default rate of 1.57%; whereas, adopting a white noise interest rate will generate an average default rate of 1.4%.

Of course, this is not the full story. This analysis completely ignores the possibility of endogenous interest rate responses to other macrovariables, which eliminates the virtue of having an interest rate rule entirely. However, this finding demonstrates something important: even if critical default amplifiers that are affected by interest rates (i.e. endogenous house prices) are ignored, adopting an interest rate rule that generates as much persistence as we see in the FFR leads to a default rate that is 12.14% higher than a white noise interest rate in the simulated data. This suggests that there could be welfare gains from making the FFR less persistent, but without introducing volatility, which we’ve also shown also generates default. Later on, I will demonstrate that the gains might be substantially larger when we add default amplifiers.

In addition to looking at default, it is important to consider how the rule fares along other dimensions. If, for instance, interest rate persistence leads to a higher rate of default, but does so by generating a higher rate of homeownership (and, thus, adding more marginal borrowers), then persistence may have an positive impact on welfare overall. In fact, as shown in Figure 15, the exact opposite is true.

Figure 16 plots normalized, average utility against the autoregressive coefficient. In the graph, average utility is maximized when the autoregressive coefficient close to zero. For comparison’s sake, consider an individual who is in the economy with a white noise interest rate and is consuming the average amount of the consumption good. In order to offset the utility loss of moving from the white noise interest rate economy to the economy with an interest rate that has the same univariate time series properties

\textsuperscript{26}Parameter values for the baseline are shown in Table 2.

\textsuperscript{27}Note that simulations were conducted close to--but not at--the boundary values in order to avoid introducing nonstationary into the model.
as the FFR, this individual will need to have her consumption increased by 11%.

When looking at the remaining simulation results, I will omit the findings for negatively autocorrelated interest rate rules, since these are less likely to be relevant for policy purposes. However, it is fair to state that interest rate volatility in general tends to push up the default rate—and this is perhaps the most important reason why negatively autocorrelated rates generate more default than white noise. Indeed, in a separate set of simulations, I found that increasing the standard deviation of the interest rate shock from .01 to .05 increased default by 20.8%.

### 5.2.2 Adding Endogenous House Prices

Next, I expand on the baseline version of the model used in the autoregressive rule simulations by making house prices endogenous. This is done by adding the capacity utilization term, $\delta(IH_t)$, as discussed in the model section. This makes the price of a unit of housing equal to $\frac{1}{\delta(IH_t)e^{Ut}}$ units of consumption. The purpose here is to match another important facet of the relationship between interest rates and default: when interest rates rise, financing new homes with a mortgage becomes more expensive, which pushes down housing investment. In the model without endogenous house prices, this effect could actually reduce the default rate by preventing marginal borrowers from obtaining homes. However, in the case with endogenous prices, rising interest rates will deteriorate housing equity positions by pushing down prices. This will tend to push up the default rate and amplify the effects of interest rates on housing cycles.

There’s one more important thing to notice about this particular form for house prices: when the default rate rises and financial intermediaries foreclose on homes, housing investment from earlier periods will be reversed when intermediaries liquidate it, pulling prices down further. This will help the model to match the observation that rising foreclosure rates often depress house prices further.

Figure 17 yields two additional insights. First, the rate of default is higher at all autoregressive coefficients. Even when the interest rate follows a white noise process, the default rate is roughly 276% higher when house prices are endogenous. This suggests that the impact of interest rates on house prices may be one of the most important channels for default.

Adding endogenous house prices not only increases the default rate, but also amplifies the effects of persistence and volatility. For example, a white noise process generates a default rate of 3.95%; whereas, an interest rate with the persistence of the FFR series generates a default rate of 4.7%—a 23% increase. This is more than twice as large proportionally as the change without endogenous house prices.

In Figure 18, we can see that the impact of interest rate persistence on utility is more pronounced. An individual in the white noise interest rate economy who is consuming the average level of the consumption good will now need a 18.18% increase in consumption in order to be willing to switch to the economy with the FFR-like interest rate rule.
5.2.3 Adding Capital

Finally, we add capital, which completes the model outlined originally. Capital will play several important roles. First, it will add another source of aggregate uncertainty to the model. Second, it will give households another asset they can use to hedge against house price drops. And third, it will add an additional channel through which interest rates may affect default. That is, if the interest rate on deposits falls relative to the return to capital, marginal households will invest in capital instead, increasing output. Conversely, an increase in interest rates will divert investment away from capital into deposits. Applying the logic of the double trigger requirement for default, lower income will not cause default by itself, but will cause more households to satisfy a necessary condition of default.

