The Financial Accelerator and the Optimal Lending Contract*

Mikhail Dmitriev† and Jonathan Hoddenbagh‡

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In the financial accelerator literature pioneered by Bernanke, Gertler and Gilchrist (1999) entrepreneurs are myopic and lenders suboptimally choose a safe rate of return on their loans. We derive the optimal lending contract for forward looking entrepreneurs and provide three main results. First, under the optimal contract we find that financial frictions do not amplify business cycle fluctuations. Second, we show that shocks to the variance of unobserved idiosyncratic productivity — so-called “risk shocks” — have little effect on the real economy under the optimal contract. Third, we find that amplification under the suboptimal contract depends on loose monetary policy.

Keywords: Financial accelerator; financial frictions; risk; optimal contract; agency costs.

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†Florida State. Email: mdmitriev@fsu.edu. Web: http://www.mikhaildmitriev.org

‡Johns Hopkins. Email: jon.hoddenbagh@gmail.com. Web: https://www2.bc.edu/jonathan-hoddenbagh/
1 Introduction

In one of the foundational papers in the literature on financial frictions in macroeconomic models, Bernanke, Gertler and Gilchrist (1999) derive a contract between risk averse lenders and risk neutral borrowers in the costly state verification (CSV) framework of Townsend (1979). Although this loan contract has become the standard contract for CSV models of financial frictions, it is not optimal because it assumes returns for lenders are predetermined and borrowers are myopic.

In this paper we relax these two assumptions and derive the optimal history-independent loan contract in the CSV model. We allow returns to the lender to be contingent on the aggregate state of the economy following early criticism of Bernanke, Gertler and Gilchrist (1999), hereafter BGG, by Chari (2003). We also introduce forward looking entrepreneurs who maximize the present discounted value of all future consumption instead of next period expected consumption.

Our analysis provides three main contributions. First, under the optimal contract we find that financial frictions do not amplify business cycles. Relative to a model with financial frictions, monetary and technology shocks generate much larger output responses when frictions are absent. Second, we develop a novel framework to study the impact of shocks to the cross-sectional variance of entrepreneurs’ idiosyncratic productivity — so-called “risk” shocks — in the CSV environment. We show that these shocks have little to no impact on the real economy when lending contracts are optimal, in contrast with the BGG contract. This is particularly important as recent work by Christiano, Motto and Rostagno (2013) emphasizes the importance of risk shocks in driving business cycles. We prove that risk shocks provide amplification only when the lending contract is suboptimal, regardless of the degree of lender’s risk aversion. Third, we demonstrate that the financial accelerator in the original BGG framework is dependent on three key characteristics: a suboptimal contract, loose monetary policy and extremely persistent technology shocks. We conduct a number of robustness checks in Section 5 and find that the removal of any one of these characteristics significantly weakens or eliminates the financial accelerator. Overall, our results cast doubt on the qualitative and quantitative importance of the financial accelerator in the CSV framework.

Our model consists of a risk averse representative household and risk neutral entrepreneurs. Entrepreneurs borrow money from the representative household and purchase capital to use in production. Entrepreneurs are identical ex ante but differ depending on the ex post realization.

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1 A non-exhaustive list of some important early contributions in this literature include Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997).

2 To be precise, we derive the optimal one-period contract with deterministic monitoring. An excellent list of references for partial equilibrium multi-period contracts includes Monnett and Quintin (2005) for stochastic monitoring, Wang (2005) for deterministic monitoring, and Cole (2013) for self-enforcing stochastic monitoring.

3 As we discuss later, the deamplification effect of the optimal contract is a general result that holds for a wide range of calibrations.
of an idiosyncratic productivity shock. Both agents have full information about the distribution of idiosyncratic shocks ex ante, so there is no adverse selection problem. Borrowers observe the realization of their idiosyncratic shock, but lenders do not: they need to pay monitoring costs to observe it.

In the BGG contract borrowers guarantee a constant safe rate of return to lenders in order to maximize returns on their equity. As a result, borrowers absorb all risk in the economy. It should be noted that this is an assumption and not an equilibrium condition. Because of this assumption, negative shocks cause a decline in entrepreneurs’ net worth which leads to a tightening of financial constraints. The subsequent fall in investment and output is stronger than the effect from the initial shock. This results in the financial accelerator: the BGG contract amplifies macroeconomic fluctuations in a dynamic stochastic general equilibrium (DSGE) model.

In BGG and the CSV literature entrepreneurs are myopic: they maximize their expected next period consumption, but expected utility depends on the expected discounted stream of all future consumption. We depart from the literature and embed forward looking entrepreneurs into an otherwise standard CSV framework. Our analysis provides a number of results that call the robustness of the financial accelerator into question under optimal and suboptimal contracts, for myopic and non-myopic entrepreneurs.

The intuition is as follows. When lenders’ returns are predetermined, we find that to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic. In period $t$, the predetermined lending rate is chosen to satisfy the lender’s Euler equation in that specific period without the possibility of revisions in period $t + 1$. As a result, it does not matter whether entrepreneurs are forward looking or not, as the lender’s stochastic discount factor determines the rate of return. In order to generate amplification however, this suboptimal contract must be combined with other ingredients. In our robustness exercise in Section 5, we show that contracts with a predetermined deposit rate only generate a financial accelerator when monetary policy deviates from price stability and when technology shocks are stationary.

On the other hand, when lender’s returns are chosen optimally and vary with the aggregate state of the economy, the presence of forward looking entrepreneurs or myopic entrepreneurs matters greatly. Myopic entrepreneurs sell as much insurance to the household as they can because insurance does not effect their next period expected consumption. During a recession, the provision of insurance leads to very tight financial constraints for entrepreneurs, as they face a higher lending rate due to the fall in household consumption. During a boom the opposite occurs: myopic entrepreneurs have too much capital and earn small returns on their capital. In other words myopic entrepreneurs miss good investment opportunities on a consistent basis because they do not take the future flow of capital returns into account when making investment decisions. Under the optimal contract however, forward looking entrepreneurs sell less insurance because they are concerned not only about next period expected consumption but also expected
consumption in all future periods, which is impacted by insurance claims. In particular, forward looking entrepreneurs desire high net worth in states of the world where the financial premium is also high.

For example, assume that ex-post there is a shock which suddenly decreases the entrepreneur’s net worth. Lower net worth today means that the financial premium today and in the future will be higher. The entrepreneur desires more net worth in states with a higher financial premium because capital returns are higher and borrowing is more costly. Forward looking entrepreneurs thus find it profitable to enter into an ex-ante agreement that stipulates a lower lending rate in these states. Correspondingly, entrepreneurs prefer to pay a higher lending rate when a shock increases net worth, because the financial premium will be lower in these states. This interplay between movements in net worth and the financial premium leads risk-neutral entrepreneurs to behave in a “risk averse” manner because they want to avoid borrowing in states with a high financial premium. In contrast, if there is no costly state verification so that financial frictions are absent, non-myopic entrepreneurs will ignore concerns about the financial premium and provide as much insurance as possible, generating large amplification.

We also find that risk shocks have little effect on the real economy and give the wrong comovement between macroeconomic aggregates when contracts are optimal. This contrasts with Christiano, Motto and Rostagno (2013), who employ the BGG contract and emphasize the importance of risk shocks in generating business cycle fluctuations. Under the BGG contract, increased idiosyncratic variance causes an increase in defaults leading to a decline in the price of capital and consequently net worth. However, if returns to lenders are not predetermined and entrepreneurs are forward looking, they realize that lower net worth implies higher financial premiums and more costly borrowing in the future. Therefore, forward looking entrepreneurs desire more net worth in these states and thus negotiate lower returns to lenders, which stabilizes the response of net worth to the shock. As a result, under the optimal contract the financial accelerator is severely dampened for risk shocks.

