Inventory, Risk Shifting, and Trade Credit

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This paper has two objectives. First, we show how debt financing distorts a retailer’s inventory decision when the retailer orders multiple items that differ in cost, revenue, or demand parameters. Taking advantage of limited liability, a debt-financed retailer favors items with a low salvage value, those with a high profit margin, and those that represent a large proportion of the total inventory investment. Second, we argue that this distortion is mitigated when the financing is provided by the supplier who can observe the actual order quantities before determining the credit terms. Borrowing goods rather than borrowing cash limits the retailer’s ability to deviate from the first-best inventory decision. On the flip side, few suppliers can access capital at the same low cost as banks. We study a combination of bank and supplier financing that allows the retailer to get the best of both worlds.

Keywords: trade credit; supplier financing; debt; agency; newsvendor model; inventory

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1. Introduction

Debt financing creates an agency conflict between the creditor and the borrower generally known as “risk shifting” or “asset substitution.” Due to limited liability, a firm maximizing the value of equity has an incentive to increase risk after issuing debt. Because the lender anticipates the firm’s risk-shifting behavior and prices the debt accordingly, the equity holders ultimately bear the agency cost of debt through a higher cost of borrowing. Whereas the seminal work of Black and Scholes (1973), Jensen and Meckling (1976), and Myers (1977) gives us an understanding of risk-shifting incentives, this paper articulates what contributes to the riskiness of an inventory item. Namely, it shows how debt financing distorts a retailer’s inventory decision when the retailer orders multiple items that differ in cost, revenue, or demand parameters. In addition, we argue that this distortion is mitigated when the financing is provided by the supplier who can observe the actual order quantities before determining the credit terms.

In reality, most firms use a combination of bank and supplier financing. The advantage of banks providing credit stems primarily from their access to capital markets and ability to diversify credit risk. A supplier, on the other hand, may have an advantage in being able to understand the buyer’s business, enforce repayment, or salvage the repossessed inventory upon the buyer’s default. Trade credit can be also used to circumvent anti-price discrimination laws, reduce precautionary money holdings, or mitigate moral hazard. Rather than attempting to capture all potential pros and cons of the two types of financing, this paper simply describes one particular benefit of trade credit, which should play a role in a firm’s choice of the optimal financing strategy, but has not yet been identified in the literature.

We model a retailer with limited wealth who orders inventory of two products while facing uncertain demand. We first consider bank financing that the retailer obtains prior to purchasing inventory and without any covenants regarding order quantities. Assuming that the retailer maximizes the value of equity, his order quantities are distorted by the limited liability effect and deviate from the first-best levels that maximize total firm value. This scenario hinges on the assumption that the bank chooses credit terms before the retailer chooses inventory. This would be the case with a standard or revolving line of credit, which the borrower can tap at its discretion. The agency problem will not arise if the bank ties financing to a particular transaction, e.g., through a letter of credit.

Under trade credit financing, the retailer’s incentives depend on the number of suppliers. When purchasing both products from the same supplier who observes the entire inventory order before determining the credit terms, the retailer has the incentive to order the first-best quantities. When each product is ordered from a different supplier, trade credit cannot eliminate the retailer’s incentive to seek risk at the expense of the first supplier when ordering from the second supplier.

Because banks have access to cheaper capital than suppliers, trade credit does not necessarily dominate...
bank financing even when the retailer sources multiple items from a single supplier. Whether the retailer prefers bank financing or trade credit depends on the magnitude of the agency problem and the difference between the cost of capital of the bank and that of the supplier. We also consider a mixed financing strategy that dominates pure trade credit financing by leveraging the bank’s access to cheap capital and the supplier’s monitoring advantage.

An important assumption of our model is that the supply market is competitive, i.e., there are many potential suppliers offering similar products, none of whom are able to influence prices or credit terms. Free entry ensures that at equilibrium suppliers make zero economic profit. This has two important implications. First, wholesale prices are equal to the unit production costs, which we assume to be constant. Second, trade credit is fairly priced, i.e., suppliers offer credit terms at which they expect to break even. Consider a powerful retailer who proposes to a potential supplier a take-it-or-leave-it contract that specifies order quantities, up-front payment, and delayed payment. In the presence of many competing suppliers, the retailer will always choose contract terms at which the supplier expects to break even, and the supplier will always accept. Such a contract is equivalent to the supplier selling at the unit production costs and providing fairly priced credit. An example is a discount department store that carries generic-brand clothing, cosmetics, electronics, or food products, and sources multiple stock keeping units from any given supplier. Wal-Mart, for instance, is known to rely heavily on trade credit financing, borrowing more from its considerably smaller suppliers than in bank and bond markets despite its outstanding credit rating (Murfin and Njoroge 2015).

Our model provides the following managerial implications. First, we demonstrate how the optimal order quantities of a multiproduct newsvendor should be adjusted in the presence of debt. In particular, we show that relative to the classical all-equity newsvendor, a shareholder-controlled but debt-financed newsvendor should favor (i) items with a low salvage value, (ii) those with a high profit margin, and (iii) those that represent a large portion of the total inventory investment.

1 The existence of many competing suppliers does not mean that a particular retailer sources from more than one.

2 This is a fairly common assumption in the trade credit literature (e.g., Biais and Gollier 1997, Wilner 2000, Burkart and Ellingsen 2004). This also means that trade credit interest reflects the actual credit risk, which seems to be supported by empirical data. Namely, Klapper et al. (2012) use a data set of almost 30,000 trade credit contracts of 56 large buyers to show that early payment discounts (trade credit interest) tend to be offered to riskier buyers.

A basic tenet of the corporate finance theory is that debt induces risk seeking. For example, Jensen and Meckling (1976, pp. 335–336) argue that a debt-financed firm is better off with a higher profit variance, ceteris paribus. Interestingly, we show that between two otherwise identical products, a debt-financed newsvendor may favor the product with less volatile demand. This occurs when the product with less volatile demand represents a larger portion of the inventory investment, and thus, a greater bankruptcy risk.

Because the second-best risk-seeking strategy is optimal after financing has been obtained, managers have a fiduciary duty to pursue it. To achieve the first best, they would have to commit to their inventory decision before obtaining financing, for example, by relying on trade credit and consolidating the number of suppliers. According to our model, a firm benefits from trade credit financing most when it sources multiple differentiated items from a single supplier, and when the bankruptcy risk and the limited liability effect are significant. Finally, we show that the firm may be best off by using a mixed financing strategy involving some bank credit, complemented with additional supplier financing.

Relation to the Literature
The agency theory dates back to Black and Scholes (1973), who observed that in the presence of debt, the value of equity increases in the volatility of the return on the firm’s assets. Jensen and Meckling (1976) introduced the notion of “agency costs” that stem from shareholders’ opportunity to extract wealth from debt holders by increasing risk. Myers (1977) showed that in addition to risk shifting, the existence of debt can also lead to underinvestment. Myers and Majluf (1984) described the agency conflict that arises between old and new shareholders when managers acting in the interest of the former know more about the firm value than potential investors.

Despite the prominent role agency theory has played in the finance literature, its operational implications have received relatively little attention. One of the few exceptions is Chod and Zhou (2014), who consider a two-product firm that chooses the optimal capital structure and, subsequently, invests in product-flexible and product-dedicated production capacity. They show that while leverage generally leads to underinvestment and less flexibility, the agency problem disappears when the firm invests in a single fully flexible resource and, thus, cannot favor one type of capacity over another. This result has a parallel in the context of our model: A retailer that sells a single undifferentiated product cannot engage in risk shifting by favoring the “riskier items” over the “safer ones.” The incremental contribution of our
paper vis-à-vis Chod and Zhou (2014) is twofold. First, we examine how debt affects stocking levels of products that differ in cost, revenue, and demand parameters, whereas Chod and Zhou study how debt affects capacity to produce symmetrical, undifferentiated products. Second, we argue that trade credit mitigates the agency cost associated with other forms of debt, whereas Chod and Zhou do not consider trade credit at all.3

Another recent paper that studies operational repercussions of financial leverage is by Iancu et al. (2016), who consider a lending contract between a retailer and a bank that is collateralized by inventory. The agency conflict arises when the retailer, after observing initial sales, must decide whether to liquidate inventory or continue to operate. Iancu et al. (2016) demonstrate that this conflict can be eliminated by a suitably designed financial covenant, which depends on the borrower’s operational characteristics such as demand distribution, inventory depreciation, or profit margin. Our work is also related to that of Van Mieghem (2007) and Chod et al. (2010), who examine the role of risk in multi-item newsvendor networks. Whereas this literature assumes risk aversion, we endogenize risk attitude by taking into account the role of external financing and limited liability. Boyabatli and Toktay (2011) examine the effect of external financing on optimal technology choice, but no agency frictions arise in their model.

The question of why most firms provide trade credit to their customers while receiving credit from their suppliers in the presence of specialized financial intermediaries, has puzzled financial economists for several decades. Among the existing explanations, our work is most related to those based on a monitoring advantage of the supplier and those based on moral hazard. The former argue that in the course of their business relationship the supplier gains superior information about the buyer’s credit worthiness (see, e.g., Smith 1987, Brennan et al. 1988, Biais and Gollier 1997, Jain 2001). The latter focus mostly on the moral hazard faced by the buyer. When product quality is not immediately observable, deferred payment provides the supplier with an incentive to exert effort and serves as a guarantee or a signal of product quality (see, e.g., Lee and Stowe 1993, Long et al. 1993, Babich and Tang 2012, Kim and Shin 2012).

The literature that links trade credit to the moral hazard faced by the lender, is much more limited. Cunat (2007) points out that the supplier of a unique input has an advantage over a bank in controlling the buyer’s behavior. According to Burkart and Ellingsen (2004), the advantage of lending goods over lending cash is that goods are more difficult for an opportunistic borrower to divert for private benefits. Similar to Burkart and Ellingsen, we link the benefit of trade credit to an agency conflict between the borrower and the lender. However, whereas Burkart and Ellingsen consider cash diversion by an opportunistic entrepreneur, we focus on asset substitution due to limited liability. Therefore, unlike Burkart and Ellingsen, we must consider multiple products and bankruptcy risk.