Figure 19 shows the results for the default rate. One important thing to notice is that the average default rate is substantially lower in this economy than in the economy with endogenous house prices, but no capital. In particular, when the interest rate is a white noise process, the economy has a default rate that is only 44.21% of the rate for the economy with endogenous house prices, but no capital.

Importantly, however, the original results have not changed. Higher interest rate persistence still leads to higher default rates. In fact, moving from a white noise interest rate rule to a rule with an autoregressive coefficient of .76 increases the default rate by 30.95%. This is similar to the economy without endogenous house prices, but is considerably smaller than the case with endogenous house prices, but no capital.

Figure 20 shows similar, but dampened results for welfare. Normalized average utility drops as the interest rate becomes more persistent. In this case, however, the decline is more gradual. Using the welfare metric invoked in the previous simulation exercises again, the average individual’s consumption would have to increase by 12.66% in the economy with $\rho_F = .76$ in order to make him as well off as he would be under a white noise interest rate.

5.2.4 Sine Wave Rules

In the section on autoregressive rules, I considered the persistence and volatility properties of interest rate rules. One major finding was that either increased persistence or increased volatility would generate a higher rate of default. This section is intended to add a small, but important, nuance to the earlier findings. Here, the importance of cyclicality will be considered explicitly, rather than using persistence as a proxy. I will do this by having the central bank use a time-based sine wave rule to set interest rates.

In particular, a rule will consist of setting the $z$, $g$, and $n$ parameters in the equation below:

\[
rt = \frac{\sin(zt)}{g} + n
\]  

(49)

Note that $n$ is the minimum interest rate, $g$ determines the maximum interest rate, and $z$ will change the wavelength of the interest rate cycle. For the purposes of this
simulation exercise, I set g to 25 and n to .02, which creates an upper bound for the interest rate at .1 and a lower bound at .02. I then vary z to generate a different wavelength for each simulation. I found that default is minimized when the wavelength of interest rate cycle is 22 years and utility is maximized when the wavelength is 17 years. For comparison’s sake, if the nominal FFR’s low frequency trend is removed, it has maintained an approximate wavelength of 5-10 years since 1956. Furthermore, if the interest rate is simulated at this approximate wavelength (7.5 years), the default rate increases by 1.27 percentage points over the minimum rate, suggesting that there could be welfare gains to increasing the wavelength of interest rate cycles.

This finding adds an important nuance to the findings from the previous section: persistence generates default because it creates cycles—not because it causes interest rates to move slowly. In fact, within the class of interest rates that exhibit cycles, slow moving interest rates (i.e. ones with long cycle wavelengths) may be preferable to fast ones. If this point seems too subtle, consider the following: if the average wavelength of a cycle were 60 years, then many households would never hold a mortgage during the turning point of an interest rate cycle. Conversely, if the average wavelength of a cycle were 5 years, then almost every homeowner will hold a mortgage during the turning point of a cycle. If these turning points are critical times for default—as we have argued thus far—then the wavelength of an interest rate cycle may be a critical consideration when constructing an interest rate rule.

Furthermore, if the cycles are longer, then house price equity declines (insofar as they are related to interest rates) will happen much more slowly. That is, if the interest rate moves very slowly, then the partial, negative effect that an increase in interest rates will have on home equity will be smaller than the size of mortgage payments. That is, in most periods, households will tend to improve their equity positions over time. This will make it easier for younger households to deal with income shocks without defaulting. It will also prevent them from experiencing sharp payment increases associated with holding an ARM during a rapid increase in interest rates.

Finally, it is important to note that OLG models with realistic life cycles and mortgage structure are uniquely qualified to solve this class of problem. Had I used infinitely-lived agents with one period mortgages, it is not clear that I could say anything at all about optimal wavelength, since it depends critically on lifecycle elements.