Related Literature

Our results are important because the CSV framework remains one of the benchmark methods for embedding financial friction in DSGE models. The literature follows the BGG framework and employs myopic entrepreneurs with suboptimal contracts. A non-exhaustive survey of recent work in this area includes Christiano, Motto and Rostagno (2013), Christensen and Dib

4Recent work by Carlstrom, Fuerst and Paustian (2013), hereafter CFP, simultaneously and independently derives the dynamically optimal contract for forward looking households that we present here. In the January 2013 version of their working paper, CFP solve for the contingent contract with myopic entrepreneurs and find that financial frictions amplify business cycles. In the October 2013 version of their paper, CFP solve for the optimal contract with forward looking entrepreneurs. CFP focus on the social planner’s problem and the relative social efficiency of the optimal contract vis-à-vis the BGG contract for technology and monetary shocks, while we focus on the following question: do optimal contracts mitigate the financial accelerator or not? In contrast with CFP, we compare the model with frictions against a frictionless benchmark to examine the role of optimal contracts in amplifying and propagating business cycle fluctuations. We also study the impact of risk shocks, which are absent in CFP.
The CSV approach is not the only way to model financial frictions. In a related paper (Dmitriev and Hoddenbagh (2014)) we investigate the effect of optimal state-contingent contracts in a model with costly state enforcement frictions a la Kiyotaki and Moore (1997). In this alternative environment, we find that optimal state-contingent contracts severely dampen the amplification response from technology and monetary shocks. Again, our results demonstrate that amplification in the costly state enforcement literature is generated via the assumption of non-contingent lending contracts (Kiyotaki and Gertler (2010) and Gertler and Karadi (2011)). An exception is presented by Jermann and Quadrini (2012), who allow both debt and equity contracts and achieve amplification by introducing adjustment costs between these instruments and ruling out other instruments. In the adverse selection framework, House (2006) extends the Stiglitz and Weiss (1981) and also shows that financial frictions amplify business cycles only when returns for lenders are non-contingent. When contracts are contingent or allow both debt and equity, financial frictions actually stabilize business cycles.

In contrast to our paper, Di Tella (2013) investigates the role of optimal state-contingent contracts in a model with no labor and finds that risk shocks do generate amplification. Our model includes labor, and amplification is severely dampened for risk shocks as households use labor supply to stabilize their income even though the optimal contract shifts risk away from the balance sheet of entrepreneurs. We believe Di Tella’s approach will face similar problems if his model is extended to include labor. To obtain large amplification in our model, labor supply must be highly inelastic to prevent households from using labor to smooth their income. But the well known problem with low labor supply elasticity is that models with such calibrations are unable to replicate basic business cycle facts.

In summary, the bulk of the evidence suggests that the ability of financial frictions to amplify business cycle fluctuations is dependent on non-contingent lending contracts across a wide class of models, including the CSV, costly state enforcement and adverse selection frameworks common in the literature.

2 The Optimal Lending Contract in Partial Equilibrium

Our main theoretical contribution in this paper is to introduce forward looking entrepreneurs into an otherwise standard CSV model of financial frictions. In this section we outline the key differences between the dynamically optimal loan contract chosen by forward looking entrepreneurs and the contingent and non-contingent loan contracts chosen by myopic entrepreneurs in a partial equilibrium setting. Here we assume that entrepreneurs take the price of capital and the expected return to capital as given. In Section 3 we endogenize these variables in general equilibrium.

At time $t$, entrepreneur $j$ purchases capital $K_t(j)$ at a unit price of $Q_t$. At time $t+1$, the entrepreneur rents this capital to perfectly competitive wholesale goods producers. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase
capital:

\[ Q_t K_t(j) = N_t(j) + B_t(j). \]  

After buying capital, the entrepreneur is hit with an idiosyncratic shock \( \omega_{t+1}(j) \) and an aggregate shock \( R^k_{t+1} \), so that entrepreneur \( j \) is able to deliver \( Q_t K_t(j) R^k_{t+1} \omega_{t+1}(j) \) units of assets. The idiosyncratic shock \( \omega(j) \) is a log-normal random variable with distribution \( \log(\omega(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_{\omega}, \sigma^2_{\omega}) \) and mean of one.

Following BGG, we assume entrepreneurs are risk neutral and die with constant probability \( 1 - \gamma \). Upon dying, entrepreneurs consume all operational equities, which are equal to net worth minus wages. If entrepreneurs survive they do not consume anything, and they supply labor and earn wages which they later reinvest. Entrepreneur \( j \)'s value function is

\[ V^e_t(j) = (1 - \gamma) E_t \left\{ \sum_{s=1}^{\infty} \gamma^s C^e_{t+s} \right\} \]  

where \( C^e_{t+s} \) is the entrepreneur’s consumption,

\[ C^e_t(j) = N_t(j) - W^e_t \]  

defined as wealth accumulated from operating firms, equal to net worth without entrepreneurial real wages \( W^e_t \). The timeline for entrepreneurs is plotted in Figure 1.

![Figure 1: Timeline for Entrepreneurs](image)

### 2.1 Borrower and Lender Payoffs

The contract between the lender and borrower follows the familiar CSV framework. We assume that the lender cannot observe the realization of idiosyncratic shocks to entrepreneurs unless he pays monitoring costs \( \mu \) which are a fixed percentage of total assets. Given this friction, the risk neutral borrower offers the risk averse lender a contract with an state-contingent interest rate \( Z^e_{t+1} \) subject to macroeconomic conditions.

The entrepreneur repays the loan only when it is profitable to do so. In particular, the entrepreneur will repay the loan only if, after repayment, he has more assets than liabilities. We define the cutoff productivity level \( \tilde{\omega}_{t+1} \), also known as the bankruptcy threshold, as the
minimum level of productivity necessary for an entrepreneur to repay the loan:

$$\frac{B_t(j)Z_{t+1}(j)}{\text{Cost of loan repayment}} = \frac{\bar{\omega}_{t+1}R_{t+1}^kQ_tK_t(j)}{\text{Minimum revenue for loan repayment}}.$$  \hspace{1cm} (4)

If $\omega_{t+1}(j) < \bar{\omega}_{t+1}$ the entrepreneur defaults and enters bankruptcy; if $\omega_{t+1}(j) \geq \bar{\omega}_{t+1}$ he repays the loan. The cutoff productivity level allows us to express the dynamics of net worth for a particular entrepreneur $j$:

$$N_{t+1}(j) = Q_tK_t(j)R_{t+1}^k \max \left\{ \omega_{t+1}(j) - \bar{\omega}_{t+1}, 0 \right\} + W^e_{t+1}. \hspace{1cm} (5)$$

The gross rate of return for the lender, $R_{t+1}$, also depends on the productivity cutoff. For idiosyncratic realizations above the cutoff, the lender will be repaid the full amount of the loan $B_t(j)Z_{t+1}(j)$. For idiosyncratic realizations below the cutoff, the entrepreneur will enter bankruptcy and the lender will pay monitoring costs $\mu$ and take over the entrepreneur’s assets, ending up with $(1 - \mu)\bar{K}_t(j)R_{t+1}^h(\omega_{t+1}(j))$. More formally, the lender’s ex post return is

$$B_t(j)R_{t+1}(j) = \begin{cases} B_t(j)Z_{t+1}(j) & \text{if } \omega_{t+1}(j) \geq \bar{\omega}_{t+1} \\ (1 - \mu)\bar{K}_t(j)R_{t+1}^h(\omega_{t+1}(j)) & \text{if } \omega_{t+1}(j) < \bar{\omega}_{t+1} \end{cases} \hspace{1cm} (6)$$

