Anarchy and Autarky:

*Endogenous Predation as a Barrier to Trade*

James E. Anderson
Boston College and NBER

Douglas Marcouiller
Boston College and Saint Louis University

This paper presents a general equilibrium two-country Ricardian trade model with endogenous transactions costs that arise from individual utility-maximizing allocation of labor to production and piracy. In the absence of institutions for risk-sharing and coordination of defense, autarky obtains over most of the parameter space. When both trade and predation are supported in equilibrium, terms of trade effects can make security immiserizing. In that case, paradoxically, predation creates trade.

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Transactions costs cast a long shadow over international trade, as shown in recent empirical work (summarized in Rodrik 2000; see also Trefler 1995). Trade modelers, however, usually brush transactions costs aside, either ignoring them entirely or incorporating them as exogenous “iceberg melting” costs. This paper puts transactions costs front and center.

We focus on costs arising from the exposure of international shipments to misappropriation. Empirically, extortion, hijacking and theft dramatically reduce trade (Anderson and Marcouiller, 2002). Incorporating these costs into a general equilibrium model poses an interesting challenge, since the trade costs associated with predatory activity depend on the endogenous allocation of resources to predation, which in turn depends on the volume of trade.

This paper develops a two-good, two-country Ricardian model in which both the terms of trade and the security of trade are endogenously determined. Trade has fixed start-up costs, so autarky is always a Nash equilibrium of our model; no agent will choose to be the only one bearing the fixed cost of trade. The surprise is that autarky is the only equilibrium for much of the parameter space explored in our simulation model. Thus coordination failure explains only a small part of the prevalence of autarky with anarchy. In our model, four parameters prove central to the existence of a trading equilibrium: the extent of the fixed costs of trade, the relative effectiveness of predatory and evasive resources, the difference between countries in autarky relative prices, and relative country size. Our simulations explore only a special case, but our qualitative results are likely to
obtain in a much wider class of models, as we explain below.

When both trade and predation do emerge in equilibrium, insecurity and the terms of trade interact in interesting ways. The terms of trade of one of the countries will deteriorate as security improves. This negative terms of trade effect may outweigh the positive effects of enhanced security so that welfare in that country falls, an effect we call "immiserizing security."\(^2\) Conversely, through the terms of trade effect, enhanced efficiency of predation may increase one country’s potential gains from trade enough to draw it out of autarky, leading to the paradox of trade-creating predation.

To keep the focus on the fundamentals of predation, we assume that anarchic conditions exclude collective action. Producers do not organize insurance markets. Predators do not organize crime. By showing how difficult trade is under anarchy, we implicitly underscore the importance of institutional development for the success of international markets.

The exclusion of collective action, while analytically very helpful, impedes immediate application of our model to real-world extortion, hijacking, and theft. The possibility of predation can make a market dry up -- try getting a pizza delivered in a tough neighborhood (Raspberry, 2002) -- but most predation seems to be at least minimally organized, and most trade is institutionally supported, as our model suggests it must be. The pirates of the Caribbean played a role in the collapse of the Spanish

\(^2\) The phrase echoes Bhagwati’s “immiserizing growth,” but the underlying mechanism is quite different.
transatlantic trade, but both the pirates and the traders had state support. Piracy continues today, but shipments can be insured. Corrupt customs officials extract illegal payments, but their power to do so flows from their position in the regulatory structure. The illegal duplication of traded software, videos and CDs comes close to our idea of free-entry predatory activity, but monopoly rents on protected intellectual property come from a world more complicated than the one we model.

The model we offer in this paper is simple, but also complex enough to capture the simultaneous determination of trade and transactions costs in general equilibrium. That, we claim, is its virtue. It moves beyond exogenous “iceberg melting” trade costs and uncovers some unexpected interactions along the way.

This paper is related to the literature on predation (Anderton, Anderton, and Carter 1999; Grossman and Kim 1995; Skaperdas and Syropoulos 1996, 2001, and 2002) as well as to the literature on various sorts of trade costs (Deardorff 2001; Hummels 2001; Rauch 1999; Rauch and Trindade 2002; Romer 1994). As far as we know, this is the first paper to offer a general equilibrium model with endogenous predation on trade.

Section 1 sketches the logic of individual choice and indicates the types of equilibrium which could emerge. Temporarily taking the number of predators as fixed, Section 2 determines the conditions under which trade can be supported in equilibrium. Section 3 sets out the full model of endogenous predation. Section 4 shows results of numerical simulation of the Cobb-Douglas form of the model. Section 5 concludes.
1. Outline of the Decision to Trade

We first sketch the logic of individual choice and indicate the types of equilibrium that could emerge. As in a classical Ricardian model, we have two goods and two groups of agents (home and foreign) with differing relative labor productivities, potentially generating gains from trade. However, agents who opt to trade run the risk of losing their merchandise to other agents who have opted for piracy. Each agent decides whether to allocate labor to certain fixed costs associated with positioning oneself to be able to trade and then allocates remaining labor across productive and predatory activities. Shippers and bandits distribute themselves across available trade routes in such a way that each shipper (bandit) faces the same probability of successful shipment (theft). Ex ante, without mechanisms for risk diversification, a trader does not know what her ex post consumption bundle will be. Depending on parameter values, the system may settle into an autarkic equilibrium, a secure trading equilibrium in which no predation occurs, or an insecure equilibrium in which predation and trade coexist.

Consider first the logic of individual choice. We assume the decision to incur the fixed cost of trade to be simultaneous with the allocation of the remaining labor across production and predation, which allows us to avoid inessential complexities of a dynamic model. The logic of the choice is more easily explained, however, in two stages.

Putting oneself in a position to trade involves fixed start-up costs. Among these start-up costs are some which enhance the shipper’s ability to evade potential predators. We assume that each agent makes a dichotomous choice, devoting either nothing or the
fixed share $\tilde{l}$ of his unit labor endowment to the start-up costs. Those who position themselves to be able to trade have three additional options: they can devote their remaining labor to autarkic production for their own use, they can produce and exchange goods internationally, or they can devote their remaining labor to predation on the international shipments undertaken by others. Those who do not bear the fixed costs are not in a position to trade. Their further options are limited to two: they can devote their unit of labor either to predation or to production for home use. By simplifying (but harmless) assumption, predation has no fixed cost.  

Let $p$ be the relative price of good 1, and let $\cup$ represent the probability that a shipment eludes capture. These two variables are endogenously determined, as described below, but both are exogenous from the point of view of the individual trader. Let $\cup$ denote the exogenous opportunity cost of good 1 in terms of good 2 for home agents along the home country’s production frontier.

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3 No fixed cost is plausible, if one’s image of predation is stealing pizza or pirating videos. Substantial fixed costs in predation are important in activities such as classic Caribbean piracy, but allowing for fixed cost has no formal consequence in our model. In equilibrium, predators devote all their labor to predation, and the need to deduct a portion of it for a setup cost simply reduces the relative effectiveness of predatory to trading labor, which is treated as a parameter in our formal model below. Fixed costs in challenging initial claims to productive resources are treated in Grossman (2001) in a setting where they are more consequential due to nonspecialization.
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The indirect utility function for a home agent who operates in autarky, \( v^A(\cdot) \), has as its arguments \( l / l \) and the labor devoted to production. The indirect utility function for a trader, \( v^T(\cdot) \), has as its arguments \( p, 1 / l, l / l \), and the labor devoted to production. We derive these below along familiar lines. The indirect utility function for a predator, \( v^P(\cdot) \), is shown below to depend on \( p, 1 / l \), the labor devoted to predation, the number of competing predators, and the amount of production for trade on either side of the market. Figure 1 represents the options facing an agent in the home country.