5.3 Conclusion

This paper examines the relationship between a central bank’s choice of interest rate rule, the mortgage default rate, and welfare. I do this by constructing a quantitative equilibrium model with rigorous microfoundations; and incorporate aggregate uncertainty, incomplete markets, overlapping-generations, credit-scoring, optimal default, inter-sectoral productivity correlation, and realistic mortgage structure. I then use this model to perform counterfactual simulations with a variety of interest rate rules, including modified Taylor rules, price-level targeting rules, autoregressive rules, and sine wave rules.
I find that the univariate time series properties implied by an interest rate rule are a critical determinant of default rates. In particular, rules that generate short interest rate cycles (i.e. have a shorter wavelength) tend to generate higher rates of default. For instance, the FFR series has an average wavelength that is substantially shorter than the optimal length implied by the simulation exercises.\footnote{I find the optimal wavelength to be 22 years for default-minimization and 17 years for utility maximization.}

In addition to looking at the univariate time series properties, I also considered the importance of inter-sectoral productivity correlation. For the class of Taylor rules, very aggressive inflation targeting increased the default rate in an economy with positively correlated housing and consumption goods sector productivities. In contrast, in an economy with negatively correlated productivities, aggressive inflation targeting pushed down the default rate after initially increasing it. Furthermore, I suggested this may be related to the properties of house prices in the model: if sectoral productivities are negatively correlated, then an interest rate that is strongly procyclical (for the consumption goods sector) will tend to stabilize house prices. Conversely, if sectoral productivities are positively correlated, then an interest rate that is strongly procyclical will destabilize house prices.

Next, I considered price-level targeting rules. In contrast to an inflation target or output-gap target, a price-level target will continue to push up interest rates if the price level is high, even if income is low and house prices are low. With that said, a price-level targeting coefficient of .5 outperformed both white noise rules and rules with higher coefficients with respect to both default-minimization and welfare-maximization. This suggests that the business cycle stabilization benefits that come from weak price level targeting may outweigh the costs (i.e. high interest rates at bad times).

In addition to price-level targeting rules, I also examined house price level targeting rules. I found that modifying the Taylor Rule to add a house price level targeting component substantially reduced the average default rate and increased average welfare; however, it might not be possible to implement such a rule, since it would require the central bank to identify a target house price level or growth rate. With this said, it may still be possible to adopt such a more limited form of this rule if central banks only activate the house price component when growth is abnormally high or low.

Finally, I explored who actually defaulted in the model: young households\footnote{See Figure 21 for the age-default profile from the baseline simulation.} with little to no capital, negative equity, and low incomes. No households that had positive equity defaulted, even if they lacked employment. Additionally, even if households had negative equity, they never defaulted unless they also had a sufficiently low income or were unemployed. This suggests that the “double trigger” mechanism for default as described by Foote, Geraradi, Goette, and Willen (2008) is an accurate way to capture the behavior of optimizing agents in a DSGE model with default. Furthermore, it indicates that central banks should be mindful of this condition when implementing policy; and should actively avoid creating situations where many households have negative equity and low incomes simultaneously. This may involve slowing down interest
rate cycles (i.e. increasing the interest rate wavelength), so that new borrowers are unlikely to enter the market at a turning point—or to experience house price declines at a rate that exceeds equity gains from mortgage payments.
6 References


7 Appendix

7.1 Figures and Tables

Figure 1: Taylor Rule vs. Effective Federal Funds Rate

![Taylor Rule vs. Effective Federal Funds Rate](image)

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![Characteristics of Defaulters and Non-Defaulters](image)
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Figure 14: Welfare with House Price Level Targeting
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Figure 16: Welfare without House Prices or Capital

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Figure 17: Default with Endogenous House Prices

Figure 18: Welfare with Endogenous House Prices
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Figure 20: Welfare with Endogenous House Prices and Capital
Figure 21: Age-Default Rate Profile

Figure 22: 150-Period AR(1) Series for Consumption Sector Productivity
Figure 23: 150-Period Rouwenhorst Approximation

Figure 24: Ergodic Employment Rate Convergence
Figure 25: 150-Period LoM-Generated Unemployment Rate
Table 1: Simulated vs. Actual (1957-2011) Moments of Log Detrended Data