Other theories attempt to explain the prevalence of trade credit based on price discrimination (e.g., Brennan et al. 1988, Mian and Smith 1992), transaction cost (Ferris 1981, Emery 1987), imperfect competition (Emery 1984, Wilner 2000), and the supplier’s advantage in salvaging the repossessed merchandise in case the buyer defaults (Frank and Maksimovic 2004, Mian and Smith 1992). Petersen and Rajan (1997) tested the various theories empirically concluding that trade credit is likely to serve multiple purposes. More recently, Giannetti et al. (2011) reported empirical evidence favoring the theories based on borrower’s opportunism and suppliers’ informational advantage.

The operations literature has studied the effect of trade credit on optimal inventory control (e.g., Haley and Higgins 1973, Jaggi et al. 2008, Gupta and Wang 2009, Luo and Shang 2013), as well as on supply chain performance (Chaharsooghi and Heydari 2010, Lee and Rhee 2010). Kouvelis and Zhao (2012) model the strategic interaction between a retailer and its supplier, both of whom are financially constrained and face bankruptcy risk. They characterize the optimal trade credit contract from the supplier’s perspective, and they show that the retailer always prefers this contract to bank financing and that trade credit improves supply chain efficiency.

Yang and Birge (2013) consider the interaction between a supplier and a retailer who finances inventories using the optimal mix of cash, trade credit, and short-term debt. They demonstrate that trade credit enhances supply chain efficiency by serving as a risk-sharing mechanism. In particular, trade credit shifts some of the inventory risk from the retailer to the supplier, which induces the retailer to increase its order quantity. Dong et al. (2016) show that a manufacturer can use trade credit to induce a retailer to carry more of the manufacturer’s products.

By identifying a new rationale for consolidating the number of suppliers, our paper also relates to the literature on optimal supplier selection. This literature generally ignores financing decisions with the notable exception of Babich et al. (2012), who study the optimal financing strategy and supplier selection, assuming that suppliers are unreliable and offer trade credit.

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3 Another difference from Chod and Zhou (2014) is our assumption that equity is given. This reflects the fact that a firm is unlikely to raise new equity and reoptimize its capital structure each time it orders inventory.
The authors show that bank loans and trade credit are complementary, and that firms should use more suppliers when bank financing is not available. Seifert et al. (2013) provide a comprehensive survey of the trade credit literature spanning all disciplines. As they point out, most existing trade credit models consider a single product. We show that considering multiple products, or “assets,” makes a difference because of the role that trade credit can play in alleviating the asset substitution problem.

2. Model

In the following, $E$ denotes expectation, bold font denotes vectors, and prime denotes transpose. We consider a retail firm that orders an inventory of two products under demand uncertainty. Let $Q = (Q_1, Q_2)'$ denote the order quantities, and let $c = (c_1, c_2)'$ denote the unit product costs, or wholesale prices. Assuming that the upstream industry is competitive and suppliers incur constant marginal costs of production, we take the wholesale prices as given.

Once demand uncertainty is resolved, the firm generates revenue $\pi(D, Q)$, where $D = (D_1, D_2)'$ represents demand shocks corresponding to the two products. The random vector $D$ is allowed to follow an arbitrary continuous distribution over $\mathbb{R}_+^2$, and we denote its mean vector and covariance matrix as $\mu = (\mu_1, \mu_2)'$ and

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix},$$

respectively. The revenue function $\pi(D, Q)$ is assumed to be continuous and nondecreasing in all arguments, and jointly concave in $Q$. Finally, we assume that $\pi_i \equiv \partial \pi/\partial Q_i$, $i = 1, 2$, exists almost everywhere. These assumptions are satisfied, among others, in the following two instances.

**Newsvendor Model.** Suppose that the random vector $D$ represents actual product demands, the firm sells as much inventory as possible at given retail prices $p = (p_1, p_2)'$, and it salvages the rest at given salvage values $s = (s_1, s_2)'$. In other words, the firm’s revenue is given by

$$\pi(D, Q) = \sum_{i=1}^2 (p_i \min(Q_i, D_i) + s_i (Q_i - \min(Q_i, D_i))).$$

To avoid trivial scenarios, we assume $s_i < c_i < p_i$, $i = 1, 2$.

**Linear Demand Model.** Suppose that the firm faces linear demand curves that are subject to additive uncertainty. Namely, the retail market-clearing price of product $i$ is given by $p_i(D, x) = D_i - x_i - \gamma x_{3-i}$, where $D$ is the vector of stochastic demand curve intercepts, $x = (x_1, x_2)'$ is the vector of quantities sold, and $\gamma \in [0, 1)$ is a measure of product substitutability, for $i = 1, 2$. Suppose further that the firm chooses the retail prices or, equivalently, how much inventory to sell, after observing the realized demand shocks. The unsold inventory is again salvaged at fixed salvage values $s$. In this case, the firm’s revenue is given by

$$\pi(D, Q) = \max(x'p(D, x) + (Q - x)'s)$$

$$\text{s.t. } x_i \leq Q_i, \quad i = 1, 2.$$  \hspace{1cm} (2)

Regardless of the functional form of $\pi$, it is useful to define the following four events: Let $\Omega_0$ be the set of demand shock realizations for which the firm ends up with excess inventory of both products; let $\Omega_1$ ($\Omega_2$) be the event in which the firm sells the entire inventory of product 1 (2), but ends up with excess inventory of product 2 (1); and let $\Omega_3$ be the event in which the firm sells the entire inventory of both products. It is also useful to define $\Omega_{ij} \equiv \Omega_i \cup \Omega_j$, $i, j \in \{0, 1, 2, 3\}$. The formal characterization of these events, as well as the functional form of $\pi$ corresponding to each of them, are given in (34) for the newsvendor model and in (35) for the linear demand model.

Let $W$ be the firm’s remaining wealth after all non-inventory costs have been incurred. Assuming that this wealth does not suffice to finance the entire inventory investment, i.e., $W < cQ$, we consider two forms of external financing: bank loan and trade credit.

2.1. Bank Financing

Suppose that before buying the inventory, the firm borrows $L$ dollars from a bank, promising to repay $L + r$ dollars once the selling season is over. Assuming that debt covenants prohibit the firm from paying out dividends before repaying the loan, the firm has no reason to borrow more than what it intends to spend on inventory, i.e.,

$$L + W = cQ.$$  \hspace{1cm} (3)

Since it is unusual for debt covenants to prescribe a specific operational policy, we assume that it is within the firm’s discretion to allocate the available capital $L + W$ between the inventory of the two products.

Depending on the realized demand, the firm may or may not be able to repay the debt in full. Thus, we distinguish two events:

(a) If $\pi(D, Q) \geq L + r$, the firm’s revenue is sufficient to repay the debt in full. The value of debt is $L + r$, whereas the value of equity is $\pi(D, Q) - L - r$.

(b) If $\pi(D, Q) < L + r$, the firm is unable to repay the debt in full, declares bankruptcy, and is taken over by the lender. Assuming a fixed bankruptcy cost...
denoted as \( b \), the payoff to the lender is \( \pi(D, Q) - b \). The equity becomes worthless.

We denote these two events as \( \Omega_a \) and \( \Omega_b \), respectively, so that subscript \( b \) is mnemonic for bankruptcy. If we further define \( \Omega_j \equiv \Omega_i \cap \Omega_j \), for \( i \in \{0, 1, 2, 3\} \) and \( j \in \{a, b\} \), we can distinguish seven events, which are illustrated in Figures 1(a) and 1(b) for the newsvendor and linear demand models, respectively.\(^5\)

Following the literature (e.g., Biais and Gollier 1997, Wilner 2000, Burkart and Ellingsen 2004), we assume that upon default, the creditor initiates the bankruptcy process before observing the retailer’s liquidation value and, thus, initiates bankruptcy even if \( b > \pi \).

We also assume that the firm is fully controlled by shareholders who maximize the expected terminal value of equity, which we denote by \( V \). Because the value of equity is \( \pi - L - r \) if \( D \in \Omega_a \), and it is zero if \( D \in \Omega_b \), we have

\[
V(Q, L, r) = \Pr(\Omega_a) \mathbb{E}(\pi - L - r | \Omega_a).
\]  

(5)

The firm chooses the amount of borrowing and inventory in two stages. In the second stage, the credit terms \( (L, r) \) are given and the firm chooses the optimal inventory levels. Using the superscript \( BF \) to denote the optimal solution under bank financing, the second-stage problem can be written as

\[
Q^{BF}(L, r) = \arg \max_{Q \geq 0} V(Q | L, r) \quad \text{subject to} \quad (3). \tag{6}
\]

In the first stage, the firm chooses the optimal loan amount, taking into account the interest determined by the bank according to (4). In this stage, both the firm and the bank anticipate the firm’s second-stage inventory choice given by (6). Formally, we have

\[
L^{BF} = \arg \max_{L \geq 0} V(L, r^{BF}(L), Q^{BF}(L)), \tag{7}
\]

where \( r^{BF}(L) \) and \( Q^{BF}(L) \) satisfy (4) together with (6). To simplify the notation, we let \( Q^{BF} \equiv Q^{BF}(L^{BF}), r^{BF} \equiv r^{BF}(L^{BF}), \) and \( V^{BF} \equiv V(Q^{BF}, L^{BF}, r^{BF}) \). The next lemma characterizes the optimal loan and inventory levels assuming they are all positive.