**Figure 1: Logic of Choice in the Home Country**

![Diagram of choice options]

Three types of equilibrium are possible. Trivially, autarky is always a Nash equilibrium; if no one else is prepared to trade, an agent knows that she will have no partner and thus cannot benefit by incurring the fixed costs of trade.

A central result of our paper is that autarky is the only equilibrium for most parameter values. Obviously, exogenously fixed start-up costs of trade may be high.
enough to wash out potential gains from trade. The surprise is that, even if perfectly secure trade would offer gains sufficient to cover fixed costs, the entry of predators can generate terms of trade and a level of security inconsistent with gains from trade for agents in one of the countries. To be precise, let $p^0$ and $\bar{p}^0$ denote the price and success rates associated with equilibrium defined exclusive of the entry condition. There is no trading equilibrium if $v^A(1,\bar{p}) > v^T(p^0,\bar{p}^0;1 \bar{I},\bar{G})$, so that home agents are unwilling to trade, or $v^{A^*}(1,\bar{p}^*) > v^{T^*}(p^0,\bar{p}^0;1 \bar{I}^*,\bar{G}^*)$, so that foreign agents are unwilling to trade. (Throughout the paper, an asterisk designates the foreign economy.)

Perfectly secure trade implies that in equilibrium, all agents bear the start-up costs of trade, and no one opts for predation on the trade. Let $p^c$ denote the secure equilibrium price. The secure equilibrium emerges if $v^T(p^c,1;1 \bar{I},\bar{G}) > v^A(1,\bar{p})$, $v^{T^*}(p^c,1;1 \bar{I}^*,\bar{G}^*) > v^{A^*}(1,\bar{G}^*)$, $v^T(p^c,1;1 \bar{I},\bar{G}) > v^p(\cdot)$, and $v^{T^*}(p^c,1;1 \bar{I}^*,\bar{G}^*) > v^p(\cdot)$. This occurs when the gains from trade are high and the gains from predation are small. What is required for this to occur will be easier to explain once the $v^p$ function has been fully delineated.

The third type of possible equilibrium, an insecure equilibrium, is one in which some agents in each country bear the start-up costs and trade while other agents opt for predation on that trade. Willingness to trade requires that $v^T(p^c,\bar{G};1 \bar{I},\bar{G}) > v^A(1,\bar{p})$ and $v^{T^*}(p^c,\bar{G};1 \bar{I}^*,\bar{G}^*) > v^{A^*}(1,\bar{G}^*)$. Predation is a free entry activity, and $v^p$ is declining in the number of predators. Agents from the country with the lower utility will enter predation until the marginal agent is indifferent between predation and production.
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for trade. Therefore, this third type of equilibrium also requires that

\[ v^I(p^*, \square; 1 \square \tilde{I}, \square) = v^P(\cdot), \quad v^{I*}(p^*, \square; 1 \square \tilde{I}^*, \square) = v^{P*}(\cdot), \] or both.

Note that some of the choices in Figure 1 can be ruled out. In equilibrium, everyone who chooses to bear the fixed costs of trade will trade. Bearing \( \tilde{I} \) but then producing autarkically solely for one’s own use is a dominated strategy, since

\[ v^A(1, \square) > v^A(1 \square \tilde{I}, \square). \] Similarly, \( v^P(1, \cdot) > v^P(1 \square \tilde{I}, \cdot) \), and bearing \( \tilde{I} \) but subsequently shifting to banditry is dominated.

Moreover, if in equilibrium anyone pays \( \tilde{I} \), no one will opt for autarky. All agents within a country are identical. If trading utility dominates autarky utility for any agent in a country, it must dominate autarky utility for all agents in that country.

Furthermore, in equilibrium no one will pay \( \tilde{I} \) unless trade can actually take place, which requires that trade dominate autarky for agents in both countries. Therefore, in equilibrium, anyone who pays the fixed costs of trade will trade, and if any agent pays the fixed costs, all agents will either trade or prey upon the trade. Consequently, in equilibrium, if some agents pay \( \tilde{I} \) and others do not, those who have not paid \( \tilde{I} \) will be predators. Due to this structure, the decision to bear the trade cost is also in equilibrium a career choice between predation and specialized production for trade.

2. Production and Trade with Exogenous Predation

Although the purpose of this paper is to present a model in which the choice to engage in predation is fully endogenous, it is helpful to begin by analyzing behavior when
the number of predators is exogenously given. We first analyze the behavior of individual agents, who take as given the level of security and the terms of trade, then turn to the analysis of the trading equilibrium. Several results emerge. Insecurity tightens the range of productivity differences which can support trade in equilibrium. In contrast to the usual Ricardian result, when predation is possible, agents in both countries will generally produce at home some of the import-competing good. Proposition 1 specifies a sufficient condition for improvements in security and in the terms of trade to lead to higher import demand. Insecurity and the terms of trade interact in interesting ways. While decreased security always lowers the welfare of one country, it can have an ambiguous effect on the welfare of the other.

2.1. Individual Production and Trade Decisions

In this section we examine output, imports and exports as functions of the terms of trade, \( p \), and the security of trade, \( / / \), variables which the individual agent takes as given. Agents who have borne the fixed start-up costs of trade allocate their remaining labor across production of the two goods. Goods may be lost in shipment and, in the absence of institutions for insurance or risk diversification, traders face an uncertain consumption bundle. This insecurity leads to incomplete specialization in production and reduces trade.

Let \((y_1, y_2)\) denote production of the two goods. The constant opportunity cost of producing good 1 is higher at home than abroad. To economize on notation, we assume
that the unit labor requirement in the export good is equal to 1. Given a decision to bear
the fixed cost of trade, $\bar{I}$, the Ricardian technology for each potential trader in the home
economy is described by:

\[(2.1) \quad a y_1 + y_2 \leq 1 - l\]

where $a$ is the opportunity cost of producing the import-competing good.

All agents have identical preferences. The representative home trader chooses the
output of each good and the amount of each to trade at price $p$ (the price of good 1 in
terms of good 2). Agents trade only once each period; we thus rule out for simplicity any
\textit{ex post} trade within countries between successful and unsuccessful agents. Let the trade
vector be $(m_1, m_2)$, where exports appear as negative quantities. Conditional on having
undertaken the fixed costs of trade, the potential trader’s choice problem is:

\[(2.2) \quad \max_{y_1, y_2, m_1, m_2} [u(y_1 + m_1, y_2 + m_2) + (1 \cdot l) u(y_1 - \min(m_1, 0), y_2 + \min(m_2, 0))] \]

\[\text{subject to} \quad \square y_1 + y_2 \leq 1 - \bar{I} \]

\[pm_1 + m_2 \leq 0\]

where $1$ gives the probability of evading capture. The representative agent maximizes
the expected utility of consumption. With probability $(1 \cdot \square)$, one’s shipment falls prey
to thieves. In that case, consumption of the imported good equals its home production,
and consumption of the exported good equals its production less the stolen exports. The
first constraint reflects the Ricardian technology, the second the balance of payments.