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Simulated</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{X}$</td>
<td>$\sigma_{\bar{X}}$</td>
<td>$\rho_{X,Y}$</td>
<td>$\bar{Y}$</td>
<td>$\sigma_{\bar{Y}}$</td>
</tr>
<tr>
<td>C</td>
<td>0.91</td>
<td>0.94</td>
<td>0.94</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>$I_H$</td>
<td>3.94</td>
<td>0.38</td>
<td>2.51</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>$I_K$</td>
<td>2.43</td>
<td>0.53</td>
<td>1.31</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.38</td>
<td>-0.34</td>
<td>0.135</td>
<td>-0.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>.97</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Replacement Ratio</td>
<td>.4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital’s Share in production</td>
<td>.33</td>
</tr>
<tr>
<td>$c_A$</td>
<td>AR(1) Technology Process Constant Term</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard Deviation of Technology Shock</td>
<td>0.03</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Technology Level AR(1) Coefficient</td>
<td>0.8</td>
</tr>
<tr>
<td>$c_H$</td>
<td>AR(1) Housing Productivity Process Constant Term</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Standard Deviation of Housing Productivity Shock</td>
<td>.1</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>Housing Productivity AR(1) Coefficient</td>
<td>0.8</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>Physical Capital Depreciation Rate</td>
<td>.07</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>Housing Depreciation Rate</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Max LTV Ratio</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fraction of Lifetime Earnings Borrowable</td>
<td>.3</td>
</tr>
<tr>
<td>$f_p$</td>
<td>Default Exclusion Period</td>
<td>1</td>
</tr>
</tbody>
</table>

These simulations are performed with $\delta_H = .1$; however, for most simulations results shown, $\delta_H = 0$. 
7.2 Extended Discussion of Topics

7.2.1 Sticky Prices

The original set of simulations was performed using the sticky price version of the model. However, after comparing the results to the flexible price version, it became clear that sticky prices play a limited role in my model. For this reason, they were replaced by the flexible price results in order to clarify the importance of critical mechanisms in the model. One reason why we might expect sticky prices to have a limited impact is because the unemployment rate in the model is pinned down by aggregate and idiosyncratic shocks. Thus, it is not possible to have a strong employment response after a monetary shock. Note that this differs substantially from the standard New Keynesian model, which features elastic labor supply, flexible wages, sticky prices, and demand-determined output in order to generate large output and employment responses to monetary shocks. At most, my model will generate mild output changes through the sticky price mechanism.

With this said, my results are “robust” to the inclusion of sticky prices. That is, when sticky prices are incorporated into the model in the way described in the paper, there are no substantial qualitative differences in my results. In some cases, the intensities of default and welfare results are slightly stronger when sticky prices are introduced; however, the magnitudes of the impacts are generally quite small.

7.2.2 Unemployment Benefits

Originally, simulations were performed without unemployment benefits in the model. This tended to push up default rates and to overstate the welfare impact of rules that caused default. Later, unemployment benefits were introduced for two reasons: 1) to ensure that consumption is always non-zero, even when a household is unemployed (and, thus, avoid the need to set an arbitrary lower bound for utility from consumption); and 2) to avoid triggering defaults that might not immediately be caused by unemployment. No major qualitative results changed after introducing unemployment benefits into the model; however, the intensity of some results have been reduced by its inclusion.

7.2.3 Non-Housing Shelter

While incorporating non-housing shelter is not equivalent to adding a rental market to the model (since no one pays for the shelter), it provides young households and low-income households in the model with an attractive alternative to becoming highly leveraged. Instead, these groups have the option accumulate assets, live in non-housing shelter, and then purchase a home with a larger downpayment.31

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31Note that this will not be an attractive option in the case without non-housing shelter because housing’s contribution to utility will be large and negative if an individual holds no housing.
The simulation results confirm the intuition above. When non-housing shelter is introduced into the model, the default rate drops by 51%. Furthermore, the homeownership rate drops from 93% to 74%, since households do not need to quickly purchase a home after entering the model. Relatedly, the median age of first time borrowers at the date of the mortgage origination rises from 27 to 31. While aggregate home equity in the economy drops by 26%, the impact is much less when we instead consider average equity among homeowners, which is only reduced by 6%.

Finally, while excluding non-housing shelter (or a rental market) will tend to magnify the intensity of my results by pushing up default rates and lowering the utility associated with not owning a house, it does not appear to have a qualitative impact on my results. At least in the case of autoregressive rules, the relationships were identical with and without non-housing shelter.

7.2.4 OER and House Prices

The Bank of England, which adopted an inflation target of 2% in 2003, recently debated whether or not to revise the CPI to incorporate house prices in order to deal with this issue. Since the the Office of National Statistics’ (ONS) measure of CPI excluded both housing prices and OER, the measure of inflation completely missed the run up in house prices prior to the Great Recession. In order to improve future results, the ONS plans to construct a separate measure of the CPI that will incorporate OER, but not house prices.