Taking into account that loans to entrepreneurs are perfectly diversifiable, the lenders return on a loan $R_{t+1}$ to entrepreneur $j$ is defined as

$$B_t(j)R_{t+1} \equiv Q_tK_t(j)R_{t+1}^h(\bar{\omega}_{t+1}, \sigma_{\omega, t}), \hspace{1cm} (7)$$

where $h(\bar{\omega}_{t+1}, \sigma_{\omega, t})$ is the share of total returns to capital that go to the lender. We define this share as

$$h(\bar{\omega}_{t+1}, \sigma_{\omega, t}) = \begin{cases} \bar{\omega}_{t+1} \left[ 1 - F(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right] & \text{Share to lender if loan pays} \\ (1 - \mu) \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega, \sigma_{\omega, t})d\omega & \text{Share to lender if loan defaults} \end{cases} \hspace{1cm} (8)$$

where $f$ is the probability density function and $F$ is the cumulative distribution function of the log-normal distribution of idiosyncratic productivity.

In order to simplify the entrepreneur’s optimization problem, we introduce the concept of leverage, $\kappa_t$, defined as the value of the entrepreneur’s capital divided by net worth:

$$\kappa_t(j) \equiv Q_tK_t(j)/N_t(j). \hspace{1cm} (9)$$
2.2 Loan Contracts: BGG, the Myopic Contingent Contract (MCC) and the Optimal Contract

The differences between the BGG contract, the myopic contingent contract (hereafter denoted MCC) and the optimal contract arise from two sources: the lender's participation constraint and the borrower's objective function.

First, the lender's participation constraint in BGG differs from the participation constraint in the MCC and the optimal contract. The participation constraint arises from the household Euler equation and stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan. In BGG, the participation constraint has the following form:

\[ \mathbb{E}_t \{ \Lambda_{t,t+1} \} R_{t+1} = 1, \]  

(10)

where

\[ \Lambda_{t,s} \equiv \beta_s U_{C,t+1} + \beta_s U_{C,t} \]  

(11)

is the household (i.e. shareholder) intertemporal marginal rate of substitution, also known as the household stochastic discount factor. Under this participation constraint, entrepreneurs pay a constant safe rate of return to the lenders, \( R_{t+1} \), which ignores the risk averse representative household's desire for consumption insurance. In contrast, the participation constraint for the MCC and the optimal contract is:

\[ \mathbb{E}_t \{ \Lambda_{t,t+1} R_{t+1} \} = 1. \]  

(12)

The above expression implies that households prefer a state contingent rate of return that is negatively correlated with household consumption. Quite simply, households like consumption insurance. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms.

Second, the borrower's objective function in BGG and MCC differs from the the objective function which gives rise to the optimal contract. Entrepreneurs in BGG and MCC maximize next period net worth, defined in equation (5). If we substitute the expression for leverage from (9) into (5), we have the entrepreneur's objective function in BGG and MCC:

\[ \max_{\kappa_t(j)} \kappa_t(j) N_t(j) \mathbb{E}_t \left\{ R_{t+1}^k \max \left[ \omega_{t+1}(j) - \bar{\omega}_{t+1}, 0 \right] \right\}. \]  

(13)

In contrast, under the dynamically optimal contract entrepreneurs maximize utility, given by (2). As we have mentioned before, utility maximizing entrepreneurs are concerned not only about current capital returns but also future capital returns and future financial premiums.

We now have all of the ingredients necessary to set up the entrepreneur's optimization problem and solve for the three different loan contracts: (1) the BGG contract; (2) the MCC
contract; and, (3) the optimal contract.

**Proposition 1** To solve for the BGG contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize next period net worth (13) subject to (5), (7) and (10). The solution to this problem is given by

$$\kappa_t \mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\mathbb{E}_t \left\{ \frac{g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}.$$  \hfill (14)

where $g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \int_{-\infty}^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1} \left[ 1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right].$

**Proof** See Appendix B. ■

**Corollary 1** Log-linearization of the BGG optimality condition (14) and the BGG participation constraint (10) gives

$$\mathbb{E}_t \hat{R}_{t+1}^k - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t}$$ \hfill (15)

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = 0$$ \hfill (16)

where the constant $\nu_\kappa = -\frac{h_{\omega\omega} - h_{\omega\sigma} g_{\omega\omega} - g_{\omega\sigma} h_{\omega\sigma}}{h_{\omega\omega} - h_{\omega\sigma} - h_{\sigma\sigma}} \frac{1}{\kappa - 1}$ and $\nu_\sigma = -\frac{h_{\omega\omega} \left( \frac{h_{\omega\omega}}{g_{\omega\omega}} - \frac{h_{\omega\sigma}}{g_{\omega\sigma}} - \frac{g_{\omega\sigma}}{g_{\omega\sigma}} \right) + h_{\sigma\sigma} \left( \frac{g_{\omega\sigma}}{g_{\omega\omega}} - \frac{g_{\omega\sigma} h_{\sigma\sigma}}{h_{\sigma\sigma}} - \frac{g_{\omega\sigma} g_{\omega\sigma}}{g_{\omega\sigma}} \right)}{h_{\omega\omega} - h_{\omega\sigma} - h_{\sigma\sigma}} \sigma_{\omega}.$

**Proof** See Appendix G. ■

Equation (15) shows that in the BGG contract the entrepreneur’s leverage depends on next period’s expected financial premium while (16) shows that lenders returns (deposit rate) are predetermined. We prove in Appendix E that when lenders’ returns are predetermined, to a first order approximation the lending contract is identical regardless of whether entrepreneurs are forward looking or myopic.\(^5\)

**Proposition 2** To solve for the MCC contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize (13) subject to (5), (7) and (12). The solution to this problem is given by

$$\kappa_t \mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}.$$ \hfill (17)

**Proof** See Appendix C. ■

\(^5\)This is also true when the lending rate $Z$ is predetermined, rather than lenders returns $R$. The general equilibrium behavior of the model under predetermined $R$ and predetermined $Z$ is similar, so we only report results for predetermined $R$. 

9
Corollary 2  Log-linearization of the MCC optimality condition (17) and the MCC participation constraint (12) gives

\[
E_t \hat{R}^k_{t+1} - E_t \hat{R}_{t+1} = \nu \hat{k}_t + \nu \hat{\sigma}_{\omega,t} \\
\hat{R}_{t+1} - E_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - E_t R^k_{t+1} - \tilde{\alpha} \sigma (\hat{C}_{t+1} - E_t \hat{C}_{t+1})
\]

where \(\tilde{\alpha} = -\frac{h_{\omega}}{h_{\omega}}\).

Proof  See Appendix G. ■

Corollary 2 clearly illustrates the differences between the BGG contract and the MCC contract. In equation (19), lender’s returns depend on capital returns and household consumption, both elements which are missing in the BGG contract. For standard calibrations, \(\tilde{\alpha}\) takes a value between five and six and the risk aversion parameter \(\sigma\) is equal to one, so that lender’s returns are very sensitive to the consumption level and the consumption insurance channel dominates the response to capital returns. When consumption is high, the lending rate declines; when consumption is low the lending rate increases. The negative covariance between the lender’s consumption and returns reflects the nature of insurance, which amplifies the impact of shocks to the economy. Note that as entrepreneurs become more risk averse (as \(\sigma\) decreases), the impact of the consumption insurance channel declines.