The maximum value function for this program is the indirect utility function for traders,
$v^T(p, \square; 1 \cdot \bar{I}, \square)$. A similar program for the foreign representative agent yields the foreign
The first order conditions of the maximization program reveal the characteristics of the choices which the agent will make. Let \( u_i = \partial u / \partial (y_i + m_i) \) denote the marginal utility of consumption of good \( i \). Denote marginal utilities when one’s shipment successfully eludes capture with a superscript \( G \) (Good state) and when the shipment is captured with a superscript \( B \) (Bad state). Then

\[
\frac{\partial u}{\partial m_i}^B = \begin{cases} 
  u_i^B & \text{for } m_i < 0 \\
  0 & \text{for } m_i > 0.
\end{cases}
\]

The derivatives with respect to trade volumes are undefined at \( m_i = 0 \), the autarky point. However, as shown above, if in equilibrium anyone pays the fixed cost of trade, no one will chose autarkic production. Thus, conditional on the entry cost being rationally paid in equilibrium, the derivatives are defined.

As in the standard model, the foreign exchange constraint will always bind. This follows from the first order conditions for maximizing utility with respect to \( m_1 \) and \( m_2 \):

\[
\mu_i^G + (1 - \mu) \frac{\partial u^B}{\partial m_i} = \mu_i^G \mu \mu_p
\]

\[
\mu_2^G + (1 - \mu) \frac{\partial u^B}{\partial m_2} = \mu_2^G + (1 - \mu)u_2^B \mu.
\]

Here, \( \mu \) is the Lagrange multiplier for the foreign exchange constraint. If the constraint does not bind, \( \mu = 0 \). However, if that were true, the above first order conditions could not be met, since at least one of the expected marginal utilities must be positive for any sensible utility function.

In contrast to the standard Ricardian model, traders will produce some of the import-competing good for themselves, reducing the amount of trade, except in a limiting
case of indifference to consumption risk explained below. The first order conditions for
the trade vector imply:

\[ \frac{\partial u_1^G}{\partial E[u_2]} = p \]

(2.3)

where \( E \) is the expectations operator such that \( E[u_2] = \mathbb{E}[u_2^G] + (1 - p)u_2^B \). Equation (2.3) can also be written as:

\[ \frac{u_1^G}{u_2^G} = p + p \frac{1}{E[u_2]} \frac{1}{u_2^G} \cdot \]

(2.4)

The first order conditions in the output vector give the conditions:

\[ E[u_1] = \mathbb{E}[u_1^G] + (1 - p)u_1^B = \]
\[ E[u_2] = \mathbb{E}[u_2^G] + (1 - p)u_2^B = \]

where \( \mathcal{L} \) is the Lagrange multiplier for the labor constraint. In the standard Ricardian model, one of these conditions usually does not bind, and hence, by complementary slackness, one of the outputs is equal to zero. Here, in contrast, both conditions will usually bind, implying that both goods will be produced. Taking the ratio of the first equation to the second at an interior solution and using (2.3):

\[ \frac{E[u_1]}{E[u_2]} = \frac{\mathbb{E}[u_1^G]}{\mathbb{E}[u_2]} + \frac{(1 - p)u_1^B}{\mathbb{E}[u_1]} \cdot \]

(2.5)

Thus, the interior solution involves a degree of specialization in which the “marginal rate of expected substitution” is equal to the marginal rate of transformation \( \mathcal{L} \), but both of these, along with the marginal rate of substitution in the good state, are greater than \( p \).

Figure 2 depicts the agent’s decision graphically. Point A is available in autarky with utility \( u^A \). Trade requires fixed costs \( \mathcal{I} \), shifting in the production possibilities
frontier to the heavy line. The trader’s optimal production bundle, Y, which no longer need be completely specialized, permits consumption at G if exchange succeeds. If exchange fails, consumption must fall to B. If exchange were always secure, the agent would specialize completely in production of good 2 and would consume at E.

**Figure 2. A Trader’s Production and Consumption**

**Place figure 2 about here**

The system (2.3), (2.5) and the two constraints of the maximization program give four equations to determine the four variables \((y_1, y_2, m_1, m_2)\). With concave utility, the solution is globally unique. The output and trade variables are implicit functions of the variables \(p\) and \([\text{ }]\), which are exogenous to the individual, along with the parameters \(\bar{l}\) and \([\text{ }]\). Intuitively, we expect the volume of imports to be an increasing function of both security and the terms of trade (the inverse of the relative price of imports). Potential difficulties arise because of the nature of consumption risk in the model. A further restriction on preferences is sufficient for the intuitive results:

**Proposition 1:** If preferences are such that goods are weak Pareto complements, that is, if \(u_{12} \geq 0\), then:

- (a) import volume rises with the security of shipments \([\text{ }]\);
- (b) import volume rises with improvements in the terms of trade.

**Proof:** At an interior solution the full employment and balance of trade constraints can be
used to substitute in the utility function for \( y_2, m_2 \) in terms of \( y_1, m_1 \). We drop the commodity subscript for neatness. The agent’s program in the region of parameter space where good 1 is imported is

\[
\max_{y,m} W(y,m;\mu,\nu,p) = \mathcal{B} \left( y + m \right) + \mathcal{B} \left( y \right) + \mathcal{B} \left( y \right).
\]

Let \( H(W) \) denote the negative definite second derivative (Hessian) matrix of \( W \). Weak Pareto complementarity can be shown to imply that \( W_{ym} = W_{my} < 0 \) and \( W_{yy} < W_{ym} \).

Concavity at an interior optimum guarantees \( W_{yy} < 0 \), \( W_{mm} < 0 \), and \( \mathcal{H} = W_{yy} W_{mm} - W_{ym}^2 > 0 \). Trade responds to security improvements according to:

\[
\frac{dm}{dp} = \frac{1}{H} \left( W_{yy} W_{my} - W_{ym} W_{yy} \right)
\]

which must be greater than zero, since it can be shown that \( W_{my} > 0 \) and \( W_{yy} < 0 \). This proves (a). Trade responds to price according to:

\[
\frac{dm}{dp} = \frac{1}{H} \left( W_{yy} W_{mp} - W_{ym} W_{yp} \right).
\]

It can be shown that \( W_{mp} < W_{yp} < 0 \). Since under the Pareto complements condition \( W_{yy} < W_{ym} < 0 \), it follows that \( dm / dp < 0 \). This proves (b).

The Pareto complements condition is sufficient, not necessary, but it is also simpler than any alternative restriction, while concavity alone does not suffice. (The reader may verify this by examining, for example, \( W_{my} \).) The Cobb-Douglas case used in our simulations satisfies the Pareto complements condition, as does the CES utility function.

As usual in trade models, export supply schedules can have a backward bending
portion, while they must be upward sloping near the vertical axis.\textsuperscript{4} Export supply is increasing in security $\frac{\partial}{\partial y}$, applying Proposition 1 and noting that $m_2 = \Box pm_1$.

Greater aversion to income risk (imposed by strictly concave transformations of utility) reduces trade. An infinitely risk averse agent (who maximizes his minimum utility) will stay at autarky no matter how favorable the price; otherwise it pays to accept some trade, given the initial commitment of fixed cost. Low elasticity of substitution between goods makes consumption risk more painful and thus reduces trade more. Indifference to consumption risk arises with linear utility --- both infinite elasticity of substitution between goods and income risk neutrality. Outside this limiting case, trade is reduced by the risk of theft. Finally, generalizing production to include diminishing returns weakens forces of specialization as in perfectly secure trade models but retains the property that trade is reduced by insecurity.