**Lemma 1.** For any given loan amount \( L \), the optimal inventory \( Q^{BF}(L) \) and the corresponding interest \( r^{BF}(L) \) must satisfy

\[
\frac{\mathbb{E}(\pi_1 | \Omega_a)}{c_1} = \frac{\mathbb{E}(\pi_2 | \Omega_a)}{c_2}, \tag{8}
\]

together with (3) and (4). The optimal loan amount \( L^{BF} \) must satisfy

\[
\frac{\mathbb{E}(\pi_1 | \Omega_a)}{c_1} = 1 + \frac{dr^{BF}}{dL}. \tag{9}
\]

The optimality condition (8) provides the rule according to which the firm allocates its available capital between the inventory of the two products.
It ensures that the last dollar invested in the inventory of each product has the same expected payoff. When evaluating this payoff, the firm takes into account only nonbankruptcy states because the revenue realized in bankruptcy states accrues to the bank, not the shareholders. The optimality condition (9) equates the expected revenue generated for the shareholders by the last dollar borrowed and the corresponding increase in the loan principal and interest that the shareholders expect to repay.

To understand exactly how debt distorts the firm’s inventory decision, we need to define the “first best.” Note that our definition of the first-best strategy has to do with the agency conflict between the shareholders and the debt holders, rather than with the misalignment of incentives between the retailer and the supplier.

2.2. First Best

In this section, we consider an alternative scenario in which the firm chooses the amount of borrowing and inventory that jointly maximize total firm value, i.e., the value of equity plus the value of debt. Otherwise, we keep all of our assumptions unchanged. Namely, we continue to assume that the firm’s wealth is limited, debt is fairly priced, and the lender’s cost of capital is \( \phi_B \). Formally, we define the first-best strategy as

\[
Q^*, L^*, r^* = \arg\max_{Q, L, r \geq 0} \mathbb{E} \pi(Q) - cQ - \phi_B L - \Pr(\Omega_B) \]

subject to (3) and (4),

\[
(10)
\]

and we let \( V^* = V(Q^*, L^*, r^*) \).

The reason why a bank-financed firm does not generally achieve the first best is that after obtaining the loan, it has the incentive to choose inventory that maximizes the value of equity rather than the total firm value. The bank can anticipate this and prices the loan accordingly. The resulting loss of total firm value, known as the agency cost of debt, \( V^* - V^{BF} \), is thus ultimately borne by the shareholders through a higher cost of borrowing. The next lemma characterizes the solution to (10) when the optimal inventory levels and loan amount are positive.

**Lemma 2.** The first-best inventory \( Q^* \) must satisfy

\[
\mathbb{E} \pi_1 = (1 + \phi_B)c_1 + \frac{d \Pr(\Omega_B)}{dQ_1} b \quad \text{and} \quad \mathbb{E} \pi_2 = (1 + \phi_B)c_2 + \frac{d \Pr(\Omega_B)}{dQ_2} b,
\]

where \( L \) and \( r \) embedded in \( \Omega_B \) are given by (3) and (4).

The optimality conditions (11) equate the expected marginal revenue and expected marginal cost of the two products, respectively. In general, the first-best inventory levels depend on the bankruptcy cost because the amount of inventory affects the probability of bankruptcy. In the special case of zero bankruptcy cost, the optimality conditions (11) can be combined into the following rule, according to which the firm allocates total capital between the two products:

\[
\frac{\mathbb{E} \pi_1}{c_1} = \frac{\mathbb{E} \pi_2}{c_2}.
\]

Recall the analogous rule that we established in Lemma 1 for the bank financing scenario:

\[
\frac{\mathbb{E}(\pi_1 | \Omega_a)}{c_1} = \frac{\mathbb{E}(\pi_2 | \Omega_a)}{c_2}.
\]

The difference between the two allocation rules, (12) and (13), is that a bank-financed firm takes into account only the revenue that accrues to shareholders, i.e., the revenue generated in nonbankruptcy states \( \Omega_a \). In the next section, we examine how exactly this affects the stocking levels.

3. Risk Shifting and Inventory

In this section, we characterize the effect of debt on optimal inventory for several special cases, and verify the robustness of our analytical findings in a series of numerical experiments. It is well known that in the presence of debt, equity is equivalent to a call option on the firm’s assets, and its value increases in volatility of the return on these assets (Black and Scholes 1973). One would, therefore, expect a debt-financed firm to favor the “riskier” of the two products. The question is what makes an inventory item risky. We address this issue in the next proposition assuming that the revenue function \( \pi \) is given by the newsvendor or linear demand model.

**Proposition 1.** Suppose that \( b = 0 \) and \( \pi \) is given by the newsvendor model (1) or by the linear demand model (2) with \( \gamma = 0 \). Suppose further that

(i) \( \Pr(\Omega_{2b}(Q^{BF})) > 0 \),
(ii) \( \Pr(\Omega_{1b}(Q^{BF})) = 0 \),
and (iii) \( s_1/c_1 \leq s_2/c_2 \).

Under bank financing, the firm overinvests in product 1, or it underinvests in product 2, i.e.,

\[
Q_1^{BF} > Q_1^* \quad \text{or} \quad Q_2^{BF} < Q_2^*.
\]

The three conditions in (14) ensure that product 1 is in some sense riskier than product 2. According to (i), low sales of product 1 can lead to bankruptcy even if product 2 is sold out. According to (ii), low sales of product 2 cannot result in bankruptcy as long as product 1 is sold out. This situation is illustrated in Figure 2 for both revenue models. Finally, condition (iii) ensures that product 1 has a lower relative salvage value. In the remainder of this section, we examine how risk shifting depends on various model parameters, focusing on the newsvendor model.
3.1. Newsvendor Model

When the revenue function is given by (1), conditions (i) and (ii) in (14) can be written as

\[(p_2 - c_2)Q_2^{BF} - (c_1 - s_1)Q_1^{BF} < r^{BF} - W, \quad (16)\]

and \[(p_1 - c_1)Q_1^{BF} - (c_2 - s_2)Q_2^{BF} \geq r^{BF} - W, \quad (17)\]

respectively. Thus, we would expect a bank-financed firm to favor product 1 as a result of one or several of the following factors:

1. product 1 has a low salvage value; product 2 has a high salvage value
2. product 1 has a high profit margin; product 2 has a low profit margin
3. product 1 represents a large proportion of total inventory, e.g., because of high mean demand.

In the following analysis, we examine how the distortions of the inventory decision depend on individual product parameters. When the products are symmetrical \((p_1 = p_2, c_1 = c_2, s_1 = s_2, \mu_1 = \mu_2, \text{and } \sigma_1 = \sigma_2)\), the firm does not have an incentive to favor either of them, thus, it follows the first-best strategy, i.e., \(Q_1^{BF} = Q_2^{BF} = Q_1^* = Q_2^*\). We start by considering this symmetrical case, and examine how the optimal inventory levels deviate from the first best, as one of the model parameters changes by a sufficiently small amount.

We complement this analysis by a series of numerical experiments in which we also begin with the fully symmetrical “base case,” and examine the impact of increasing asymmetry in various model parameters on the inventory decision, cost of borrowing, and the agency cost of debt. In these experiments, we assume that demand follows a bivariate lognormal distribution, and we use the following base-case parameter values: \(c_1 = 1, p_1 = 1.2, s_1 = 0, \mu_1 = 100, \text{and } \sigma_1 = 75\) for \(i = 1, 2; \ W = 0, \ \phi_B = 0, \ b = 5, \text{and } \rho \in \{-0.5, 0, 0.75\}\).

We consider a relatively low profit margin and a relatively high coefficient of variation of demand to capture situations in which the bankruptcy risk, and therefore, the agency cost of debt, are significant. With our base-case parameter values, the probability of bankruptcy ranges between 8%–11%, depending on demand correlation. Setting \(W = 0\) means that the firm’s internal capital covers all noninventory costs, whereas the inventory is financed entirely by debt. When \(W > 0\), our results are qualitatively the same, but of a smaller magnitude. Our choice of bankruptcy cost, \(b = 5\), means that at the base-case parameter values, the bankruptcy cost represents, depending on demand correlation, 6.8%–8.2% of the expected firm value at default \(\mathbb{E} (\pi | \Omega_b)\). This is in line with empirical data. For example, Bris et al. (2006) estimate the default cost to vary between 0%–20% of the firm value at default. Although we ran all of our numerical experiments for three different values of demand correlation, we only show results for the correlation at which the agency cost is most significant, and which we indicate in the figure captions. The numerical study is based on simulating 100,000 bivariate demand scenarios, and the optimization is done directly using “Global Search” algorithm in Matlab.
Salvage Value. The next proposition characterizes the effect of salvage value assuming that $\Omega_{12b}(Q^*) = \emptyset$, i.e., the firm cannot go bankrupt when at least one of two products is sold out.

**Proposition 2.** Assume that $b = 0$, all product parameters are symmetrical, and $\Omega_{12b}(Q^*) = \emptyset$. Now suppose that the salvage value of product 2 increases by $\epsilon$. Under bank financing, the proportion of product 2 in the inventory mix increases less than it does under the first-best strategy, i.e.,

$$\left[ \frac{d}{ds_2} \frac{Q_{2}^{BF}}{Q_{1}^{BF} + Q_{2}^{BF}} \right]_{s_2=s_1} \leq \left[ \frac{d}{ds_2} \frac{Q_{2}^*}{Q_{1}^* + Q_{2}^*} \right]_{s_2=s_1}. \quad (18)$$

A debt-financed firm ignores the salvage revenue generated in bankruptcy states. As a result, the product with a higher salvage value represents a smaller portion of the firm’s total inventory than in the first-best scenario. This effect is illustrated in Figure 3. When $s_2 = s_1 = 0$, the products are symmetrical and the firm orders the same quantity of each. As $s_2$ increases, product 2 becomes more attractive and its optimal inventory level increases (Figure 3(a)). The higher salvage value of product 2 also reduces credit risk and, hence, the interest rate, $r/L$ (Figure 3(b)). The lower cost of capital, together with the higher salvage value of one of the products, results in a larger total inventory investment and the corresponding loan, $cQ = L$ (Figure 3(c)). The lower cost of capital means that the firm increases the order quantities of both products, but the increase is more significant for product 2, whose salvage value has gone up (Figure 3a).
Although all of these effects are similar under bank financing and the first-best scenario, they have different magnitudes. Consistent with Proposition 2, a bank-financed firm underinvests in the high-salvage-value product, $Q_{1B} > Q_{2B}$, and overinvests in the low-salvage-value product, $Q_{2B} > Q_{1B}$ (Figure 3(a)). Because the bank anticipates this distortion of the firm's inventory decision, it charges a higher interest rate compared to the first best (Figure 3(b)), to which the firm responds by borrowing and investing less (Figure 3(c)). Finally, Figure 3(d) shows the agency cost of debt as a percentage of shareholder value, $(V^* - V_{BF})/V_{BF}100\%$. As the salvage value of product 2 increases, so does the difference in the riskiness of the two products. As a result, risk shifting becomes more significant and the agency cost of debt increases.