Now that we have described the BGG and MCC contracts in detail, we turn our attention to the optimal contract. As we discussed above, the optimal contract takes the consumption insurance channel from MCC and adds forward looking entrepreneurs.

Proposition 3  To solve for the optimal contract, entrepreneurs choose their state contingent cutoff \(\tilde{\omega}_{t+1}\) and leverage \(\kappa_i(j)\) to maximize (2) subject to (3), (5), (7) and (12). The solution to this problem is given by

\[
\kappa_i E_t \left\{ \Psi_{t+1} R^k_{t+1} g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g_{\omega}(\tilde{\omega}_{t+1}, \sigma_{\omega,t})}{h_{\omega}(\tilde{\omega}_{t+1}, \sigma_{\omega,t})} \Lambda_{t,t+1} \\
\Psi_{t} = 1 + \gamma \kappa_i E_t \left\{ g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1} \Psi_{t+1} \right\}
\]

where

Proof  See Appendix D. ■

Corollary 3  Log-linearization of the optimal contract, (20) and (21), and the participation
constraint (12) gives
\[ E_t \hat{R}^{k}_t - \hat{R}^{k}_{t+1} = \nu_\kappa \hat{R}_t + \nu_\sigma \hat{\sigma}_{\omega,t} \]  \hspace{1cm} (22)
\[ \hat{R}_t + \hat{R}^{k}_t - \hat{R}^{k}_{t+1} = \hat{R}^{k}_{t+1} - \nu_\Psi \hat{\Psi}_{t+1} \]  \hspace{1cm} (23)
\[ \hat{\Psi}_{t+1} = \epsilon N E_{t+1} \left\{ (\kappa - 1)(\hat{R}^{k}_{t+2} - \hat{R}_{t+2}) + \hat{R}^{k}_{t+2} + \nu_\Psi \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2} \right\} \]  \hspace{1cm} (24)

where \( \nu_\Psi = \frac{g_\sigma - h_\sigma \tilde{m}_\omega}{g} \sigma_\omega \).

**Proof** See Appendix G. □

We see from (23) that under the optimal contract, the surprise to lender’s returns depends not only on surprises to capital returns and consumption, as in the MCC contract, but future capital returns and future financial premiums as well. If entrepreneurs are more optimistic about expected future financial premiums or future returns to capital, they prefer to pay the lender a lower interest rate because one unit of net worth becomes more valuable. Corollary 3 thus illustrates the strong stabilizing mechanism of the optimal contract. When a crisis hits and decreases entrepreneur’s net worth, expected future financial premiums will rise. But entrepreneurs will also pay lenders a smaller deposit rate, which stabilizes their net worth. As a result, the main channel for the financial accelerator, the volatility in net worth, is diminished when entrepreneurs are forward looking.

Although we have taken a partial equilibrium view here, Corollaries 1-3 are identical in the general equilibrium setting. In both partial and general equilibrium, leverage and the deposit rate are determined by the paths of capital returns and consumption. Therefore, the intuition provided by Corollaries 1-3 holds in general equilibrium.

3 The Model in General Equilibrium

We now embed the three loan contracts in a standard dynamic New Keynesian model. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. Entrepreneurs buy capital from capital producers and then rent it out to perfectly competitive wholesalers, who sell their goods to monopolistically competitive retailers. Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. Retailers have price-setting power and are subject to Calvo price rigidities. Households bundle the retail goods in CES fashion into a final consumption good. A graphical overview of the model is provided in Figure 2. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).
3.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

\[
\max E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \zeta \log \left( \frac{M_{t+s}}{P_{t+s}} \right) - \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\},
\]

(25)

where \( C_t \) is household consumption, \( M_t/P_t \) denotes real money balances, and \( H_t \) is household labor effort. The budget constraint of the representative household is

\[
C_t = W_t H_t - T_t + \Pi_t + R_t \frac{D_t}{P_t} - \frac{D_{t+1}}{P_t} + \frac{M_{t-1} - M_t}{P_t} + \frac{B_{t-1} R^n_t - B_t}{P_t}
\]

(26)

where \( W_t \) is the real wage, \( T_t \) is lump-sum taxes, \( \Pi_t \) is profit received from household ownership of final goods firms distributed in lump-sum fashion, and \( D_t \) are deposits in financial intermediaries (banks) that pay a contingent nominal gross interest rate \( R_t \), and \( B_t \) are nominal bonds that pay a gross nominal non-contingent interest rate \( R^n_t \).

Households maximize their utility (25) subject to the budget constraint (26) with respect to deposits, labor, nominal bonds and money, yielding four first order conditions:

\[
U_{C,t} = \beta E_t \left\{ R_{t+1} U_{C,t+1} \right\},
\]

(27)

\[
U_{C,t} = \beta R^n_t E_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}
\]

(28)

\[
W_t U_{C,t} = \chi H^n_t
\]

(29)

\[
U_{C,t} = \zeta \frac{1}{m_t} + \beta E_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}
\]

(30)

We define the gross rate of inflation as \( \pi_{t+1} = P_{t+1}/P_t \), and real money balances as \( m_t = M_t/P_t \).

3.2 Retailers

The final consumption good is made up of a basket of intermediate retail goods which are aggregated together in CES fashion by the representative household:

\[
C_t = \left( \int_0^1 c_i^\frac{1}{\varepsilon} d\bar{i} \right)^{-\varepsilon}.
\]

(31)

Demand for retailer \( i \)'s unique variety is

\[
c_{it} = \left( \frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t
\]

(32)
where \( p_{it} \) is the price charged by retail firm \( i \). The aggregate price index is defined as

\[
P_t = \left( \int_0^1 p_{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}.
\]  

(33)

Each retail firm chooses its price according to Calvo (1979) in order to maximize net discounted profit. With probability \( 1 - \theta \) each retailer is able to change its price in a particular period \( t \). Retailer \( i \)'s objective function is

\[
\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s E_t \left\{ \Lambda_{t,s} \left( p_{it}^* - P_{w,t+s} \right)^{-\epsilon} Y_{t+s} \right\},
\]

(34)

where \( P_{w,t+s} \) is the wholesale goods price. The first order condition with respect to the retailer’s price \( p_{it}^* \) is

\[
\sum_{s=0}^{\infty} \theta^s E_t \left\{ \Lambda_{t,s} (p_{it}^*/P_{t+s})^{-\epsilon} Y_{t+s} \left[ p_{it}^* - \frac{\epsilon}{\epsilon - 1} P_{w,t+s} \right] \right\} = 0.
\]

(35)

From this condition it is clear that all retailers which are able to reset their prices in period \( t \) will choose the same price \( p_{it}^* = P_t^* \forall i \). The price level will evolve according to

\[
P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.
\]

(36)

Dividing the left and right hand side of (36) by the price level gives

\[
1 = \left[ \theta \pi_{t-1}^{\epsilon-1} + (1 - \theta)(p_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}},
\]

(37)

where \( p_t^* = P_t^*/P_t \). Using the same logic, we can normalize (35) and obtain:

\[
p_t^* = \frac{\epsilon}{\epsilon - 1} \sum_{s=0}^{\infty} \theta^s E_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\epsilon} Y_{t+s} p_{w,t+s} \right\},
\]

(38)

where \( p_{t+s}^w = \frac{p_{w,t+s}}{R_t} \) and \( p_{t+s} = P_{t+s}/P_t \).