While this choice might not be as important in the U.S., where the FOMC is permitted to use discretion, it might be substantially more important for central banks that claim to target only inflation or are legally-required to keep inflation below a target threshold. In particular, re-defining the CPI to incorporate house prices, rather than OER, would have resulted in tighter monetary policy in many countries in the years leading up to the Great Recession.

It is important to note, however, that constructing an alternative version of the CPI that incorporates house prices, rather than OER, might not be a conceptually accurate measure of the “price level,” even if it yields better results for inflation-targeting central banks. Cecchetti (2007), for instance, points out that OER captures the opportunity cost of living in a house; whereas, a measure of the level of house prices confuses increases in cost with increases in the value of assets.

Thus, redefining the CPI in countries that have a legal mandate to target inflation could be a useful strategy for preventing the inflation of housing bubbles, even if doing so is not entirely conceptually accurate. It will permit the central bank to continue to communicate a clear target for a highly visible measurement of inflation; and it will make the choice to continue with rate hikes easier, since the public will see that the rate of inflation is high, even if non-house price growth is contained. If house prices are not explicitly included in the index, adding OER could still provide a significant

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32Note that the comparison simulations were run using an autoregressive interest rate rule with a coefficient of .76
33See Osborne’s (2007) letter to King.
improvement for central banks that target an inflation measure that does not include the cost of housing at all.

7.3 Solution Algorithms

7.3.1 The Rouwenhorst Method

I use a number of routines to generate and calibrate the exogenous processes in the model. Central to most of these routines is the Rouwenhorst method (1995), which is used to discretize AR(1) processes into conditional Markov processes. Kopecky and Suen (2009) argue that this method is more accurate than other approximation algorithms for highly persistent AR(1) processes. Additionally, once the algorithm is constructed, we may generate a Markov process (i.e. the series of TFP shocks) by setting four parameters only: ρ, q, σ, and µ, where ρ is the probability of the highest state, q is the probability of the lowest state, σ is the desired standard deviation of the process, and µ is the mean of the process.

The Rouwenhorst method works by creating an initial Markov transition matrix for a 2-state approximation to an AR(1) process. It then uses that matrix as an input for the next step, where a 3-state approximation is created. This recursion continues until the desired level of discretization is achieved. In our case, exogenous processes are approximated with 7-state chains, which is standard in the literature. As outlined in Kopecky and Suen (2009), the process works as follows:

**Step 1:** Create the initial 2-state Markov transition matrix.

\[ P_2 = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \]

**Step 2:** Next, construct the 3-state Markov matrix as follows:

\[
P_3 = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix}
\]

**Step 3:** Construct the nth-state Markov matrix in two steps. First, compute the following:

\[
\begin{bmatrix} P_{n-1} & 0 \\ 0' & 0 \end{bmatrix} + (1-p) \begin{bmatrix} 0 & P_{n-1} \\ 0 & 0' \end{bmatrix} + (1-q) \begin{bmatrix} 0' & 0 \\ P_{n-1} & 0 \end{bmatrix} + q \begin{bmatrix} 0 & 0' \\ 0 & P_{n-1} \end{bmatrix}
\]

Next, divide all rows 2 through (n-1) by 2 to complete the nth-state Markov matrix. Note that 0' denotes a (n-1) row vector and 0 denotes a (n-1) column vector. This will yield an AR(1) process with an autoregressive coefficient of p+q-1.

Note that the Markov chain associated with the probability matrix is constructed by discretizing the interval \([μ - v, μ + v]\) into the same number of states as the matrix.
where \( v = \sqrt{\frac{n-1}{(p+q-1)^2}-1} \sigma \). This will generate a process with a standard deviation of \( \sigma \). In our case, the mean is 1 the standard deviation is .05.

As an example of the Rouwenhorst Method, consider Figures 22 an 23. The first is a 150-period series with an autoregressive coefficient of .8 and a mean of 1. The second graph is a discrete, 7-state Rouwenhorst approximation.

Note also that the standard deviation of the approximated series over 2000 periods is .0491 and the mean is .9959, which suggests that the 7-state chain provides a reasonably accurate approximation to an actual AR(1) process with the same mean, standard deviation, and autoregressive coefficient.

### 7.3.2 Unemployment Rate

The computation of individual unemployment spells and aggregate unemployment is done in the following steps:

**Step 1:** Draw a series of consumption productivity shocks.

**Step 2:** For each individual and time period, use the conditional Markov chain to compute the probability of becoming employed given the current consumption productivity shock and the previous period’s employment state. Note that the probability of becoming employed will be higher if the agent was employed last period and if the consumption productivity shock is larger.