**Unit Revenue (Retail Price).** The next proposition characterizes the effect of asymmetry in unit revenues, assuming that demands are independent and follow IFR distributions. Now suppose that the unit revenue of product 2 increases by $\varepsilon$. Under bank financing, the proportion of product 2 in the inventory mix increases more than it does under the first-best strategy, i.e.,

$$\frac{d}{dp_2} \left( \frac{Q_{2B}^p}{Q_{1B}^p + Q_{2B}^p} \right)_{p_2=p_1} \geq \frac{d}{dp_2} \left( \frac{Q_{2}^*}{Q_{1}^* + Q_{2}^*} \right)_{p_2=p_1}. \quad (19)$$

The product with a higher unit revenue represents a larger inventory investment and a larger risk of excess inventory. As a result, sales of this product are more critical to whether the firm goes bankrupt or not. In this sense, the high-margin product is riskier. Therefore, it is favored by a debt-financed firm. This effect is illustrated in Figure 4(a). When $p_2 = p_1 = 1.2$, the order quantities of the two products are equal and are not distorted by the agency problem. When $p_2 \neq p_1$, a bank-financed firm overinvests in the high-margin, high-inventory product, and underinvests in the low-margin, low-inventory product, as predicted by Proposition 3. The larger the asymmetry in profit margins, the larger the distortion of the inventory decision, and the larger the agency cost of debt (Figure 4(b)). Our numerical experiments also indicate that the agency problem increases the cost of borrowing, $r_{BF}/L_{BF} > r^*/L^*$, and decreases the inventory investment, $cQ_{BF}^p \leq cQ^*.$

**Unit Cost (Wholesale Price).** The effect of the unit cost is similar to that of the unit revenue in the sense that a debt-financed firm favors, once again, the high-margin product.

**Proposition 4.** Assume that $b = 0$, $s_1 = s_2 = 0$, all product parameters are symmetrical, and demands are independent and follow IFR distributions. Now suppose that the cost of product 2 increases by $\varepsilon$. Under bank financing, the proportion of product 2 in the inventory mix decreases more than it does under the first-best strategy, i.e.,

$$\frac{d}{dc_2} \left( \frac{Q_{2B}^p}{Q_{1B}^p + Q_{2B}^p} \right)_{c_2=c_1} \leq \frac{d}{dc_2} \left( \frac{Q_{2}^*}{Q_{1}^* + Q_{2}^*} \right)_{c_2=c_1}. \quad (20)$$

A debt-financed firm favors the low-cost product because it represents a larger portion of total inventory, and its sales are the main driver of the bankruptcy risk. To examine the effect of asymmetry in demand parameters, we resort directly to numerical results.
**Mean Demand.** The effect of asymmetry in mean product demands is shown in Figure 5. At $\mu_2 = \mu_1 = 100$, the products are fully symmetrical and there is no agency cost of debt. When $\mu_2 \neq \mu_1$, the low-demand product has less inventory, and thus, is less risky, in the sense that its low sales are unlikely to cause bankruptcy as long as the other product sells out. Therefore, consistent with Proposition 1, a bank-financed firm overinvests in the high-demand, high-inventory product, and underinvests in the low-demand, low-inventory product, as shown in Figure 5(a). Figure 5(b) illustrates how this distortion of the inventory decision impacts firm value.

**Demand Volatility.** As shown in Figure 6(a), a bank-financed, risk-seeking firm overinvests in the low-volatility product and underinvests in the high-volatility product. However surprising this may seem, it is consistent with Proposition 1 and conditions (16)–(17), according to which a debt-financed firm favors the item that represents a large portion of total inventory. When profit margins are relatively low, as is the case in our numerical experiments, the high-volatility product represents a smaller portion of total inventory. Therefore, its low sales are unlikely to cause bankruptcy as long as the low-volatility, high-inventory product sells out. Thus, the low-volatility, high-inventory product turns out to be riskier. As such, it is favored by a bank-financed retailer.

In newsvendor-like models and under Gaussian demand, the optimal inventory level depends on the product of the demand standard deviation and the $z$-value. Thus, the fact that higher demand volatility leads to lower inventory is true only for negative $z$-values, which correspond to low profit margins. Therefore, we expected to observe the opposite effect for high-margin products whose optimal inventory *increases* in demand volatility. However, in our numerical experiments based on lognormal demand distribution, optimal inventory does not increase in demand volatility unless the newsvendor ratio $(p_i - c_i)/\sigma_i \gtrsim 0.8$, i.e., $p_i \gtrsim 5$. With such high profit margins, the firm cannot go bankrupt with one of the products being sold out ($\Omega_{i2} = \emptyset$), in which case asymmetry in demand volatility alone does not result in any risk shifting. In other words, we did not find any parameter combination for which we would observe results contrary to those shown in Figure 6.

As we have seen, when any parameter of one inventory item changes, the optimal order quantities of both items change in response. This is because both order quantities depend on the cost of borrowing, which in turn depends on characteristics of both products. As product 2 becomes more attractive (say, its salvage value, $s_2$, or unit revenue, $p_2$, increases), the marginal cost of borrowing decreases, which provides the firm with an incentive to increase both order quantities. At the same time, the firm has an incentive to reallocate some of its inventory budget away from product 1 toward product 2. As a consequence of these two opposing effects, the optimal order quantity of product 1 can increase or decrease. For example, $Q_{1*}^{BF}$ increases in $s_2$ (Figure 3(a)), but mostly decreases in $p_2$ (Figure 4(a)). When the products are substitutes or complements from the consumers’ point of view, their optimal order quantities are further linked on the revenue side through cross-price elasticity. This is the case under the linear demand model.

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7 These two effects are analogous to the income and substitution effects in the standard consumer choice model.
3.2. Linear Demand Model

The electronic companion (available as supplemental material at http://dx.doi.org/10.1287/mnsc.2016.2515) provides a similar numerical analysis for the linear demand model (2). It examines the effect of asymmetry in product-specific parameters, $c_i$, $\mu_i$, and $\sigma_i$, under different levels of product substitutability $\gamma$. The results are consistent with the newsvendor model in the following sense. As we have seen in Figures 4(a), 5(a), and 6(a), when the two products differ in profit margin, mean demand, or demand volatility, a bank-financed newsvendor overinvests in the product that represents a larger portion of the total inventory, and underinvests in the other product, i.e.,

$$Q_i^{BF} \geq Q_i^* \geq Q_j^* \geq Q_j^{BF}.$$  

We observe exactly the same phenomenon under the linear demand model. Moreover, in all of our numerical experiments, the relative agency cost, $(V^* - V^{BF})/V^{BF}$, increases with product substitutability $\gamma$. As the products become closer substitutes, their profit margins shrink, bankruptcy risk increases, and so does the risk-shifting problem.

4. Trade Credit

4.1. Single Supplier

Suppose, once again, that the firm orders inventory $Q$ at wholesale prices $c$, while its wealth $W$ is less than the total inventory cost $cQ$. Suppose further that both items are ordered from the same supplier who provides the entire residual financing, $L = cQ - W$, in the form of trade credit. The interest is again denoted as $r$, so the firm pays the supplier $W$ up front and $L + r$ at the end of the selling season.\footnote{This contract corresponds to a common form of trade credit terms known as “two-part terms,” which specify the early payment discounted price, the discount period, the payment due date, and the effective interest rate (Klapper et al. 2012). In our model, these correspond to $c$, the delivery time, the end of the selling season, and $r/L$, respectively. It is fairly common for firms to delay payments to their suppliers beyond the terms stipulated contractually (e.g., Giannetti et al. 2011). Although the greater leniency of suppliers relative to banks may be an important consideration in practice, it is outside the scope of our static model.} Consistent with our assumption of a competitive upstream industry and the finance literature (e.g., Biais and Gollier 1997, Wilner 2000, Burkart and Ellingsen 2004), we assume that, similar to bank credit, trade credit is fairly priced.

Trade credit differs from bank credit in two ways. First, following Burkart and Ellingsen (2004), we assume that the supplier faces a higher cost of capital than do banks. Denoting the interest rate at which the supplier obtains funds as $\phi_S$, fair pricing of trade credit requires

$$(1 + \phi_S)L = \Pr(\Omega_1)(L + r) + \Pr(\Omega_2)\mathbb{E}(\pi - b | \Omega_2).$$  \hspace{1cm} (21)

Second, we assume that unlike a bank, the supplier sets the interest rate after observing the actual order quantities. Using superscript TC for the optimal solution under trade credit financing, the firm’s decision problem can be written as

$$Q^{TC} = \arg\max_{Q \geq 0} V(Q, L^{TC}(Q), r^{TC}(Q)), \hspace{1cm} (22)$$

where the value of equity $V$ is given by (5), and the credit terms, $L^{TC}(Q)$ and $r^{TC}(Q)$, are given by (3) and (21). Let $L^{TC} \equiv L^{TC}(Q^{TC})$, $r^{TC} \equiv r^{TC}(Q^{TC})$, and $V^{TC} \equiv V(Q^{TC}, L^{TC}, r^{TC})$ be the optimal amount...
of trade credit, and the corresponding interest and equity value, respectively. The next proposition formalizes the advantage of using trade credit.

**Proposition 5.** Under trade credit financing, the firm chooses the first-best inventory levels, for which it receives the first-best credit terms. Formally, $(Q^{TC}, L^{TC}, r^{TC})$ solves (10) with $\phi_B$ replaced by $\phi_S$.