Institutions for risk-sharing matter in this setup. If insurance were readily available, predation would still generate an endogenous transactions cost, but there would be no uncertainty in the consumption bundle. The incentive for incomplete specialization in a trading equilibrium would disappear. Consumption would be somewhere on the locus of tangencies of specialized budget lines and isoutility curves above A and below E, since the certainty equivalent price $\frac{p}{\Box}$ would have to incorporate the insurance premium. Predation would still shrink the gains from trade as compared to secure trade, but the

\textsuperscript{4} When good 1 is imported, the exports of good 2 are equal to $pm_1$ and the slope of the export supply schedule is given by $\frac{dm_2}{d(1/p)} = \Box p^2 [m_1 + pdm_1 / dp]$. 
availability of insurance allows more scope for offsetting the fixed cost of trade.\footnote{The insurance-type solution obtains in anarchy if each agent can enter trade paying the fixed cost once and then dividing exports and imports across many small independent trips to the market. To keep consumption risk in the model and maintain incomplete specialization, we assume away this possibility. Real world trade achieved risk diversification without formal insurance through the practice of fractional cargoes, but this practice requires property rights enforcement to assure that the owner of the ship will pay off the other shippers at the end of a successful voyage.}

Summarizing results: Insecurity reduces trade. Despite Ricardian technologies, producers generally will produce some of each good instead of completely specializing according to comparative advantage. Conditional on paying the fixed costs of trade, if the goods are weak Pareto complements, the import volume rises with the security of trade and falls with increases in the relative price of the imported good.

### 2.2. Exogenous Predation and the Trading Equilibrium

The previous section identified the individual trader’s choices as implicit functions of security, $/\backslash$, and the price of traded goods, $p$. Now we determine the equilibrium $p$, conditional on $/\backslash$ and on the allocation of labor to production and predation. In this partial equilibrium setting, by parametrically driving $/\backslash$ down we can push into a range in which there exists no $p$ consistent with a trading equilibrium. In the full general equilibrium...
model presented in Section 3, \( \parallel / \) is endogenous and driven down by changing the primitive parameters of the model.

It is convenient to work with the comparative labor productivity for each country’s import good. For the foreign economy, \( \square^* \) is the opportunity cost of good 2. Whether predation exists or not, trade equilibrium constrains the relative price to the range \( \square > p > 1/\square^* \), where the strong inequality is due to the requirement that the fixed cost of trade must be covered.

The international equilibrium of the model is determined by the market-clearing condition for the home country’s imported good:

\[
(2.6) \quad (N \parallel N^r)m_i(p, \parallel; 1 \parallel \tilde{I}, \square) + (N^* \parallel N^{*r})m_i^*(p, \square^*; 1 \parallel \tilde{I}^*, \square^*) = 0.
\]

Here \( m_i \) denotes per producer excess demand for good 1 in the home country while \( m_i^* \) denotes the per producer excess demand for good 1 in the foreign country. The per producer excess demands are scaled up by the number of nonpredatory agents in each country, that is, by the total number of agents \( N \) or \( N^* \) less the number who enter predation, \( N^p \) and \( N^{*p} \), numbers assumed in this section of the paper to be fixed. We also assume a complete separation between legitimate trade and the thieves’ market in which stolen goods are exchanged.

The analysis of existence, uniqueness and stability of equilibrium follows standard lines, assuming that the import demand functions are downward sloping (see Proposition 1(b)). Equilibrium need not be unique, since supply curves can bend backward. The sole question of existence arises from the effect of lower \( \parallel / \) in reducing and eventually eliminating the range of potential equilibrium prices.
Consider the incipient autarky price at which the home country is just barely willing to trade. By (2.3) and (2.5) evaluated at $m_1 = m_2 = 0$,

$$u_1(y_1, y_2) / u_2(y_1, y_2) = a = p^a / p^f \quad p^a = p^f.$$ For the foreign economy, the incipient autarky price is similarly defined by $1 / p^a = a^* / p^f$, or $p^a = 1/a^*$. Solving the two equations simultaneously, $\|p = (a^*)^{1/2}$ defines the critical level of security below which no trading equilibrium can exist. This demonstrates the importance of the technologically based arbitrage margin, $a^*\|$, for the existence of trade under autarky. Assume that the opportunity cost of each country’s imported good is 1.1. The arbitrage margin is then .21 or 21%. With this arbitrage margin, the probability of successful exchange must exceed 90% to permit trade. Even at the very large arbitrage margin of 300%, which we will use in later simulations, trade can exist only if the probability of successful exchange exceeds 50%.

**Figure 3. Security and Trading Equilibrium**

Place figure 3 about here

Figure 3 illustrates. The equilibrium at E represents mutually beneficial trade despite some predation. With lower values of $L$, the range of prices between $L$ and $1 / L^*$ shrinks and eventually disappears, destroying the possibility of trade. For fixed $L$, reductions in the arbitrage margin similarly reduce trade. The need to cover the fixed
costs of trade imposes still tighter limits on the range of prices which support trade in equilibrium.

**Proposition 2:** For \( \bar{I} \left( \bar{I}^* \right)^{3/2} \), autarky prevails.

Proposition 2 shows that, even leaving aside the fixed cost of trading \( \bar{I} \), when traders are faced with exogenous predation, the market may not find a price at which voluntary exchange will occur. To enable trade either the probability of successful exchange or the arbitrage margin must be high. Finally, we note that Proposition 2 extends to non-Ricardian production models: \( \bar{I} \) and \( \bar{I}^* \) simply refer to the autarky relative price of the incipiently importable good.

The diagrammatic analysis links our model to the familiar iceberg transactions costs, with \( 1/\bar{I} \) equal to the melting rate. The key differences are that in our model the melting rate is (or soon will be) endogenous and that, away from the autarky point, behavior is modified by the consumption risk imposed by predation.

By application of standard comparative static methods to (2.6), \( dp/d\bar{I} \) has the sign of \( m_{1[\bar{I}]} \left( \frac{1}{p} \right) m_{2[\bar{I}]} \frac{N^* \bar{I} N^p}{N \bar{I} N^p} \). By Proposition 1, the condition \( u_{i2} > 0 \) is sufficient although not necessary to guarantee that \( m_{1[\bar{I}]} > 0 \). When \( m_{1[\bar{I}]} = m_{2[\bar{I}]} / p \), a natural benchmark case, \( dp/d\bar{I} \) will be negative if \( N \bar{I} N^p < N^* \bar{I} N^p \). Stated more generally, in the benchmark case, lowering \( \bar{I} \) improves the terms of trade of the larger country. In Figure 3, the excess demand function of the larger country shifts to the left by more.
**Proposition 3:** Lower probability of successful exchange improves the terms of trade of the larger country if the asymmetry of country size dominates other asymmetries, in the case of weak Pareto complements.

For more intuition we refer to the Cobb-Douglas form used in our simulations. The parametric expenditure share for good 1 (the home country import) is denoted by $\beta$. The benchmark result holds in the symmetric Cobb-Douglas case of $\beta = \beta^*$ and $\beta = 1/2$ (see the Appendix). The prediction of an improvement in terms of trade for the larger country as $\beta$ falls also holds for asymmetric parameters in the Cobb-Douglas case as long as the asymmetry in country size is sufficiently great.