### 3.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage \( W_t \) and \( W_e^t \) and rent capital from entrepreneurs at rental rate \( R_t^e \). Note that capital purchased in period \( t \) is used in period \( t + 1 \). Following BGG, the production function of the representative wholesaler is given by

\[
Y_t = A_t K_{t-1}^\alpha (H_t)^(1-\alpha)\Omega (H_e^t)^(1-\alpha)(1-\Omega),
\]

(39)
where $A_t$ denotes aggregate technology, $K_t$ is capital, $H_t$ is household labor, $H^e_t$ is entrepreneurial labor, and $\Omega$ defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to
\[
Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}.
\] (40)

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be
\[
\frac{P^w_t}{P_t} = \frac{1}{X_t},
\] (41)
where $X_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by
\[
\max_{H_t, H^e_t, K_{t-1}} \frac{1}{X_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H^e_t)^{(1-\alpha)(1-\Omega)} - W_t H_t - W^e_t H^e_t - R_t K_{t-1}.
\] (42)

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are
\[
\frac{1}{X_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H^e_t)^{(1-\alpha)(1-\Omega)} = R_t,
\] (43)
\[
\frac{\Omega}{X_t} (1 - \alpha) \frac{Y_t}{H_t} = W_t,
\] (44)
\[
\frac{\Omega}{X_t} (1 - \alpha) \frac{Y_t}{H^e_t} = W^e_t.
\] (45)

### 3.4 Capital Producers

The perfectly competitive capital producer transforms final consumption goods into capital. Capital production is subject to adjustment costs, according to
\[
K_t = I_t + (1 - \delta) K_{t-1} - \frac{\phi_K}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1},
\] (46)
where $I_t$ is investment in period $t$, $\delta$ is the rate of depreciation and $\phi_K$ is a parameter that governs the magnitude of the adjustment cost. The capital producer’s objective function is
\[
\max_{I_t} K_t Q_t - I_t,
\] (47)
where $Q_t$ denotes the price of capital. The first order condition of the capital producer’s optimization problem is
\[
\frac{1}{Q_t} = 1 - \phi_K \left( \frac{I_t}{K_{t-1}} - \delta \right).
\] (48)
3.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which
costlessly intermediates between households and borrowers. The role of the lender is to diversify
the household’s funds among various entrepreneurs. The bank takes nominal household deposits
\( D_t \) and loans out nominal amount \( B_t \) to entrepreneurs. In equilibrium, deposits will equal
loanable funds \( (D_t = B_t) \). Households, as owners of the bank, receive a state contingent real rate
of return \( R_{t+1} \) on their “deposits” — which equals the rate of return on loans to entrepreneurs.\(^6\)
Households choose the optimal lending rate according to their first order condition with respect
to deposits:

\[
\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} R_{t+1} \right\} = \mathbb{E}_t \left\{ \Lambda_{t+1} R_{t+1} \right\} = 1.
\]

As we discussed above, the lender prefers a return that co-varies negatively with household
consumption, which amplifies the financial accelerator.

3.6 Entrepreneurs

We have already described the entrepreneur’s problem in detail in Section 2. Entrepreneurs
choose their cutoff productivity level and leverage according to: (14) in BGG; (17) in MCC;
and (20) and (21) in the dynamically optimal contract.

Wholesale firms rent capital at rate \( R_{t+1} = \frac{\alpha_Y}{K_{t-1}} \) from entrepreneurs. After production
takes place entrepreneurs sell undepreciated capital back to capital goods producers for the
unit price \( Q_{t+1} \). Aggregate returns to capital are then given by

\[
R^k_{t+1} = \frac{\frac{1}{K_t} \alpha_{Yt+1} + Q_{t+1}(1-\delta)}{Q_t}.
\]

(49)

Consistent with the partial equilibrium specification, entrepreneurs die with probability \( 1-\gamma \),
which implies the following dynamics for aggregate net worth:

\[
N_{t+1} = \gamma N_t k_t R^k_{t+1} g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) + W^c_{t+1}.
\]

(50)

3.7 Goods market clearing

We have goods market clearing

\[
Y_t = C_t + I_t + G_t + C^e_t + \mu G(\tilde{\omega}_t, \sigma_{\omega,t-1}) R^k_t Q_{t-1} K_{t-1},
\]

(51)

where \( \mu G(\tilde{\omega}) = \int_0^{\tilde{\omega}} \mu f(\omega) \omega d\omega \) is the fraction of capital returns that go to monitoring costs,
paid by lenders.

\(^6\)Note that lenders are not necessary in the model, but we follow BGG and MCC in positing a perfectly
competitive financial intermediary between households and borrowers.
3.8 Monetary Policy

We assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate $R^n_t$. In Section 4 we employ the nominal interest rate rule in BGG:

$$\log(R^m_t) - \log(R^n_t) = \rho^R \left( \log(R^m_{t-1}) - \log(R^n_t) \right) + \xi \pi_t - \epsilon^R_t \quad (52)$$

where $\rho^R$ and $\xi$ determine the relative importance of the past interest rate and past inflation in the central bank’s interest rate rule. Shocks to the nominal interest rate are given by $\epsilon^R_t$. It should be noted that the interest rule in BGG differs from the conventional Taylor rule, which targets current inflation rather than past inflation.

In Section 5, we consider the conventional Taylor rule with interest rate smoothing

$$\log(R^m_t) - \log(R^n_t) = \rho^R \left( \log(R^m_{t-1}) - \log(R^n_t) \right) + \rho^Y \left( \log(Y_t) - \log(Y_{t-1}) \right) + \epsilon^R_t + \rho^Y \log(Y_t) \quad (53)$$

3.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon^A_t \quad (54)$$

$$\log(G_t/Y_t) = (1 - \rho^G) \log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon^G_t \quad (55)$$

$$\log(\sigma_{\omega,t}) = (1 - \rho^\sigma) \log(\sigma_{\omega,ss}) + \rho^\sigma \log(\sigma_{\omega,t-1}) + \epsilon^\sigma_t \quad (56)$$

where $\epsilon^A$, $\epsilon^G$ and $\epsilon^\sigma$ denote exogenous shocks to technology, government spending and idiosyncratic volatility, and $G_{ss}$ and $\sigma_{\omega,ss}$ denote the steady state values for government spending and idiosyncratic volatility respectively. Recall that $\sigma^2$ is the variance of idiosyncratic productivity, so that $\sigma$ is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (52) or the Taylor rule in (53).

3.10 Equilibrium

The model has 20 endogenous variables and 20 equations. The endogenous variables are: $Y$, $H$, $C$, $\Lambda$, $C^e$, $W$, $W^e$, $I$, $Q$, $K$, $R^n$, $R^k$, $R$, $p^*$, $X$, $\pi$, $N$, $\bar{\omega}$, $k$ and $Z$. The equations defining these endogenous variables are: (9), (27), (29), (30), (11), (37), (38), (41), (40), (44), (45), (46), (48), (49), (50), (51), (52), (28), (A.8) and (E.3). The exogenous processes for technology, government spending and idiosyncratic volatility follow (54), (55) and (56) respectively. Nominal interest rate shocks are defined by the Taylor rule in (52).
4 Quantitative Analysis

4.1 Calibration

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$ so that utility is logarithmic in consumption, and the elasticity of labor is $3 (\eta = 1/3)$. The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Investment adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly depreciation is $\delta = 0.025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = 0.0272$, yielding an annualized business failure rate of three percent. The idiosyncratic productivity term, $\log(\omega(j))$, is assumed to be log-normally distributed with variance of 0.28. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we assume the Calvo parameter $\theta = 0.75$, so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho_R = 0.9$ and the parameter on past inflation to $\xi = 0.11$. Note that in Section 5 we also consider a conventional Taylor rule where the central bank targets current inflation rather than past inflation. For the conventional Taylor rule, we set $\rho_R = 0$, $\xi = 1.5$ and $\rho_Y = 0.5$ as a benchmark, and consider an inertial interest rate rule with smoothing parameter $\rho^\sigma = 0.5$, $\xi = 0.75$ and $\rho_Y = 0.25$. We follow BGG and set the persistence of the shocks to technology and government spending at $\rho_A = 0.999$ and $\rho_G = 0.95$. We follow Christiano, Motto and Rostagno (2013) and set the persistence of idiosyncratic volatility at $\rho^\sigma = 0.9706$ and the distribution of the shocks equal to $\epsilon_t \sim N(0, 0.0283)$.