There is no agency cost associated with trade credit because the trade credit interest is contingent on the inventory decision. The retailer knows that taking too much risk when ordering inventory would automatically result in less favorable credit terms. The order quantities that maximize the value of equity under supplier financing are those that maximize total firm value.

At the same time, the supplier faces a higher cost of capital than a bank, $\phi_S > \phi_B$. This means that for any given order quantities, the supplier requires a higher interest rate from the retailer than does a bank. Obviously, if there is no agency cost associated with bank financing, such as when the two products are completely symmetrical or there is no demand risk, the retailer prefers the cheaper bank credit. In the more interesting case in which the agency cost is positive, $V^{BF} < V^*$, there is a trade-off between using trade credit and bank financing, which we characterize in the next corollary to Proposition 5.

**Corollary 1.** Suppose that $V^{BF} < V^*$. The retailer prefers trade credit over bank financing, i.e., $V^{TC} > V^{BF}$, as long as the difference between $\phi_S$ and $\phi_B$ is sufficiently small. When this difference is sufficiently large, the retailer prefers bank financing over trade credit, i.e., $V^{TC} < V^{BF}$.

Because it eliminates risk-shifting incentives, supplier financing can dominate bank financing despite the supplier’s cost of capital being higher than that of the bank, as long as the difference is not too large. A natural question arises whether a retailer can combine the benefit of trade credit with that of bank financing by using a mixed financing strategy.

### 4.2. Mixed Financing

Suppose that the retailer first obtains a fairly priced bank loan, and let $L_B$ and $r_B$ denote its principal and interest, respectively. As is common is practice (e.g., Longhofer and Santos 2000 and references therein), we assume that this bank loan is given seniority with respect to all subsequent creditors. The retailer then orders inventory, using trade credit for the remaining amount $L_S = cQ - L_B - W$. The supplier provides the goods and the financing, and charges fair interest $r_S$. At the end of the selling season, one of two events, denoted again as $\Omega_s$ and $\Omega_b$, takes place.

(a) If $\pi(D, Q) \geq L_B + L_S + r_B + r_S$, both creditors are fully repaid, and the value of equity is $V = \pi - L_B - L_S - r_B - r_S$.

(b) If $\pi(D, Q) < L_B + L_S + r_B + r_S$, the retailer defaults, the bank receives $\min(\pi - b, L_B + r_B)$, the supplier receives $\max(0, \pi - b - L_B - r_B)$, and the value of equity is zero.

The expected value of equity is therefore $V = \Pr(\Omega_s)E(\pi - L_B - L_S - r_B - r_S | \Omega_s)$. Using superscript $MF$ for the optimal mixed financing strategy, the retailer’s second-stage problem is

$$Q^{MF}(L_B, r_B) = \arg\max_{Q \geq 0} V(Q, L_S, r_S | L_B, r_B),$$

where $L_S = cQ - L_B - W$, and $r_S$ is given by the following fair pricing condition:

$$(1 + \phi_S)L_S = \Pr(\Omega_s)(L_S + r_S) + \Pr(\Omega_b)E(\max(0, \pi - b - L_B - r_B) | \Omega_b).$$

The retailer’s first-stage problem is

$$L_B^{MF} = \arg\max_{L_B \geq 0} V(Q, L_S, r_S, L_B, r_B),$$

where $Q$ is given by (23), $L_S = cQ - L_B - W$, $r_S$ is given by (24), and $r_B$ is given by the following fair pricing condition:

$$(1 + \phi_B)L_B = \Pr(\Omega_b)(L_B + r_B) + \Pr(\Omega_s)\cdot E(\min(\pi - b, L_B + r_B) | \Omega_b).$$

Let $V^{MF} \equiv V(Q^{MF}, L_B^{MF}, L_S, r_B^{MF}, r_S^{MF})$. The next proposition confirms our intuition about the advantage of mixed financing.

**Proposition 6.** The retailer prefers mixed financing over trade credit financing, i.e., $V^{MF} \geq V^{TC}$. If $\pi$ is given by the newsvendor model (1) or by the linear demand model (2), $s > 0$, $W > 0$, and $b = 0$, the preference is strict and

$$V^{MF} - V^{TC} \geq (\phi_S - \phi_B)\frac{W\min(s_1/c_1, s_2/c_2)}{1 + \phi_B}.$$
including the fair interest, regardless of demand realization. Because this loan is not risky, it does not distort the retailer’s incentives when he orders inventory and requests additional credit from the supplier. The retailer thus orders the same first-best quantities as in the trade credit scenario, while enjoying a lower interest rate applied by the bank to the secured portion of total debt. In practice, a firm can use a similar mixed financing strategy even if \( s = 0, W = 0, \) and \( b > 0, \) as long as it has some assets to collateralize the bank loan.

The benefit of trade credit hinges on the assumption that both products are ordered from the same supplier. In the next section, we examine what happens if we depart from this assumption.

### 4.3. Two Suppliers

Suppose that the retailer buys each product from a different supplier and relies entirely on trade credit financing. We consider the following sequence of events. The retailer first orders \( Q_1 \) units of product 1 from supplier 1 at unit cost \( c_1. \) Supplier 1 provides the goods as well as full trade credit financing \( L_1 = c_1 Q_1, \) and charges fair interest \( r_1. \) Subsequently, a similar transaction takes place between the retailer and supplier 2.

Let \( \Omega_2 \equiv \{ D: \pi(D, Q) \geq L_1 + r_1 + L_2 + r_2 \} \) be the event in which the retailer repays both suppliers in full, and let \( \Omega_b \equiv \mathbb{R}^2 \setminus \Omega_2 \) be the complementary event, in which the retailer goes bankrupt. We assume that upon bankruptcy, the suppliers split the liquidation value of the retailer proportionally to the face value of their claims.\(^9\) Assuming that both suppliers incur the same cost of capital \( \phi, \) fair pricing of trade credit requires

\[
(1 + \phi) L_i = \Pr(\Omega_a) (L_i + r_i) + \Pr(\Omega_b) \pi(\pi - b | \Omega_b) \cdot \frac{L_i + r_i}{L_i + r_1 + L_2 + r_2}, \quad \text{for } i = 1, 2. \tag{28}
\]

Interestingly, conditions (28) imply that both suppliers charge the same interest rate, i.e., \( r_i / L_i = r_2 / L_2, \) regardless of the amounts of credit provided, product characteristics, or the lending sequence.

The retailer maximizes the value of equity \( V = \Pr(\Omega_a) \pi(\pi - L_1 - r_1 - L_2 - r_2 | \Omega_a) \) in two stages. In the second stage, the retailer chooses how much to order from the second supplier, taking the inventory of product 1 and the corresponding trade credit contract as given, i.e.,

\[
Q_1^{TC}(Q_1, L_1, r_1) = \arg \max_{Q_1 \geq 0} V(Q_2, L_2, r_2 | Q_1, L_1, r_1), \tag{29}
\]

where \( L_2 = Q_2 c_2 \) and \( r_2 \) satisfies (28) for \( i = 2. \) In the first stage, the retailer chooses how much to order from the first supplier, i.e.,

\[
Q_1^{TC} = \arg \max_{Q_1 \geq 0} V(Q, L, r), \tag{30}
\]

where \( L_i = Q_i c_i \) and \( r_i \) satisfies (28) for \( i = 1, 2, \) and where \( Q_2 = Q_2^{TC}(Q_1, L_1, r_1) \) is the anticipated solution to the second-stage problem (29).

In the next lemma, we compare the optimal quantity ordered from the second supplier under trade credit financing with its first-best counterpart.

**Lemma 3.** The first-best inventory of product 2, \( Q_2^*, \) and the optimal inventory of product 2 chosen under trade credit financing, \( Q_2^{TC}, \) must satisfy the following conditions, respectively:

\[
\Pr(\Omega_a) \pi(\pi_2 | \Omega_a) + \left( \Pr(\Omega_b) \pi(\pi_2 | \Omega_b) - b \frac{d \Pr(\Omega_b)}{d Q_2} \right) = (1 + \phi) c_2, \tag{31}
\]

and

\[
\Pr(\Omega_a) \pi(\pi_2 | \Omega_a) + \left( \Pr(\Omega_b) \pi(\pi_2 | \Omega_b) - b \frac{d \Pr(\Omega_b)}{d Q_2} \right) \cdot \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} + \Pr(\Omega_b) \pi(b | \Omega_b) \cdot \frac{d}{d Q_2} \left( \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right) = (1 + \phi) c_2. \tag{32}
\]

The first-best condition (31) is analogous to the one provided in Lemma 2, but it is written so that we can distinguish the payoffs in nonbankruptcy and bankruptcy states. The optimality condition (32) differs from its first-best counterpart in two ways. First, the retailer takes into account only a portion of the payoff realized in bankruptcy states, namely, the portion that is allocated to the second supplier, \( (L_2 + r_2)(L_1 + r_1 + L_2 + r_2). \) This gives the retailer an incentive to *underinvest*. Second, the retailer takes into account the effect of the order quantity on this allocation, which is captured by the term \( (d/d Q_2)((L_2 + r_2)/(L_1 + r_1 + L_2 + r_2)) \). The larger the order from the second supplier, the larger this supplier’s claim on the retailer’s liquidation value. This provides an incentive to *overinvest*. It is difficult to analytically determine which of the above two effects dominates, except for the following special case.

**Proposition 7.** Suppose \( b = 0, \) \( \partial \pi_i / \partial Q_j = 0 \) for \( i \neq j, \) and \( \pi_2(D, Q^{TC}) = 0 \) for any \( D \in \Omega_2(Q^{TC}). \) The retailer overorders from the second supplier in the sense that \( Q_2^{TC} > Q_2^*. \)
The assumptions $b = 0$ and $\partial \pi_i / \partial Q_i = 0$ (no cross-price effect) are made for tractability. The condition that $\pi_2 = 0$ for any $D \in \Omega_b$ (the last unit of product 2 has no value in bankruptcy states) eliminates the underinvestment incentive. The retailer has an incentive to overorder product 2 because it increases the claim of the second supplier on the retailer’s assets in case of bankruptcy. In general, trade credit aligns incentives between the retailer and the supplier who provides this particular credit. However, it cannot eliminate the retailer’s incentive to seek risk at the expense of the suppliers who provided trade credit earlier.