**Step 3:** For each individual and time period, draw a real number from a uniform distribution on the interval \([0,1]\). If the number drawn is greater than the conditional probability of employment, then the agent becomes unemployed. Otherwise, he becomes employed.

**Step 4:** Compute the unemployment rate as the fraction of agents who are employed in a given period.

**Step 5:** Adjust the conditional Markov transition matrix probabilities and re-simulate until individual-level unemployment spell profiles and the unemployment rate match calibration targets.

Note that this model is calibrated to generate an unemployment rate that fluctuates between 4% and 12%. Figure 25 shows a 50-period sample of the unemployment rate generated using this method.

### 7.3.3 Ergodic Rate of Unemployment

Since the ergodic rate of unemployment is used as an input in the solution algorithm, it is necessary to compute it prior to solving the model. With Markov processes, we can typically compute the steady state analytically; however, since there is a different
Markov transition matrix for each TFP state, I instead solve for the ergodic rate of unemployment by simulating an individual employment path for over 50,000 periods and computing the fraction of periods unemployed.

I re-simulated and recalibrated to achieve a target ergodic unemployment rate of 6.3%. Figure 24 shows convergence in the employment rate after 5,000 simulation periods.

7.3.4 Krusell and Smith (1998) for Unemployment

In the model, the future unemployment rate affects factor prices, which means that agents must be able to form expectations about future unemployment rates. We use the Krusell and Smith algorithm in the following way to compute the law of motion for unemployment:

**Step 1:** Compute the unemployment rate in each period for a 5,000 period simulation of the economy.

**Step 2:** Update the law of motion coefficients. Construct a set of regressors by interacting the previous period unemployment rate and a consumption sector shock state-specific constant with a binary variable that equals one if the current shock state is j and 0 otherwise.

In this case, the estimating equation is is the following, where UR(t) denotes the unemployment rate at time t, S(t) denotes the index of the consumption sector state at time t, C(j) denotes a state-specific constant, G(j) denotes the coefficient on capital, 1{ } denotes an indicator function, and ns denotes the number of states:

\[
UR(t) = \sum_{j=1}^{ns} C(j) 1\{S(t) = j\} + \sum_{j=1}^{ns} G(j) UR(t-1) 1\{S(t) = j\} \tag{50}
\]

This law of motion is then checked for fit. If \(R^2\) is sufficiently high, then the law of motion is adopted—and agents use it to form expectations about future factor prices. If the fit is poor, then a longer simulation is performed and the law is re-estimated.

Figure 25 shows the predicted unemployment rate for 150 periods.

7.4 Solution Method

Since the model contains many individual-level and aggregate-level continuous states, I employ three methods to make it computational feasible. First, I use GPU computing as outlined in Aldritch et. al (2010) to perform all matrix operations in my solution method.\(^{34}\) Next, I use an approximate dynamic programming algorithm for simulations that would otherwise have prohibitively long run times. And, finally, I use

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\(^{34}\)These operations are performed using an Nvidia GeForce GTX 560 Ti 2b with 384 CUDA cores.
a modification of Judd, Maliar, and Maliar (2010) to limit the household’s choice set to the ergodic set in the cases where I do not solve using backwards recursion.

The DDP is solved for households in one of two ways. First, in the cases where we have no capital or endogenous house prices, I solve the problem for the household using backwards recursion with cubic polynomial interpolation. For the more involved simulations, I use an approximate dynamic programming algorithm that is a modification of an algorithm in Powell (2007). It involves the following steps:

### 7.4.1 ADP Algorithm

**Step 1:** Draw a sequence of exogenous shocks for 5,000 periods and agents.

**Step 2:** Initialize the future value of all post-decision states and simulate the values of the aggregate variables using the initial laws of motion.

**Step 3:** For each household, step forward in time by choosing consumption, housing, capital, deposits, mortgage debt, and whether or not to default.

**Step 4:** After each household’s choice, update the value of the post-decision state using the following equation:

\[
V(s) = \alpha_j V_{new}(s) + (1 - \alpha_j)V_{old}(s) \tag{51}
\]

Note that \( j \) denotes the number of times we have iterated through all households.