Under the condition of Proposition 3, the impact of decreased security on the welfare of producers in the larger country is ambiguous. In contrast, producers in the smaller country lose from decreased security both directly and through a deterioration of their terms of trade. The analysis suggests that producers in the two countries may have opposing interests in security arrangements when terms of trade effects are powerful. To complete the welfare analysis of changes in security requires simulation, as the welfare of producers changes according to the magnitudes of $\beta$ and $dp/d\beta$, both of which are deeply nonlinear functions of the fundamental parameters. We will return to the welfare analysis after developing the full model, endogenizing the probability of safe shipment by endogenizing the allocation of labor across production and predation.
3. Production and Trade with Endogenous Predation

We now move to the full model, in which the number of producers, the number of predators, and the probability of eluding capture are endogenously determined. We first specify the probability of safe shipment, \( \beta \), as a function of the number of agents choosing piracy over production. We then derive the indirect utility function for the individual predator and, armed with the appropriate utility functions, examine the equilibria which can emerge when predation is endogenous.

The probability of a safe shipment depends on random encounters as predators hunt and shippers evade. We assume that shipments originate in a diffuse region and go to a market point for exchange. Predators and prey spread themselves evenly on the approaches to the market. It is quite reasonable to assume that the probability of evasion is a decreasing function of the ratio of predators to prey. We simplify the round trip shipment success rate to the logistic functional form which has been widely used in the previous predation literature:

\[
\beta = \frac{1}{1 + \frac{N^p + N^p^*}{\bar{l} \cdot N + N^* \cdot N^p + N^p^*}}.
\]

The number of pirates is \( N^p + N^p^* \). The number of shippers is \( N + N^* \cap N^p \cap N^p^* \). The parameter \( \bar{l} \) captures the relative effectiveness of predatory and evasive activity. Recall that \( \bar{l} \) is the exogenously fixed cost of entering trade, which includes the investment which must be made in evasive capacity. For Equation (3.1) it does not
matter that the fixed costs of trade also include elements unrelated to security; one could account for these by simply rescaling $f$. With equal numbers of predators and prey and equal effectiveness, $f = 1/2$.

Our simple structure for the individual trader’s fixed costs ensures that all shippers will devote exactly $\bar{T}$ to evasive capacity, bandits will attack shipments randomly, and the probability of loss will be the same in every case, invariant to individual decisions. This is a strong but harmless simplifying assumption for research into the likelihood of autarky and the market channels through which predation and trade interact.\(^6\)

The pay-off to the predator is the captured shipment, which can be exchanged in a thieves’ market. The assumed separation of legal and illegal exchange simplifies the structure and affects the model only inessentially, since prices in the two markets are closely related in any case.\(^7\) The relative price of good 1 on the thieves’ market, $p^p$, is determined by the marginal rate of substitution between the goods, which itself depends

\(^6\) In contrast, the predation literature has been interested in precisely the interaction of defensive effort and the opposing offensive effort and on the various externalities generated by defensive effort of any one agent.

\(^7\) Alternatively, we could assume that stolen goods find their way into legitimate commerce again, the appropriate setup when the household is treated as an integrated producing and predating agent. Allowing some members of the household to enter banditry complicates the notation but adds nothing essential to the analysis.
on the availability of each good in the thieves’ market. Successful thieves wish to diversify their consumption, trading their loot for an optimal bundle based on the common utility function of all agents.

It is convenient to simplify by restricting tastes to be homothetic, so that the marginal rate of substitution depends only on the ratio of the quantities consumed of the two goods. The aggregate prize vector is \( (1 - p)M_1, (1 - p)M_2^* \) where \( M_i = (N - N^*_p)m_i \) denotes the aggregate quantity of home excess demand for good \( i \). The ratio of goods available for thieves’ consumption is thus \( M_1 / M_2^* = \frac{M_1}{M_2} = p \), where the last step follows from the balanced trade constraint \( pM_1 = M_2^* \). Then the thieves’ market relative price is given by \( p^p = \frac{(1 - p)u(p)}{M_2^*} \), where \( u(\cdot) \) is the marginal rate of substitution. The aggregate value to thieves of the stolen goods is \( (1 - p)(p^pM_1 - M_2) = (1 - p)(p^p + p)M_1 \). The expected income of each predator is:

\[
\zeta^p = \frac{(1 - p)(p^p + p)m_1(p, \mathbb{1} - p, N^*_p)}{N^p + N^*_p}.
\]

Further restricting the utility function to homogeneity of degree one imposes risk

\[ ^8 \text{This specification is equivalent to pooled shares in banditry, where the aggregate proportion of goods stolen is certain and all individual risk is removed. With income-risk neutral bandits, as assumed in the Cobb-Douglas utility function, such pooling is irrelevant as the agent is indifferent between the expected per capita income with certainty and the uncertain stream with the same expected value. We prefer the individual uncertain return interpretation, as risk pooling presumes coordination.} \]
neutrality. This structure avoids the need to specify the detailed states of bandit income; only the mean matters. In this case, the indirect utility function is linear in income and the expected predator indirect utility function is:

\[
\nu^p = \left( \frac{N^p N^p}{N^p + N^p} \right) \eta p, p, l, a \right) \equiv \left( \left( \eta p, p, l, a \right) / \eta c(\eta p, 1) \right)
\]

where \( \eta p, \eta, l, a \) = \( \left( \eta p, \eta, l, a \right) / \eta c(\eta p, 1) \) and \( c(\eta p, 1) \) is the true cost of living index for the homogeneous of degree one utility function. See the Appendix for explicit closed form solutions for the trade and predator indirect utility functions for the Cobb-Douglas case.

Section 1 of this paper sketched three types of possible equilibria. Perfectly secure trading equilibria arise endogenously when \( q/l \) is low enough to deter entry into banditry and Ricardian complete specialization generates mutual gains from trade sufficient to cover the fixed costs of trade. Autarky is always a Nash equilibrium. It will be the only equilibrium if trade has sufficiently high fixed costs. It will also be the only equilibrium if potential trade is choked off by endogenous predation.

The most interesting and complex class of equilibria are the interior solutions or insecure equilibria in which trade and predation coexist. Predation is an international free entry activity, and \( \nu^p \) is declining in the aggregate number of predators, \( N^p + N^*p \). Agents from the country with the lower utility will enter predation until the marginal agent is indifferent between predation and production for trade. Domestic entry into predation requires that:

\[
\nu^p = \nu^T > \nu^A.
\]
Foreign agents will engage in predation if:

\[(3.5) \quad v^p = v^*T \geq v^*A. \]

For a small range of parameter values both equalities can hold, implying entry into banditry by both countries. We restrict attention to the more usual case, in which predators are drawn only from the country with the lower utility for the representative trader.