In sum, supplier financing can eliminate the asset substitution problem only if the assets are purchased from the same supplier. Interestingly, there are also circumstances under which trade credit provided by multiple suppliers introduces risk-shifting incentives that would not exist under bank financing. Recall that bank financing does not lead to any agency conflict when the products have symmetrical cost, revenue, and demand parameters. As is apparent from conditions (31) and (32), parameter symmetry, or equal riskiness of the two products, does not eliminate agency frictions when financing is provided by two suppliers. This is because sequential financing is, in itself, a form of asymmetry, which leaves the retailer with risk-shifting opportunities.

There are limited circumstances under which different rules for liquidating the firm’s assets may apply, e.g., a supplier may have the “reclamation right” to repossess the goods sold (see, e.g., Ayer et al. 2004). This does not change the fact that there are risk-shifting incentives present when sourcing from two or more suppliers. When there are two suppliers, the agency problem can be eliminated only if the credit from the first supplier is fully secured with inventory or any other collateral.

5. Model Implications and Limitations

5.1. Managerial Implications

The comparative statics in Section 3 provide guidelines for managing an inventory portfolio under debt financing. Relative to a classical equity-financed newsvendor, a shareholder-controlled but debt-financed newsvendor should favor items (i) with a low salvage value, (ii) with a high profit margin, and (iii) those that represent a large portion of total inventory because of high or relatively predictable demand.

This strategy is optimal after debt has been issued and managers have a fiduciary duty to pursue it. Yet, it creates an agency cost that is ultimately borne by the shareholders through the higher cost of external financing. Risk-seeking behavior and the resulting agency cost can be avoided when inventory and credit terms are determined simultaneously, as is often the case when financing is provided by a single supplier in the form of trade credit. This has several implications. First, a firm benefits from supplier financing most (i) when sourcing multiple items from a single supplier, and (ii) when bankruptcy risk and the limited liability effect are significant. Second, when buying multiple items on trade credit, consolidating the number of suppliers is likely to alleviate the agency problem associated with sourcing from multiple suppliers.

This is not to suggest that trade credit always dominates bank financing, even when sourcing multiple items from a single supplier. Because banks tend to face lower cost of capital than suppliers, bank credit can ultimately be cheaper than trade credit. We study a mixed financing strategy that allows the firm to enjoy the best of both worlds. It involves obtaining bank credit up to an amount that can be secured or nearly secured by the firm’s assets, and using trade credit for the remaining financing.

5.2. Empirical Implications

In our model, there is an agency cost of debt only if the retailer offers multiple products; and the greater the asymmetry in product parameters, the greater this cost (see Figures 3(b), 4(b), 5(b), and 6(b)). Therefore, we would expect firms that offer high product variety to rely less on debt financing.\footnote{Although our focus has been on bank credit, publicly traded debt leads to similar, if not greater, agency frictions.}

We have also observed that a debt-financed retailer tends to favor products that already represent a large portion of total inventory (see Propositions 3 and 4 and Figures 4(a), 5(a), and 6(a)). By extrapolation, we would expect a debt-financed retailer to prefer a less diversified product portfolio. This further corroborates our conjecture regarding the negative relationship between debt and product variety. We formalize it as our first empirical prediction.

**Prediction 1.** A firm’s reliance on debt financing, excluding trade credit, is negatively related to its product mix variety.

Our remaining predictions are related to the use of trade credit. In the absence of demand uncertainty, the retailer has no risk-shifting opportunities and, according to our model, no reason to use trade credit. In the presence of demand uncertainty, bank financing creates agency frictions, which favors the use of trade credit. Our numerical experiments indicate that when demand volatilities are increased proportionally, the relative agency cost, $(V^* - V^{BF}) / V^{BF}$, increases under...
both the newsvendor and linear demand model.\textsuperscript{12} Therefore, we expect higher overall demand risk to result in greater use of supplier financing.

**Prediction 2.** Higher demand risk leads to greater reliance on trade credit financing.

The challenge in testing this prediction is that demand volatility is difficult to measure directly, and its natural proxy—sales volatility—is endogenous in our model. (Sales volatility depends on inventory levels, which in turn depend on the financing method.) However, there is some evidence that the use of trade credit increases with various measures of exogenous risk. For example, Elliehausen and Wolken (1993) find that probability of using trade credit increases with the amount of trade credit outstanding. This is consistent with our theory that ties the advantage of using trade credit to bankruptcy risk, which for most firms increases during recessions. Finally, Huyghebaert and Van de Gucht (2007) find evidence that risk-shifting incentives measured by industry-wide asset liquidity multiplied by historical bankruptcy rate, have a negative impact on bank debt and a positive impact on leasing and trade credit as proportions of total debt.

Admittedly, the aforementioned empirical evidence is consistent with several existing trade credit theories, and thus, does not prove that our explanation is at work.\textsuperscript{13} The distinguishing feature of our theory is that the benefit of trade credit hinges on multiple items being procured from a given supplier. When managing a single product, the retailer cannot favor one item over another, and therefore, no agency problem arises in inventory management. This leads to the following prediction.

**Prediction 3.** A firm is more likely to use trade credit when sourcing multiple items from a given supplier.

Finally, we have seen that trade credit eliminates the agency problem in the single supplier scenario (Proposition 5), but not in the multiple supplier scenario (Proposition 7). We formalize this observation in our last prediction.

**Prediction 4.** A firm is less likely to use trade credit when sourcing from multiple suppliers.

Testing Predictions 3 and 4 would be a good way to validate our theory as a distinct motivation for the use of supplier financing.

### 5.3. Model Limitations

Our model relies on two important assumptions, fixed wholesale prices and fairly priced trade credit, both of which can be justified by perfect competition among suppliers. In the following, we discuss robustness of our results under alternative market structures.

**Price-Setting Supplier.** Suppose that any type of credit is fairly priced, but the supplier strategically chooses wholesale prices as the first mover. When choosing inventory and financing, the retailer faces the same problem as in our model. With bank financing, the best-response order quantities, \( Q^{BF} (e) \), must satisfy the optimality conditions given in Lemma 1, and they are obviously distorted by risk shifting. With supplier financing, the best-response order quantities, \( Q^{TF} (e) \), maximize total firm value, and they are given by Lemma 2. However, even though supplier financing eliminates the agency problem associated with bank financing, it does not produce the first-best outcome because the retailer’s inventory decision is still distorted by double marginalization (see Lariviere and Porteus 2001). To instate the first-best outcome in this situation, trade credit terms would have to be part of a larger supply chain coordinating contract.

**Monopolistic Lender.** Now suppose that the wholesale prices are given exogenously, but the credit market is monopolistic. Suppose further that the lender, whether it is the supplier or a bank, sets the interest rate \( z \) as the first mover. The retailer responds by ordering \( Q(z) \), borrowing \( (c'Q - W) \), and promising to repay \((1 + z)(c'Q - W) \). Regardless of who provides financing, the retailer chooses inventory to maximize the value of equity \( V = \text{Pr}(\Omega_z)E(\pi - (1 + z) \cdot (c'Q - W) | \Omega_z) \). The best-response inventory vector, \( Q(z) \), must satisfy the first-order conditions, \( \partial V / \partial Q_i = 0, i = 1, 2 \), which together imply

\[
\frac{\mathbb{E}(\pi_1 | \Omega_z)}{c_1} = \frac{\mathbb{E}(\pi_2 | \Omega_z)}{c_2}.
\]

This is the same condition that characterizes the optimal inventory under bank financing in Lemma 1 with \( r/L \) replaced by \( z \). In other words, when the lender has the power to set the interest rate as the first mover, there is no difference between trade credit and bank financing. Trade credit can induce the first-best inventory decision only if the supplier sets the credit terms after observing the order quantities, fairly reflecting the actual credit risk.

\textsuperscript{12} Interestingly, increasing demand volatility of only one of the products can reduce the relative agency cost. Suppose the products are identical in all parameters except that \( \sigma_2 < \sigma_1 \). As \( \sigma_2 \) increases and approaches \( \sigma_1 \), the products become more similar and the agency cost diminishes, vanishing completely when \( \sigma_1 = \sigma_2 \). This situation is captured in Figure 6(b), but the effect appears to be too small to have any practical significance.

\textsuperscript{13} In particular, it is consistent with explanations based on information asymmetry regarding the buyer’s default risk (Smith 1987, Biais and Gollier 1997).
**Other Limitations.** Our model assumes that the retailer maximizes shareholder value. If a retailer is controlled jointly by shareholders and debt holders, and their objective is to maximize total firm value, the benefit of trade credit described in this paper disappears. We also assume that it is the retailer who makes the inventory decision. That may not be the case under “vendor managed inventory,” when the supplier manages stocking levels. Finally, our model does not capture all of the potential differences between trade credit and bank financing, most of which have been described in the literature. Rather, its purpose is to demonstrate one additional benefit of trade credit that the literature has not yet identified.

**Supplemental Material**
Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2016.2515.

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**Appendix**
We use \( f(x, y) \) and \( f_i(x) \), \( i = 1, 2 \), to denote the joint and marginal p.d.f. of \( \mathbf{D} \), respectively. When assuming symmetrical demand distribution, we replace \( f_i(x) \) with \( f(x) \). We also define \( x^* = \max(x, 0) \).