Following BGG, we consider a one percent technology shock and a 25 basis point shock (in annualized terms) to the nominal interest rate. For the risk shock, we allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.28 to 0.29.

4.2 Quantitative Comparison: BGG, MCC and the Optimal Contract

In our quantitative analysis we compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the MCC contract; and the competitive equilibrium under the optimal contract. Impulse responses for shocks to technology, the nominal interest rate and idiosyncratic volatility are found in this section.

Figure 3 shows impulse responses for a extremely persistent one percent technology shock when prices are sticky. Notice the impact of consumption insurance. Lenders in the MCC contract allocations will settle for a lower rate of return in a boom in order to ensure a higher rate of return in a recession, which amplifies the response of the economy. However, this does not occur under the optimal contract because entrepreneurs are forward looking: they act as a stabilizing influence on the economy. Forward looking entrepreneurs are reluctant to invest in new capital when a positive technology shock hits because financial premiums will be low. Asset
prices will decline back to their steady state value, so entrepreneurs offer higher deposit rates to lender’s in order utilize financial resources in states that promise higher capital returns. The stabilizing influence of forward looking entrepreneurs cancels out the consumption insurance channel under this calibration, such that the optimal contract and BGG output responses coincide almost exactly. In general, this coincidence does not hold outside of the particular calibration employed here.

The difference between the three allocations is very noticeable in Figure 4, which plots impulse responses for a one percent shock to the nominal interest rate when prices are sticky. Because the monetary shock is less persistent than the technology shock, the price of capital depreciates back to its steady state value very quickly after an initial rise. As a result, capital returns are positive in the first period, but negative thereafter. This leads to an even sharper difference between the response of entrepreneurs in the three models. Under the BGG contract the deposit rate does not respond to the shock at all because it is predetermined; under the MCC contract the deposit rate falls because household consumption increases in response to the shock; and under the optimal contract the deposit rate increases, because the financial premium goes down after the shock. Forward looking entrepreneurs thus stabilize consumption and output, leading to small amplification. In contrast, the MCC contract with consumption insurance leads to a decline in the lending rate following the rise in consumption, which amplifies the response of output, consumption and other macroeconomic aggregates to the interest rate shock.

In Figure 5 we plot impulse responses for a one standard deviation increase in unobserved idiosyncratic volatility $\sigma_\omega$. This is what we defined earlier as a risk shock. In all three models, the household consumption response on impact is close to zero, but slightly positive for MCC and BGG and slightly negative for the optimal contract. The consumption insurance channel in MCC and the optimal contract leads to a decline in the lending rate following a risk shock. An additional factor is at work under the optimal contract: the financial premium rises because, other things equal, higher idiosyncratic variance makes default more likely. Therefore, borrowing is more expensive and returns to capital are higher. Net worth thus increases on impact under the optimal contract. Overall, risk shocks have a very small impact on the real economy in the optimal contract equilibrium, and may even boost output over a longer time horizon. Also note the negative correlation between output and consumption under the optimal contract, unlike the BGG contract where output and consumption both fall.

5 How Robust is the Financial Accelerator? Comparison with the Frictionless Model

To truly measure the strength of the financial accelerator, we need to compare the CSV model with financial frictions against a frictionless benchmark. We employ two frictionless benchmarks:

1. The basic New Keynesian model; and
2. The financial accelerator model of this paper with monitoring costs and the variance of idiosyncratic productivity set to zero.

First, we compare the amplification response of the model with frictions to the basic New Keynesian model. Figure 6 shows that all three models with frictions generate more amplification than the basic New Keynesian model for very persistent technology shocks. In this case, forward looking entrepreneurs forecast higher capital returns in the future, which makes one unit of net worth more valuable today, leading to a large increase in net worth following the shock. For less persistent technology shocks ($\rho^A = 0.99$ for example) amplification under the optimal contract is actually lower than in the basic New Keynesian model.

Figure 6 also shows that the optimal contract delivers slightly smaller volatility than the New Keynesian model for monetary shocks. The intuition is very simple. In the wake of a positive monetary shock net worth increases and cash is abundant, so one additional unit of net worth generates a smaller consumption flow. Therefore entrepreneurs want to increase their payments to lenders and pay back their increase in net worth. Net worth thus does not react to the shock, which stabilizes expenditures relative to the basic New Keynesian model. Overall, we see that amplification under the optimal contract is slightly smaller for technology shocks with persistence equal to or lower than $\rho^A = 0.99$ as well as for monetary shocks.

Second, we take the model described in Section 3 and set monitoring costs and the variance of idiosyncratic productivity to zero. This frictionless model is similar to Carlstrom and Fuerst’s (1997), which also sets monitoring costs equal to zero, but different from the BGG frictionless model. BGG’s frictionless model assumes a constant positive financial premium. We choose our definition because a constant positive financial premium in different aggregate states implies different profits for entrepreneurs, which distorts their decisions. We focus here only on the frictionless model with zero monitoring costs, but all of our main results hold relative to both frictionless cases.

We employ a second frictionless benchmark for the following reason. The basic New Keynesian sticky price model deviates from the CSV framework in two dimensions: (1) it abstracts from heterogeneity between lenders and borrowers because there are no entrepreneurs, and (2) it has no CSV frictions. As a result, if we use only the basic New Keynesian model as a frictionless benchmark, it is difficult to isolate the impact of the CSV friction on volatility from the impact of heterogeneity. In order to perfectly isolate these two effects, we need a model that incorporates heterogeneity between lenders and borrowers but which eliminates the CSV friction. The second frictionless benchmark does exactly that, isolating the role of the CSV friction in generating volatility.

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7The model without monitoring costs generates a different steady state relative to the model with monitoring costs, but the difference between the steady states is small for all variables except leverage. We correct steady state leverage in the frictionless model by increasing the share of entrepreneurs in the production function from 0.01 to 0.1. These modifications have a very small effect on equilibrium dynamics and do not alter our conclusions in any way.
Now, let us consider the amplification response of the model with frictions relative to the frictionless benchmark with no monitoring costs. As we discussed earlier, the amplifying effect of the CSV framework depends on three characteristics: a suboptimal lending contract, extremely persistent technology shocks and loose monetary policy. The removal of any one of these characteristics eliminates the financial accelerator or even reverses the accelerator, such that financial frictions stabilize the economy in the presence of shocks.

Figure 7 plots the output response to a variety of technology shocks in a CSV model with and without monitoring costs and demonstrates the fragility of the financial accelerator to these three characteristics. The first row of plots in Figure 7 shows the response of output to an extremely persistent technology shock when prices are sticky. The model with frictions provides slightly more amplification in the suboptimal BGG lending contract, while the frictionless model generates more amplification for the MCC contract and the optimal contract.