Inserting (3.3) into (3.4) and (3.5) and inverting, if \( v^p = v^*T < v^T \) and a trading equilibrium exists, then:

\[(3.6.a) \quad N^{*p} = \left( \frac{p, q, 1, I, 0}{v^T} \right) N; \quad N^p = 0. \]

If all predators come from the home country (\( v^p = v^T < v^*T \)), then:

\[(3.6.b) \quad N^p = \left( \frac{p, q, 1, I, 0}{v^T} \right) + v^T N; \quad N^{*p} = 0. \]

In equilibrium, the realized probability of successful exchange, \( \tilde{\cdot} \), must be equal to the anticipated \( \hat{\cdot} \) on which forward-looking agents based their calculations:

\[(3.7) \quad \tilde{\cdot} = \hat{\cdot} = \frac{1}{1 + \left( \frac{N^p + N^{*p}}{I} \right) N + N^* \left( N^p \right) N^{*p}.} \]

---

9 The case where utilities are equal between the two countries and both supply predators is not a knife edge equilibrium. We are indebted to a referee for showing us this. The intuition is that entry into predation plays a role somewhat like migration would in equalizing utilities. We have not focused on such an equilibrium, since our interest is to explore the more interesting equilibria where one economy could be immiserized.
The interior equilibrium with insecure trade is the value of \( \left( p, N^P, N^{*P} \right) \) which satisfies (3.6) and the exchange equilibrium condition (2.6) when the right hand side of (3.7) is substituted for \( /\!/ \) in all expressions.

Interior solutions give rise to a rich set of comparative statics which cannot be derived as special cases of previously known results. We turn to simulation to show these effects, using the Cobb-Douglas specification.

4. Simulated Equilibria with Endogenous Predation

Simulation of the Cobb-Douglas model reveals two key results. First, under anarchy, which excludes risk-sharing and coordinated defense, autarky is the only equilibrium over most of the parameter space. Second, where insecure trading equilibria do exist, changes in the effectiveness of predatory resources have a non-monotonic effect on welfare in one of the countries. The general equilibrium interaction between predation and the terms of trade can generate “immiserizing security” or, conversely, the paradox of trade-creating predation. Although these results are demonstrated only for the Cobb-Douglas model, we argue that they are likely to be robust to generalizations of technology and preferences.

The existence and shape of a trading equilibrium depend on four parameters: the fixed cost of trade, the effectiveness of predatory resources for given evasive resources, the difference between countries in autarky relative prices, and relative country size. We have simulated the Cobb-Douglas model (see Appendix) with \( \epsilon_0 = \epsilon_{0*} = 2 \) so that autarky
price ratios differ by a factor of 4. The foreign country is larger, with N=1000 and N*=1500, and poorer, in the sense that at an interior equilibrium $v^T < v^T$. We set $\bar{t}$, the expenditure share for the home country’s import, at .45.

The parameters of most interest are the fixed cost of trade, $\bar{t}$, including investments in evasive capacity, and the effectiveness of predatory resources for given evasive resources, $\bar{q}/\bar{t}$. Holding country size and autarky price ratios constant, we vary the levels of $\bar{t}$ and $\bar{q}$ and then solve the Cobb-Douglas forms of the international market clearing condition (2.6) and the predatory labor supply conditions (3.6) for the equilibrium values of $p$, $N^p$, and $N^{*p}$, if they exist. We can then trace out the corresponding security level, trade volume, and gains from trade.

Figure 4 plots the $(\bar{t}, \bar{q})$ pairs for which a trading equilibrium exists. Squares denote parameters supporting secure trading equilibria, with endogenously determined $N^p = N^{*p} = 0$. Triangles denote insecure trading equilibria, where trade coexists with predation. The fixed costs of trade, $\bar{t}$, range from 0.5% to 10% of the trader’s labor endowment. The effectiveness of predatory labor, $\bar{q}$, ranges from 0 to 0.2. No trading equilibria were found with $\bar{q} > 0.2$ or $\bar{t} > 10\%$. 
Note the small range of parameters which support trading equilibria. It is no surprise that trade can be choked off by increases in the fixed start-up costs, $\bar{I}$, even if predatory labor is completely ineffective ($q = 0$). However, we were surprised by the huge impact of endogenously generated insecurity on the system’s ability to sustain trade when fixed start-up costs are low. We explored the robustness of this result with respect to changes in the other exogenous parameters. Eliminating the terms-of-trade effect associated with country size by equating populations ($N = N^*$) expanded the scope for trade, as expected, but only to 6.5% of the space bounded by $.01 \bar{I} 1$ and $0 \bar{I} 1$. Increasing the arbitrage margin to an outlandish 1500% by setting $\bar{I} = \bar{l} = 4$ still generated trading equilibria in only 14% of that parameter space.

**Simulation Result 1:** *In the Ricardian Cobb-Douglas model, anarchy implies autarky for most parameter values.*

The “overhang” in Figure 4 points to another surprise: the ambiguous role of improvements in predatory technology, $\bar{I}$, for given $\bar{I}$. Highly effective predatory labor will choke off trade even if the fixed costs of trade are low, as indicated by the upper
bound of the shaded area in Figure 4. Before that upper bound on $\mathcal{I}$ is reached, however, improvements in predatory technology may actually encourage trade. Consider, for example, a vertical line drawn where the fixed costs are 6% of labor. At $\mathcal{I}=0$, where predatory effort is completely ineffective, there is no trade in equilibrium. The foreign country’s gains from trade would not cover its fixed costs. As $\mathcal{I}$ rises, however, eventually a region is reached in which foreign producers can cover the fixed costs of trade, and an insecure trading equilibrium emerges.

This paradox of trade-creating predation reflects underlying changes in the terms of trade as security improves. Flipping the effect on its head, one recognizes the possibility of “immiserizing security"\textsuperscript{10} as the effectiveness of predatory labor declines. Predators are drawn exclusively from the country with lower producer welfare. A decline in the effectiveness of predatory labor tends to drive some agents out of predation and back into production, and this endogenous reallocation of labor lowers the relative price of the country’s exported good. If the poorer country is also the larger of the two, the adverse turn in the terms of trade is exacerbated by the “large country” effect noted in Proposition 3. Simulations show that, as long as a trading equilibrium continues to exist, the security and volume of trade increase as the effectiveness of predatory labor falls. Welfare effects are another story. Enhanced security can have such a negative impact on

\textsuperscript{10} As with immiserizing growth, the ability to tax trade removes immiserizing security. Anarchy suggests a government too weak to control its borders.
the terms of trade of the larger, poorer country that the positive welfare effect of increased trade is swamped. In this case, security is immiserizing.

This can be clearly seen in Figure 5. We set $\tilde{I} = .05$ and raised $\bar{q}$ from 0 to .15 in increments of .001, then solved for the equilibrium values of $p$ and $\bar{q}$. Using those values, we solved for the foreign gains from trade in equilibrium (the percentage increase in utility relative to its autarky level). The foreign country gains nothing from trade when $\bar{q}$ is less than .08, but as the effectiveness of predatory labor continues to rise, gains from trade emerge. Some of the foreign agents are being drawn out of production and into predation, lowering the supply of the foreign country’s export on the world market and raising its price. As $\bar{q}$ approaches .13, insecurity-associated costs dominate the positive terms of trade effect, and gains from trade vanish once again. Exactly this effect can also be seen in Figure 4. If one traces a vertical line upward at $\tilde{I} = .05$, one finds that trade emerges in equilibrium only for $.8 \bar{q} \leq .13$. Note in passing that the supply of predators is infinitely elastic at the level of utility enjoyed by producers in the poorer country. Thus, foreign producers and predators share whatever gains from trade accrue to their homeland in equilibrium.
**Fig 5. Foreign Gains from Trade as Effectiveness of Predatory Labor Rises**