**Newsvendor Model.** If \( \pi \) is given by (1), it can be also written as

\[
\pi = \begin{cases} 
    pD + s(Q-D) & \text{if } \mathbf{D} \in \Omega_0 = \{\mathbf{D} : D_i \leq Q_i, i = 1, 2\}, \\
    p_1Q_1 + p_2D_1 + s_1(Q_2 - D_2) & \text{if } \mathbf{D} \in \Omega_1 = \{\mathbf{D} : D_1 > Q_1, D_2 \leq Q_2\}, \\
    p_2Q_2 + p_1D_1 + s_1(Q_1 - D_1) & \text{if } \mathbf{D} \in \Omega_2 = \{\mathbf{D} : D_2 > Q_2, D_1 \leq Q_1\}, \\
    p'Q & \text{if } \mathbf{D} \in \Omega_3 = \{\mathbf{D} : D_i > Q_i, i = 1, 2\}.
\end{cases}
\]

**Linear Demand Model.** If \( \pi \) is given by (2) with \( \gamma = 0 \), it can be also written as

\[
\pi = sQ + \sum_{i=1}^{2} \left( \frac{(D_i - s_i)^+}{2} \right)^2
\]

**Proof of Lemma 1.** The optimal \( \mathbf{Q}^B(L, r) > 0 \) solving problem (6) must satisfy the first-order condition, which can be written as (8), together with constraint (3). The optimal \( L^B > 0 \) must satisfy the first-order condition \( dV/dL = 0 \), where \( \mathbf{Q}^B(L) \) and \( r^B(L) \) are given jointly by (8) together with (3) and (4). We have

\[
\frac{dV}{dL} = \frac{\partial V}{\partial L} + \frac{\partial V}{\partial r} \frac{dr}{dL} + \frac{\partial Q_1}{\partial L} \frac{dQ_1}{dQ_2} + \frac{\partial Q_2}{\partial L} \frac{dQ_2}{dQ_1} = -Pr(\Omega_s) - Pr(\Omega_s) \frac{dr}{dL} + \frac{dQ_1}{dQ_2} Pr(\Omega_s) E(\pi_1 | \Omega_s)
\]

Using (8) and (3), the optimality condition \( dV/dL = 0 \) can be written as in (9).

**Proof of Proposition 2.** The desired inequality \( (18) \) can be written as

\[
\mathbb{E} \pi_1(\mathbf{Q}^B) > \mathbb{E} \pi_1(\mathbf{Q}) \quad \text{or} \quad \mathbb{E} \pi_2(\mathbf{Q}^B) < \mathbb{E} \pi_2(\mathbf{Q}).
\]

Because \( \partial \pi_i/\partial Q_j = 0 \), inequalities (40) can be written as

\[
\mathbb{E} \pi_1(Q_i^*) > \mathbb{E} \pi_1(\mathbf{Q}^B_i) \quad \text{or} \quad \mathbb{E} \pi_2(Q_i^*) < \mathbb{E} \pi_2(\mathbf{Q}^B_i).
\]

Because \( \mathbb{E} \pi_i(Q_i) \) is decreasing in \( Q_i \), inequalities (41) imply (15).

**Proof of Proposition 2.** The desired inequality (18) can be written as

\[
\left( \frac{dQ_2^B/ds_2}{dQ_2^*/ds_2} \right) \left( \frac{Q_1^* + Q_2^*}{Q_1^* + Q_2^*} \right) - \left( \frac{dQ_1^*/ds_1}{dQ_2^*/ds_2} \right) \left( \frac{Q_1^* + Q_2^*}{Q_1^* + Q_2^*} \right) \bigg|_{s_2 = s_1}.
\]
When all parameters are symmetrical, \( Q_{1}^{\text{BF}} = Q_{2}^{\text{BF}} = Q_{1}^{*} = Q_{2}^{*} \), and the above inequality simplifies into

\[
\left[ \frac{dQ_{1}^{\text{BF}}}{ds_{2}} - \frac{dQ_{2}^{\text{BF}}}{ds_{2}} \right]_{s_{2} = s_{1}} \geq \left[ \frac{dQ_{1}^{*}}{ds_{2}} - \frac{dQ_{2}^{*}}{ds_{2}} \right]_{s_{2} = s_{1}}. \tag{42}
\]

Recall that when \( b = 0 \), the optimal \( Q^{*} \) and \( Q^{\text{BF}} \) satisfy the first-order conditions (12) and (13), which can be written, respectively, as

\[
\text{FOC}^{*} = \text{Pr}(\Omega_{1a}) \frac{p_{1}}{c_{1}} + \text{Pr}(\Omega_{2a}) \frac{s_{1}}{c_{1}} - \text{Pr}(\Omega_{2a}) \frac{p_{2}}{c_{2}} - \text{Pr}(\Omega_{1a}) \frac{s_{2}}{c_{2}} = 0 \tag{43}
\]

\[
\text{and} \quad \text{FOC}^{\text{BF}} = \text{Pr}(\Omega_{1a}) \frac{p_{1}}{c_{1}} + \text{Pr}(\Omega_{2a}) \frac{s_{1}}{c_{1}} - \text{Pr}(\Omega_{2a}) \frac{p_{2}}{c_{2}} - \text{Pr}(\Omega_{1a}) \frac{s_{2}}{c_{2}} = 0. \tag{44}
\]

Taking the total differential of (43) and (44) with respect to \( s_{2} \), we get,

\[
\frac{d\text{FOC}^{*}}{ds_{2}} = \frac{\partial \text{FOC}^{*}}{\partial s_{2}} + \sum_{i=1}^{2} \frac{dQ_{1}^{*}}{ds_{2}} \frac{\partial \text{FOC}^{*}}{\partial Q_{1}^{*}} + \frac{\partial \text{FOC}^{*}}{\partial L} \frac{dL}{ds_{2}} + \frac{\partial \text{FOC}^{*}}{\partial r} \frac{dr}{ds_{2}} = 0, \tag{45}
\]

and

\[
\frac{d\text{FOC}^{\text{BF}}}{ds_{2}} = \frac{\partial \text{FOC}^{\text{BF}}}{\partial s_{2}} + \sum_{i=1}^{2} \frac{Q_{1}^{\text{BF}}}{ds_{2}} \frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{1}^{\text{BF}}} + \frac{\partial \text{FOC}^{\text{BF}}}{\partial L} \frac{dL}{ds_{2}} + \frac{\partial \text{FOC}^{\text{BF}}}{\partial r} \frac{dr}{ds_{2}} = 0. \tag{46}
\]

Taking the partials of (43) and (44), and using the parameter symmetry, we can write

\[
\frac{\partial \text{FOC}^{*}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}} = -\frac{\partial \text{FOC}^{*}}{\partial Q_{2}} \bigg|_{s_{2} = s_{1}} = \left( \frac{p_{2}}{c_{2}} - \frac{s_{2}}{c_{2}} \right) \left( \frac{\partial \text{Pr}(\Omega_{1})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2})}{\partial Q_{1}} \right), \tag{47}
\]

\[
\frac{\partial \text{FOC}^{*}}{\partial L} \bigg|_{s_{2} = s_{1}} = 0, \quad \text{and} \quad \frac{\partial \text{FOC}^{*}}{\partial r} \bigg|_{s_{2} = s_{1}} = 0.
\]

\[
\frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}} = -\frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{2}} \bigg|_{s_{2} = s_{1}} = \left( \frac{p_{2}}{c_{2}} - \frac{s_{2}}{c_{2}} \right) \left( \frac{\partial \text{Pr}(\Omega_{1a})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2a})}{\partial Q_{1}} \right), \quad \text{and}
\]

\[
\frac{\partial \text{FOC}^{\text{BF}}}{\partial L} \bigg|_{s_{2} = s_{1}} = \frac{\partial \text{FOC}^{\text{BF}}}{\partial r} \bigg|_{s_{2} = s_{1}} = 0. \tag{47}
\]

Plugging these expressions into (45) and (46), and applying some algebra, we obtain

\[
\frac{dQ_{1}^{*}}{ds_{2}} - \frac{dQ_{2}^{*}}{ds_{2}} \bigg|_{s_{2} = s_{1}} = -\frac{\partial \text{FOC}^{*}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}}^{-1} \left( \frac{\partial \text{Pr}(\Omega_{1})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2})}{\partial Q_{1}} \right) \tag{48}
\]

\[
\text{and} \quad \frac{dQ_{1}^{\text{BF}}}{ds_{2}} - \frac{dQ_{2}^{\text{BF}}}{ds_{2}} \bigg|_{s_{2} = s_{1}} = -\frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}}^{-1} \left( \frac{\partial \text{Pr}(\Omega_{1a})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2a})}{\partial Q_{1}} \right). \tag{49}
\]

Using (48) and (49), the desired inequality (42) can be re-written as

\[
\frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}}^{-1} \left( \frac{\partial \text{Pr}(\Omega_{1a})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2a})}{\partial Q_{1}} \right) \geq \frac{\partial \text{FOC}^{*}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}}^{-1} \left( \frac{\partial \text{Pr}(\Omega_{1})}{\partial Q_{1}} - \frac{\partial \text{Pr}(\Omega_{2})}{\partial Q_{1}} \right), \tag{50}
\]