Why do financial frictions stabilize business cycles in the latter two cases? First, entrepreneurs sell insurance to the household in order to smooth household consumption. The resulting decline in household consumption volatility leads to a rise in the volatility of entrepreneurial consumption and net worth, and entrepreneurs become a driver of the business cycle. Second, when entrepreneurs are forward looking they behave in a risk averse manner by trying to tighten the financial constraint during booms, when the financial premium is low, in order to relax it during recessions, when the financial premium is high. States with positive technology shocks promise falling asset prices in the short run after the initial reaction of asset prices on impact, and higher dividends in the long run from non-stationarity. This leads entrepreneurs to lever up and increase their net worth by a large amount, generating massive amplification in the frictionless model under the optimal contract. We also find that under the MCC contract, financial frictions stabilize business cycle, although the stabilizing effect is much smaller than under the optimal contract.

The sensitivity of the financial accelerator to stationary technology shocks is illustrated clearly in the second row of Figure 7, where we consider technology shocks with lower persistence ($\rho^A = 0.95$). In this calibration, financial frictions stabilize business cycles not only for the optimal contract and MCC but also for BGG. Why is it the case? We know, that for flexible prices model with and without financial friction deliver very similar results under BGG. Therefore, amplification should exacerbate fluctuation of markups. However, we know that for stationary technology shocks even in standard New Keynesian models, markups move procyclically and stabilize business cycles. If financial frictions exacerbate fluctuation for markups, they stabilize the model response to stationary technology shocks, since markups become even more procyclical.

Rows three and four of Figure 7 demonstrate the output response for extremely persistent technology shocks with a conventional Taylor rule and under flexible prices, respectively. The impulse responses show that the financial accelerator is not robust to more conservative monetary policy or flexible prices. As in the previous cases, the frictionless model for the MCC and
optimal contracts generates higher amplification than the model with CSV frictions. Under the BGG contract, the accelerator disappears when the central bank follows a conventional Taylor rule. It is still present under flexible prices, but is extremely small quantitatively. In other words, the magnitude of the financial accelerator in the BGG case is negligible when monetary policy is more aggressive or when prices are flexible.

5.1 Sensitivity of the Financial Accelerator to Different Monetary Policy Rules

How sensitive is the financial accelerator to different monetary policy rules? Gilchrist and Leahy (2002) investigate the role of monetary policy for non-contingent contracts in the BGG framework. Figure 8 plots output responses to a 25 basis point shock to the nominal interest rate for the BGG monetary policy rule (Row 1), the inertial Taylor rule (Row 2) and the conventional Taylor rule (Row 3). Here we see the sensitivity of the financial accelerator to different monetary policy specifications. Under the BGG policy rule, the coefficient on past inflation is $\xi = 0.11$, while the interest rate smoothing parameter is $\rho = 0.9$. Following the initial 25 basis point decrease in the nominal interest rate, there is little subsequent change in the interest rate under the BGG policy rule because the central bank is targeting past inflation, and also smoothing the interest rate. Any increase in inflation on impact is not taken into account until the next quarter. Under the BGG policy rule, monetary shocks are thus quite persistent, and entrepreneur’s increase their net worth in the first period, which amplifies the shock. On the other hand, under conservative monetary policy asset prices and net worth are more stable, and there is no amplification.

We also calculate the quarterly inflation response to a monetary shock for the BGG policy rule and the conventional Taylor rule. For the conventional Taylor rule with a weight $\rho = 0.5$ on the previous interest rate, a two percent surprise to the Fed funds rate in annual terms leads to a one percent inflation response, while for the BGG monetary policy rule a one percent surprise to the Fed funds rate will lead to four percent inflation response, which significantly deviates from the flexible price equilibrium.

Overall, our simulations show that under the MCC contract and the optimal contract, financial frictions do not amplify business cycles for any calibration, while under the BGG contract they amplify business cycles only when technology shocks are extremely persistent and monetary policy is loose.

5.2 Sensitivity of the Financial Accelerator to Household Risk Aversion

One might expect that an increase in household risk aversion would shift aggregate risk onto the balance sheet of entrepreneurs, causing the financial accelerator to reappear. However, we find that households sell insurance to entrepreneurs and use labor supply to smooth their consumption regardless of the degree of risk aversion. Although this may seem counterintuitive, it is natural for entrepreneurs to increase their returns by shifting aggregate risk to households,

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8Gilchrist and Leahy (2002) investigate the role of monetary policy for non-contingent contracts in the BGG framework.
9Although not reported here, we conduct experiments for coefficients of risk-aversion, $\sigma$, from 1 to 500 and find that the financial accelerator is not present for all values.
as households can smooth their consumption through variation in labor while enjoying higher returns from insuring aggregate risk. The only way to motivate households to buy insurance from entrepreneurs is to make labor supply elasticity very small, which counterfactually reduces business cycle fluctuations to a minimum. Thus, we find that financial frictions with state-contingent contracts and forward looking entrepreneurs stabilize business cycles even when households are extremely risk averse.

6 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by introducing forward looking entrepreneurs into the costly state verification framework. In the literature, lending contracts are suboptimal and entrepreneurs are myopic. We solve for the optimal contract with forward looking entrepreneurs and show that financial frictions neither amplify nor propagate business cycles when lending contracts are optimal. In addition, we show that shocks to the variance of the unobserved productivity of entrepreneurs — so-called “risk shocks” — have little effect on the economy and generate the wrong comovement between macroeconomic aggregates when contracts are optimal.

We also investigate the robustness of the financial accelerator under the standard BGG contract, which assumes that lenders receive a constant safe rate of return and borrowers are myopic. In this setup, we find that the accelerator depends on a combination of three things: a suboptimal lending contract, extremely persistent technology shocks, and loose monetary policy. Stationary technology shocks or a standard Taylor rule eliminate the financial accelerator or even reverse the accelerator such that financial frictions stabilize macroeconomic fluctuations. We thus conclude that the amplifying effect of financial frictions is present only under very restrictive conditions in costly state verification models.

Our result that amplification requires a combination of a suboptimal non-contingent lending rate and asymmetric information between borrowers and lenders appears to be quite general and not at all specific to the CSV framework. Dmitriev and Hoddenbagh (2014) obtain a similar result in a model with collateral constraints, while House (2006) obtains a similar result in a model with adverse selection. Generating amplification from financial frictions in an optimal contractual environment remains an area for future research.
References


Figure 2: Overview of the Model

- Retailers
- Wholesalers
- Entrepreneurs
- Households
- Financial Intermediaries
- Capital Producers

- Payments for Consumption
- Retail Goods
- Payments
- Wholesale Goods
- Dividends
- Payment
- Loans
- Repayment
- Deposits
- Wages
- Labor
- Capital, Entrepreneural Labor
- Capital Rent, Entrepreneural Wage
- Capital
- Payments for Capital
Figure 3: Technology Shock

Output

Aggregate Consumption

Labor

Net Worth

Household Consumption

Price of Capital

Returns to Lenders

Financial Premium

Returns to Capital

Note: All impulse responses are plotted as percent deviations from steady state.
Figure 4: Monetary Policy Shock

Note: All impulse responses are plotted as percent deviations from steady state.
Figure 5: Idiosyncratic Volatility Shock

Note: All impulse responses are plotted as percent deviations from steady state.
Figure 6: Output Response Relative to the Basic New Keynesian Model