\( \bar{l} = .05 \)

place Figure 5 about here

**Simulation Result 2:** *In the Ricardian Cobb-Douglas model, there exist parameter values for which improvements in security through a decline in the effectiveness of predatory labor are immiserizing for the larger, poorer country.*

The home country’s gains from trade over the same range, with \( \bar{l} = .05 \) and \( l \in [.05, .13] \) are shown in Figure 6. Over the range of trading equilibria, the smaller, richer country’s gains from trade diminish monotonically as the effectiveness of predatory labor rises. However, the home country does gain from trade in *any* of these insecure trading equilibria.
Fig 6. Home Gains from Trade as Effectiveness of Predatory Labor Rises

\[ (\bar{t} = .05) \]

place figure 6 about here

Return to Figure 4 once more. For \( \bar{t} = .05 \), the enhancement of the foreign country’s terms of trade as \( \bar{q} \) approaches .8 permits foreign producers to cover the fixed start-up costs of trade. Both countries then emerge from autarky into an insecure trading equilibrium, and, paradoxically, both countries are made better off by reductions in security which move from the system from “under the overhang” into the equilibrium set. By paying a higher price for imports under insecure trade, the home country brings the relatively less favored foreign country into the world market. In this narrow parameter range, predation creates trade.

Anarchy does not permit coordination of predators or shippers. However, staying with Figure 4, it is interesting to note that where predatory effort is relatively effective (say, \( \bar{q} = .1 \)), coordination among potential traders to raise evasive effort (say, from \( \bar{t} = .01 \) to \( \bar{t} = .03 \)) would permit trade to emerge.

Uniqueness of equilibrium conditional on being in the interior is too difficult to prove analytically. Nevertheless, all interior equilibria we have found appear to be unique because grid searches with varying starting values failed to turn up any others.

Our conclusions about the shape and size of the predation/evasion parameter space which supports trade are model-specific and dependent on the values of \( \bar{\eta}, \bar{\eta}^* \) and
N/N*. Nevertheless, we believe that the conclusions would be robust to changes in the specification of preferences and production technology. Increases in the elasticity of substitution in consumption (equal to one above) will raise the volume of trade, while decreases in the elasticity of transformation in production (infinite above) will lower the volume of trade. The net effect of these changes is likely to be trade-reducing on balance. Risk aversion (in the sense of concave transformations of utility functions) is likely to shrink trade and also to shrink predation, with a net effect which is ambiguous but probably small.

The effect of changes in the specification of predation is less certain. Altering the functional form of the success rate from the logistic to some other cumulative density function of the limit value of the predator to prey ratio seems unlikely to change the conclusions. Switching to a multi-factor model of production and predation opens up more possibilities, including one in which the richer country may tend to benefit from insecurity.\textsuperscript{11} Switching to a model in which the fixed cost of trade becomes endogenous

\textsuperscript{11} In the Heckscher-Ohlin model we can sketch one plausible scenario. Assume that \textit{ex ante} identical agents bring their per capita share of capital with them into banditry. We have to assume some sort of cooperation to get both factors their competitive factor reward in banditry. If the factor intensity of banditry lies between those of the two goods, for some factor endowments there will be bandits from both countries in equilibrium. Improvements in security will result in relative endowments being pushed further apart, so trade volume is more sensitive to security than in the Ricardian model.
through a link to defensive effectiveness opens up the potential for multiple equilibria, but does not substantially affect the properties of the model focused on here.\textsuperscript{12}

5. Conclusion

Trade theory usually brushes transactions costs aside or models them simply as exogenous “iceberg” costs. This paper has put transactions costs at the center of a general equilibrium trade model.

We focus on theft, which in practice could range from the hijacking of shipments to the extraction of bribes by corrupt customs officials. Starting from individual choices

\begin{quote}
The possibility of immiserizing security in the larger, poorer country will be reduced, as both countries experience increases in their endowments which are trade creating. Indeed, it seems possible that producers in the smaller, richer country could prefer less security. This result will be even more likely if predation is the most capital intensive industry, as in this case the rich country has a comparative advantage in it. Immiserizing security will certainly be possible if the richer country is larger. In contrast, if predation is the least capital intensive industry, we return to the Ricardian result that predators come from the poorer country. We regard the latter specification as more plausible for banditry, but for some problems it may be useful to think of predation as capital intensive.
\end{quote}

\textsuperscript{12} The authors will supply an appendix supporting this claim upon request. Multiple equilibria were pointed out to us by Costas Syropoulos.
about the allocation of labor to productive and predatory activities, our model
simultaneously determines the extent of trade and the security of trade.

When both trade and predation are supported in equilibrium, security has a non-
monotonic effect on welfare in one of the trading partners. Enhanced security has a
negative terms of trade effect on one country, amplified by the migration of labor from
predation to production. Over some parameter range, this negative effect can dominate the
positive effects of security, so that enhanced security proves to be immiserizing. In fact,
enhanced security could cause one of the countries to prefer autarky to trade.
Conversely, insecurity may induce that country to enter trade, leaving us with the paradox
of trade-creating predation. These effects hold only over limited parameter ranges.

Our model assumes anarchy, excluding coordinated effort for risk-sharing or
defense of trade. Assuming anarchy, the most striking result of our numerical simulations
is that autarky is the only equilibrium over most of the parameter space. The economic
logic of this result has been set out in Sections 2 and 3 of the paper, which show that,
under anarchic conditions, endogenous predation truly can be a barrier to trade. The result
underscores the importance of institutions for the support of trade.
APPENDIX: Agents’ Decisions in the Cobb-Douglas Case

A closed form solution for production and trade obtains if we assume that utility is a Cobb-Douglas function of the consumption bundle: \( u = x_1^G x_2^G \). Here, \( x_i \) denotes consumption. With some judicious substitution, we obtain a closed form solution for the quantities in four steps.

First, we obtain a solution for the import share. The combination of the efficiency conditions (2.4) for trade and (2.5) for production implies:

\[
\frac{\partial u_1^G}{(1 \square) \partial u_1^B} = -\frac{p}{\square p}.
\]

For the Cobb-Douglas case this implies

\[
\frac{\square (x_2^G / x_1^G)^{\square}}{(1 \square) (x_1^B / x_1^G)^{\square}} = -\frac{p}{\square p}.
\]

Here, we have used the fact that \( x_2 = y_2 + m_2 \) in each state. Now note that

\[
x_1^B / x_1^G = y_1 / (y_1 + m_1).
\]

Solving this expression for the import share \( m_1 / y_1 \) we obtain

\[
(A.1) \quad \frac{m_1}{y_1} = \frac{\square (1 \square) p \square^{\square}}{\square (\square \square \square p) \square^{\square}} \quad \square = f(p, \square, \square).
\]

This import share is undefined at \( \square = 1 \); in that case the classic Ricardian model obtains and production will either be equal to zero or indeterminate. The import share is defined everywhere else, which means that with Cobb-Douglas preferences, complete specialization is never optimal in the presence of predation.
Second, we obtain the consumption ratio in the two states in terms of the import share and the production ratio. We substitute into the ratio of consumption in the two states using $m_2 = -pm_1$ to solve in terms of $m_1 / y_1$ and $y_2 / y_1$.