Using the assumption \( \text{Pr}(\Omega_{1a}) = 0 \), and taking advantage of the parameter symmetry, we can obtain the following expressions:

\[
\frac{\partial \text{FOC}^{*}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}} = -\text{Pr}(\Omega_{a}) \frac{1}{c_{2}}, \quad \text{and} \quad \frac{\partial \text{FOC}^{\text{BF}}}{\partial Q_{1}} \bigg|_{s_{2} = s_{1}} = -\text{Pr}(\Omega_{a}) \frac{1}{c_{2}}. \tag{51}
\]

\[
\text{Using the assumption Pr}(\Omega_{1a}) = 0, \text{ and taking advantage of the parameter symmetry, we can obtain the following expressions:}
\]

\[
\text{Pr}(\Omega_{1a}) = \int_{Q_{1}}^{0} \int_{Q_{2}}^{0} f(x, y) \, dx \, dy,
\]

\[
\text{Pr}(\Omega_{2a}) = \int_{Q_{2}}^{0} \int_{Q_{1}}^{0} f(x, y) \, dx \, dy,
\]

\[
\text{Pr}(\Omega_{1}) = \int_{Q_{1}}^{0} \int_{0}^{\infty} f(x, y) \, dx \, dy,
\]

\[
\text{Pr}(\Omega_{2}) = \int_{0}^{\infty} \int_{0}^{Q_{1}} f(x, y) \, dx \, dy.
\]
and using some algebra, we obtain
\[
\frac{\partial \Pr(\Omega_{1a})}{\partial q_2} = \frac{L + r - p_1 Q_1 - s_2 Q_2}{(p_2 - s_2)^2} \left[ \int_0^\infty f(x) \left( L + r - p_1 Q_1 - s_2 Q_2 \right) dx \right],
\]
\[
\frac{\partial \Pr(\Omega_{2a})}{\partial q_2} = \frac{Q_2}{p_1 - s_1} \left[ \int_0^\infty f(\frac{L + r - p_2 Q_2 - s_1 Q_1}{p_2 - s_2}, y) dy \right],
\]
\[
\frac{\partial \Pr(\Omega_{1a})}{\partial q_1} = \frac{L + r - p_1 Q_1 - s_2 Q_2}{(p_2 - s_2)^2} \left[ \int_0^\infty f(x) \left( L + r - p_1 Q_1 - s_2 Q_2 \right) dx \right]
\]
\[
- \int_0^{Q_2} f(Q_1, y) \left( L + r - p_1 Q_1 - s_2 Q_2 \right) dx.
\]
Substituting (53) and (54) into the desired inequality (52) and using some algebra, this inequality simplifies into
\[
\left( \Pr(\Omega_{2a}) - \frac{L + r - p_1 Q_1 - s_2 Q_2}{p_2 - s_2} \right)
\]
\[
\cdot \left[ \int_0^\infty f(x) \left( L + r - p_1 Q_1 - s_2 Q_2 \right) dx \right]
\]
\[
+ \left( \int_0^{Q_2} f(Q_1, y) \left( L + r - p_1 Q_1 - s_2 Q_2 \right) dx \right)
\]
\[
\geq \frac{\Pr(\Omega_{2a})}{\int_0^\infty f(Q_1, y) dy},
\]
where \((L, r, Q) = (L^*, r^*, Q^*) = (L^{BF}, r^{BF}, Q^{BF})\). Using the assumptions of symmetry and demand independence, inequality (55) further simplifies into
\[
\left( 1 - \frac{L + r - p_1 Q_1 - s_2 Q_2}{p_2 - s_2} \right)
\]
\[
\cdot \left( h(Q_1) - h\left( \frac{L + r - p_1 Q_1 - s_2 Q_2}{p_2 - s_2} \right) \right) \geq \frac{1}{h(Q_1)}.
\]
Proof of Proposition 4. Using the same procedure that we used to convert (18) into (50) in the proof of Proposition 2, we can convert the desired inequality (20) into
\[
\frac{\partial FOC^{BF}}{\partial c_2} \left( \frac{\partial \Pr(\Omega_{1a})}{\partial q_1} - \frac{\partial \Pr(\Omega_{2a})}{\partial q_1} \right)^{-1}
\]
\[
\leq \frac{\partial FOC^*}{\partial c_2} \left( \frac{\partial \Pr(\Omega_{1a})}{\partial q_1} - \frac{\partial \Pr(\Omega_{2a})}{\partial q_1} \right)^{-1},
\]
where \(\{L, r, Q\} = (L^*, r^*, Q^*) = (L^{BF}, r^{BF}, Q^{BF})\). Using the assumption that \(h(x)\) is increasing and the fact that \(Q_1 > (L + r - p_1 Q_1)/(p_2 - s_2)\), we know that the desired inequality (60) must hold if
\[
\Pr(\Omega_{2a}) \geq \Pr(\Omega_{2a}) \Pr(D_1 > \frac{L + r - p_1 Q_1}{p_2}).
\]
With independent demands, inequality (61) is clearly satisfied as an equality. □

Proof of Proposition 5. Because \(L^{TC}(Q)\) and \(r^{TC}(Q)\) are determined by (3) and (21), problem (22) can be rewritten as
\[
Q^{TC}, L^{TC}, r^{TC} = \arg \max V(Q, L, r)
\]
\[
\text{subject to } (3) \text{ and } (21).
\]
Using the definition of V in (5) and constraints (3) and (21), we can rewrite the objective in (62) as
\[
V = \Pr(Q) E(\pi - L - r | \Omega_c)
\]
\[
= E \pi - c Q - \phi L - \Pr(\Omega_c) b + W.
\]
Thus, we observe that formulation (62) is equivalent to the first-best formulation (10) with \(\phi_b\) replaced by \(\phi_L\). □

Proof of Corollary 1. It follows from Proposition 5 that when \(\phi_3 = \phi_4\), \(V^{TC} = V^* > V^{BF}\). By continuity, \(V^{TC} > V^{BF}\) for any \(\phi_3 \in (\phi_3, \phi_4 + \epsilon)\). Because \(\pi_L\) is finite, for a sufficiently high \(\phi_3\), \(L^{TC} = 0\), and the trade credit scenario is equivalent to the bank financing scenario with an additional constraint \(L = 0\). Because we are assuming \(L^{BF} > 0\), we have \(V^{TC} < V^{BF}\). □

Proof of Proposition 6. The inequality \(V^{MF} \geq V^{TC}\) follows from the fact that the trade credit scenario is equivalent to the mixed financing scenario with an additional
constraint $L_g = 0$. Now suppose that the premise of the second part of the proposition holds, and consider the mixed financing scenario with $L_g = (W \min(s_1/c_1, s_2/c_2))/(1 + \phi_b)$, and $r_g = \phi_b L_g$. With probability one, we have $\pi \geq s_0 \geq \min(s_1/c_1, s_2/c_2)|Q \geq \min(s_1/c_1, s_2/c_2)|W = L_g + r_g$. Therefore, the bank credit is riskless and fairly priced. In this case, the retailer’s second-stage problem (23) simplifies into $Q_{MF}(L_g) = \arg \max_{Q > 0} V(Q | L_g)$, where

$$V(Q, L_g) = E \pi - (1 + \phi_b)(cQ - W) + (\phi_b - \phi L_g).$$

In the trade credit scenario as presented in (22), we have $Q^{TC} = \arg \max_{Q > 0} V(Q)$, where

$$V(Q) = E \pi - (1 + \phi_b)(cQ - W).$$

Therefore, $Q_{MF}(L_g) = Q^{TC}$, and the value of equity under mixed financing equals $V^{TC} + (\phi_b - \phi)(W \min(s_1/c_1, s_2/c_2))/(1 + \phi_b)$. The value of equity under mixed financing and the optimal $L_g$ cannot be lower than that, and inequality (27) follows.

Proof of Lemma 3. The optimality condition (31) for $Q_2^2$ is equivalent to the second condition in (11) except that the lender’s cost of capital is $\phi$. The optimal $Q_{TC}^2 > 0$ solving problem (29) must satisfy the first-order condition

$$\frac{dV}{dQ^2} = \frac{\partial V}{\partial Q^2} + \frac{d}{dQ^2} \frac{\partial V}{\partial r^2} = 0.$$ (65)

Taking the partial derivatives, we get

$$\frac{dV}{dQ^2} = \Pr(\Omega) E(\pi | \Omega) - \Pr(\Omega) c_2$$ and

$$\frac{\partial V}{\partial r^2} = -\Pr(\Omega).$$ (66)

Implicitly differentiating condition (28) for $i = 2$, and applying some algebra gives

$$\frac{d}{d\Pr(\Omega)} \left[ \Pr(\Omega) c_2 + \phi_b c_2 - \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right] \cdot \left[ \Pr(\Omega) E(\pi | \Omega) - b \frac{d\Pr(\Omega)}{dQ^2} \right]$$

$$= -\Pr(\Omega) E(\pi - b | \Omega) \frac{d}{dQ^2} \left[ \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right].$$ (67)

Plugging (66) and (67) into (65), we get the optimality condition (32).

Proof of Proposition 7. When $b = 0$ and $E(\pi | \Omega) = 0$ at $Q^{TC}$, the optimality conditions for $Q_1^{TC}$ and $Q_2^{TC}$, (31) and (32), can be written as

$$E \pi = (1 + \phi_b) c_2,$$ and

$$E \pi + \Pr(\Omega) E(\pi | \Omega) \frac{d}{dQ^2} \left( \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right) = (1 + \phi_b) c_2,$$ (68)

respectively. Taking the total derivative, we get

$$\frac{d}{dQ^2} \left( \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right) = \frac{L_1 + r_1}{(L_2 + r_2 + L_1 + r_1)^2} \left( c_2 + \frac{d}{dQ^2} \right).$$ (69)

Implicitly differentiating condition (28) for $i = 2$ yields

$$\frac{d}{dQ^2} \left( \frac{L_2 + r_2}{L_1 + r_1 + L_2 + r_2} \right) = \frac{L_1 + r_1}{(L_2 + r_2 + L_1 + r_1)^2} \left( c_2 + \frac{d}{dQ^2} \right).$$ (70)

Using (70), (71), and (28), the optimality condition (69) can be written as

$$E \pi = (1 + \phi_b) c_2 - \frac{(1 + \phi_b) c_2 \Pr(\Omega) E(\pi | \Omega)}{L_1 + r_1} \frac{L_1 + r_1}{L_2 + r_2 + L_1 + r_1 + r_2}.$$ (71)

Thus, we have $E \pi(Q^2) = (1 + \phi_b) c_2$, whereas $E \pi(Q^{TC}) < (1 + \phi_b) c_2$, which means $E \pi(Q^{TC}) < E \pi(Q^2)$. Using the fact that $\partial \pi(Q^2)/\partial Q^2 = 0$ for $i \neq j$, the last inequality is equivalent to $E \pi(Q^{TC}) < E \pi(Q^2)$. Because $E \pi_2$ is decreasing in $Q_2$, we have $Q_{TC}^2 > Q_2^2$. □

References


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