**Step 5:** After each household’s problem is solved using the forward-pass portion of the algorithm, we perform the back-propagation step. That is, we use the updated, post-decision state values to perform backwards recursion. After each household’s step, we update the value of being in each post-decision state. Notice that—unlike the forward step—this gives us an unbiased estimate of being in each of these states.

**Step 6:** For all states, evaluate the post-decision value and compute the change from the previous iteration. If the maximum absolute change falls below the specified tolerance, terminate the algorithm and update the laws of motion for the aggregate states. If the tolerance criterion is not satisfied, then return to step 1.

### 7.4.2 Verification

In all cases were I use the ADP algorithm to solve the household’s problem, I also solve the problem once for a baseline case using backwards recursion and cubic polynomial interpolation.

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\(^{35}\)Note that the new value of the post-decision state will always be different, since it will depend on the particular set of shocks drawn; however, \( V(s) \) will start to approximate the expectation of the pre-decision state as we visit \( s \) more times and with different sets of shocks. This is why a variable step size \( \alpha_j \) is used, which declines as the iteration number increases.
interpolation. This allows me to verify the quality of the approximation without solving and simulating 150 times using the more computationally-intensive approach.

7.4.3 Krusell & Smith (1998) Algorithm

After the optimal path for agents is determined in each step, the aggregate variables are updated, and their laws of motion are estimated using a modification the Krusell and Smith algorithm that uses neural networks.

7.4.4 State-Contingent Pricing Algorithm for Mortgage Market

In order to select a value of $\xi_t$ that generates zero net cashflows for the financial intermediary, I use state-contingent pricing. That is, at each point in time, the financial intermediary observes $Z_t$ and sets a corresponding $\xi(Z_t)$ to set cashflows equal to zero.\(^{36}\)

The algorithm consists of the following steps:

**Step 1**: Set $\xi(Z_t)$ equal to the risk-free rate plus a fixed premium in all periods.

**Step 2**: Solve and simulate the model. Compute the average net cashflow for each set of aggregate states, $Z_t$.

**Step 3**: Modify the state-contingent mapping by setting $\xi(j) = \xi(j) + CO \ast NCF(j)$, where $CO > 0$ and $NCF(j)$ is the net cash-flow in state $j$. That is, if net cashflows are positive, then reduce $\xi(j)$ by a small number that is proportional to the net cashflow. If they are negative, then increase $\xi(j)$ by a small number that is proportional to the net cashflow.\(^{37}\)

**Step 4**: Repeat step 2. Check whether the maximum absolute net cashflow in each state falls below the tolerance value.

**Step 5**: Repeat steps 3 and 4 until all states have a maximum absolute net cashflow that falls under the tolerance value.

Note that this algorithm is nested within the main solution method; and will guarantee that deviations from market clearing in the mortgage market are small.

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\(^{36}\)Note that cashflows deviate from zero in some periods, but deviations are at least two orders of magnitude smaller than the aggregate housing stock, which we use for comparison.

\(^{37}\)As mentioned in the section on financial intermediaries—the primary impact of $\xi_t$ will be on borrowing at time $t$. Thus, increasing $\xi_t$ will tend to increase net cashflows by reducing borrowing.
7.4.5 Price Level Approximation Using a Single Layer, 20-Node Neural Network

In order to compute the current price level, we must be able to solve for the expected value of a function of the next period price level, unit cost, and output. In order to simplify the computation, I apply approximation methods. In particular, I compute the price level as a state-contingent mapping using the following algorithm:

**Step 1:** Initialize the price level for all states.

**Step 2:** Solve the household’s problem and perform the aggregation step using the state-contingent mapping for $P_t$.

**Step 3:** Compute $\bar{P}_t$ and then $P_t$ in each period, given the values of $P_{t+1}$, $V_{t+1}$, and $Y_{t+1}$ from the previous iteration.

**Step 4:** Train a single-layer, 20-node neural network to approximate the nonlinear relationship between the aggregate states of the economy and $P_t$. Use the neural network to update the state-contingent mapping for $P$.

**Step 5:** Compute the maximum absolute difference between the new and old $P$ in each state. If the maximum absolute difference is below the tolerance threshold, terminate this step and continue with the main algorithm. Otherwise, go back to Step 2.

Note that this part of the algorithm could be done by regressing $P_t$ on basis functions. I find that neural networks work particularly well because they have a low computational burden, provide a highly accurate nonlinear approximation, and do not require a substantial amount of restrictions the function being approximated.