\[
\begin{align*}
\frac{x_1^B}{x_2^B} &= \frac{y_1}{y_2 + m_2} = \frac{1}{y_2 / y_1 \cdot pm_1 / y_1} \quad \text{and} \\
\frac{x_1^G}{x_2^G} &= \frac{y_1 + m_1}{y_2 + m_2} = \frac{1 + m_1 / y_1}{y_2 / y_1 \cdot pm_1 / y_1}.
\end{align*}
\]

Third, we solve for the production ratio. Substituting the preceding expressions for the consumption ratios into the efficiency condition for imports and using $f(p)$ for the import share $m_1 / y_1$ we obtain:

\[
\frac{\mu_1^G}{\mu_2^G + (1 \cdot \alpha)u_2^G} = p = \frac{1 + f(\cdot)}{y_2 / y_1 \cdot pm_1 / y_1} + (1 \cdot \alpha)(1 \cdot \alpha) \frac{1}{y_2 / y_1 \cdot pm_1 / y_1}.
\]

This expression may be solved for $y_2 / y_1$ to yield:

\[
\begin{align*}
y_2
\frac{y_2}{y_1} &= \frac{p\{1 + f(\cdot)\} + (1 \cdot \alpha)(1 \cdot \alpha)}{1 + f(\cdot) + \alpha} + pf(\cdot) \\
&= p \frac{1 + f(\cdot) + \alpha}{1 + f(\cdot) + \alpha} + \alpha (1 + f) \frac{1 + f(\cdot) + \alpha}{1 + f(\cdot) + \alpha} \\
&= p \frac{f(\cdot) + \alpha}{1} + \alpha.
\end{align*}
\]

Finally, in combination with the full employment constraint $y_1 + y_2 \leq 1 - \alpha$, the production ratio yields the closed form solution for $y_1, y_2, m_1, m_2$ as functions of the variables $p$ and $\alpha$ and the parameter $\alpha$.

(A.2) $y_1 = \frac{\alpha(1 \cdot \alpha)}{pf(p, \alpha, \alpha, \alpha) + \alpha}$. 
Then in turn:

(A.3) \[ m_1 = \frac{\partial \mathcal{G}(p, \bar{p}, \bar{a}, \bar{g})}{\partial f(p, \bar{p}, \bar{a}, \bar{g})} + \left(1 \bar{g} \bar{i}\right) \]

(A.4) \[ y_2 = \left(1 \bar{g} \bar{i}\right) \frac{\partial \mathcal{G}(p, \bar{p}, \bar{a}, \bar{g})}{\partial pf(p, \bar{p}, \bar{a}, \bar{g})} + \left(1 \bar{g} \bar{i}\right) \]

(A.5) \[ m_2 = \frac{\partial \mathcal{G}(p, \bar{p}, \bar{a}, \bar{g})}{\partial f(p, \bar{p}, \bar{a}, \bar{g})} + \left(1 \bar{g} \bar{i}\right) \]

The Cobb-Douglas form of the trade indirect utility function \( v^T \) is found by substituting the equilibrium values (A.2)-(A.5) into the utility function:

\[ v^T(p, \bar{p}, 1 \bar{l}, \bar{g}) = \frac{1 + f}{pf + \bar{a}1} + \left(1 \bar{g} \bar{i}\right) + \frac{1}{pf + \bar{a}1} \left(1 \bar{g} \bar{i}\right). \]

In autarky, a home producer’s utility depends solely on \( \bar{g} \); in the Cobb-Douglas case autarky utility is given by \( v^A(\bar{g}) = \bar{g} \).

Deriving the foreign economy’s excess demand functions in the Cobb-Douglas case simply replicates the steps above, recognizing that the role of goods 1 and 2 is switched. The relative price of imports for the foreigner is \( 1/p \) and the marginal rate of transformation relevant to the steps above is that for the import good in terms of the export good. All properties are the same, \( \textit{mutatis mutandis} \). Similar steps characterize the utility of the foreign trader in the Cobb-Douglas case:

\[ v^*(p, \bar{p}, \bar{a}, 1 \bar{l}^*) = \frac{1 + f^*}{f^*/p + \bar{a}^*} + \left(1 \bar{g} \bar{i}\right) + \frac{1}{f^*/p + \bar{a}^*} \left(1 \bar{g} \bar{i}\right). \]

The foreign agent’s autarky utility is \( v^* = \bar{g} \).
The predators’ indirect utility function in the Cobb-Douglas case is derived as follows. First, the relative price of good 1 on the thieves’ market (see Section 3) is:

$$ p^B = \frac{M_2}{1 \cdot M_1} = \frac{1}{1 \cdot p} $$

and the utility associated with predation is:

$$ v^p = \left( p^p \right)^{\frac{M_1}{N_1} \cdot \frac{M_2}{N_2}} = \frac{1}{1 \cdot p} \left( 1 \right) m_1 \cdot \frac{N^p}{N^p + N^* p} $$

with the reduced form import demand function

$$ m_1 = \frac{f(p, p, p, a)}{p f(p, p, p, a) + 1}, $$

where

$$ f(p) = \frac{m_1}{y_1} = \frac{1}{y(1 \cdot p) p} \cdot \left( 1 \cdot p \right) = \frac{1}{1}. $$

The above equations define the Cobb-Douglas version of the model. The properties of the full Cobb-Douglas model are explored in Section 4 of the paper through simulation. Closed form partial equilibrium comparative static results for the system (A.2)-(A.5) are available if we take $p$ to be an exogenous variable, as in Section 2 of the paper. It is immediate that a rise in “effective size” $\left( 1 \cdot \hat{L} \right)$ will raise trade volume, as is intuitive. We anticipate that a rise in $\hat{L}$ will raise the level of trade $m_1$ and the degree of specialization measured by $y_2$. A rise in $\hat{L}$ should also raise trade as it increases the gap between the autarky price ratio and the price available through trade.

First differentiate the import share function $f(p, p, p)$. 
\[ f(p, \square, \square, \square) = \frac{p(1 - p) + \frac{1}{1 - p}}{\frac{1}{a - p} - \frac{1}{1 - p}} \quad \square 1, \text{ hence} \]

\[ f_p = \frac{1 + f}{1 - p} \left[ \frac{1}{p} + \frac{1}{a - p} \right] < 0 \quad (A.6) \]

\[ f_\square = \frac{1 + f}{1 - p} \left[ \frac{1}{\square} + \frac{1}{1 - p} \right] > 0 \]

\[ f_\square = \frac{1 + f}{1 - p} \left[ \frac{1}{\square \square} \right] > 0. \]

Now we are in a position to analyze the properties of the per capita import demand function \( m_i(p, \square, a) \). Differentiating (A.3) with respect to \( p \):

\[ (A.7) \quad m_i = m_i \left[ \frac{f_p}{f} \right] - \frac{pf}{pf + \square} - \frac{1}{pf + \square} < 0. \]

The negative sign follows from noting that the square bracket term is negative for positive imports. As for the response of \( m_i \) to a rise in \( \square \), we can show that this is positive and approaches zero as complete specialization is approached:

\[ (A.8) \quad m_{1/} = m_i \left[ \frac{p}{pf + \square} \frac{f_\square}{f} \right] > 0. \]
REFERENCES


Hummels, D., “Time as a Trade Barrier,” mimeo, Purdue University, 2001.


A diagram illustrating the relationship between price (p) and trade in good 1. The diagram shows two lines intersecting at point E, with arrows indicating changes in the price ratio (\(1/\square\)) and trade in good 1. The text suggests a fall in the price ratio.
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