Note: All impulse responses are plotted as percent deviations from steady state.
Figure 7: Output Response Relative to the Frictionless Model, Technology Shocks

**Extremely Persistent Technology Shock**

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<th>Optimal Contract</th>
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**Stationary Technology Shock**

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**Extremely Persistent Technology Shock, Conventional Taylor rule**

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**Extremely Persistent Technology Shock, Flex Prices**

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**Note:** All impulse responses are plotted as percent deviations from steady state.
Figure 8: Output Response Relative to the Frictionless Model, Monetary Shocks

**BGG Monetary Policy Rule**

\[
\log(R^n_t) - \log(R^n) = 0.9 \left( \log(R^n_{t-1}) - \log(R) \right) + 0.11 \pi_t + \epsilon_R^n
\]

**Inertial Taylor rule**

\[
\log(R^n_t) - \log(R^n) = 0.5 \left( \log(R^n_{t-1}) - \log(R) \right) + 0.5 \left[ 1.5 \pi_t + 0.5 \left( \log(Y_t) - \log(Y_{t-1}) \right) \right] + \epsilon_R^n
\]

**Conventional Taylor rule**

\[
\log(R^n_t) - \log(R^n) = 1.5 \pi_t + 0.5 \left( \log(Y_t) - \log(Y_{t-1}) \right) + \epsilon_R^n
\]

Note: All impulse responses are plotted as percent deviations from steady state.
A Value Function Transformation

We can also formulate the optimal contract using normalized variables. For example, one can substitute leverage into the right hand side of equation (5) to obtain

\[ N_{t+s}(j) = N_{t+s-1}(j)\kappa_{t+s-1}(j)R_{t+s}^e \max \{\omega_{t+s}(j) - \bar{\omega}_{t+s}, 0\} + W_{t+s}^e = N_{t+s-1}(j)R_{t+s}^e(j) + W_{t+s}^e, \]  

(A.1)

where \( R_{t}^e(j) = \kappa_{t-1}(j)R_{t}^e \max \{\omega_{t}(j) - \bar{\omega}_{t}, 0\} \) is the entrepreneur’s ex post realized return. Iterating this equation backward generates

\[ N_{t+s}(j) = N_{t+s-1}(j)R_{t+s}^e(j) + W_{t+s}^e \]
\[ = N_{t}(j)\tilde{R}_{t,t+s}^e + W_{t+1}^e\tilde{R}_{t+1,t+s}^e + \ldots + W_{t+s}^e \]
\[ = N_{t}(j)\tilde{R}_{t,t+s}^e + \sum_{i=1}^{s} W_{t+i}^e\tilde{R}_{t+i,t+s}^e, \]  

(A.2)

where \( \tilde{R}_{t,t+s}^e = R_{t+1}^eR_{t+2}^e\ldots R_{t+s}^e \) and \( \tilde{R}_{t,s,t+s} = 1 \). Intuitively, \( \tilde{R}_{t,t+s}^e \) is the entrepreneur’s ex post accumulated rate of return on projects from period \( t \) through period \( t + s \). For example, suppose the entrepreneur invests one dollar in period \( t \) and continues to reinvest his profits in new projects in each subsequent period. In period \( t + s \), the entrepreneur will have accumulated \( \tilde{R}_{t,t+s}^e \) from his initial one dollar investment. We can substitute (A.2) into the value function (2) and obtain

\[ V_t^e(j) = (1 - \gamma)\mathbb{E}_t \left\{ N_{t}(j)R_{t+1}^e + \sum_{s=2}^{\infty} \gamma^{s-1} \left( N_{t}(j)\tilde{R}_{t,t+s}^e + \sum_{i=1}^{s-1} W_{t+i}^e\tilde{R}_{t+i,t+s}^e \right) \right\}. \]  

(A.3)

Because the entrepreneur will optimize with respect to leverage and the productivity cutoff we want to express (A.3) as a function of the leverage and the productivity cutoff. In the first step, we separate terms to get

\[ V_t^e(j) = (1 - \gamma)N_{t}(j)\mathbb{E}_t \left\{ \tilde{R}_{t,t+1}^e \sum_{s=1}^{\infty} \gamma^{s-1}\tilde{R}_{t+1,t+s}^e \right\} + (1 - \gamma)\mathbb{E}_t \left\{ \sum_{s=1}^{\infty} \gamma^{s-1} \left( \sum_{i=1}^{s-1} W_{t+i}^e\tilde{R}_{t+i,t+s}^e \right) \right\}. \]  

(A.4)

where we used \( \tilde{R}_{t,t+s}^e = \tilde{R}_{t,t+1}^e\tilde{R}_{t+1,t+s}^e \). Net worth enters the value function as a constant multiplicative term and has no effect on the entrepreneur’s choice of leverage \( \kappa_t(j) \) or cutoff \( \bar{\omega}_{t+1} \); both enter only through \( \tilde{R}_{t,t+1}^e \). Using the law of iterated expectations \( \mathbb{E}_t(x_{t+1}) = \mathbb{E}_t[\mathbb{E}(x_{t+1}|\Omega_{agg,t+1})] \) and the independence of idiosyncratic productivity from aggregate productivity, we can replace the realizations of idiosyncratic productivity with their expectation and get
\[ V_t^e(j) = (1 - \gamma) N_t(j) \mathbb{E}_t \left[ \hat{R}^e_{t,t+1} \sum_{s=1}^{\infty} \gamma^{s-1} \hat{R}^e_{t+1,t+s} \right] + (1 - \gamma) \mathbb{E}_t \left[ \sum_{s=2}^{\infty} \gamma^{s-1} \left( \sum_{i=1}^{s-1} W_{t+1}^e \hat{R}^e_{t+1,t+s} \right) \right], \]  

(A.5)

where \( \hat{R}^e_{t,t+s} = R^e_{t+1} R^e_{t+2} \ldots R^e_{t+s} \) with \( \hat{R}^e_{t+1} = 1 \), and \( R^e_{t+1} = \kappa_t R^k_{t+1} g(\tilde{\omega}_{t+1}) \) is the entrepreneur’s ex post realized rate of return expressed as a function of aggregate productivity and leverage, with \( g(\tilde{\omega}_{t+1}) = \int_{\tilde{\omega}_{t+1}}^{\infty} [\omega - \tilde{\omega}_{t+1}] f(\omega) d\omega \).

Now we can reexpress value function as

\[ V_t^e(j) = (1 - \gamma) N_t(j) (\Psi_t - 1) + (1 - \gamma) \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \gamma^s W_t^e (\Psi_{t+s} - 1) \right], \]  

(A.6)

where \( \Psi_t = 1 + \gamma \mathbb{E}_t \left[ \kappa_t R^k_{t+1} g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} \right] \). If we divide the lender’s ex post returns in equation (7) by \( N_t(j) \) we get

\[ \left[ \kappa_t(j) - 1 \right] R_{t+1}(j) = \kappa_t(j) R^k_{t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}). \]  

(A.7)

Now, if we substitute (A.7) into the Euler equation for the representative household (12), we have

\[ \beta \mathbb{E}_t \left\{ U_{C,t+1} \kappa_t(j) R^k_{t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = \left[ \kappa_t(j) - 1 \right] U_{C,t}. \]  

(A.8)

Before looking at the first order conditions to the optimization problem, it is important to notice that all entrepreneurs will choose the same leverage and state-contingent interest rate regardless of their net worth, due to the homotheticity of the problem. Thus, the entrepreneur index \( (j) \) is omitted below. We use the following notation: \( BGG \) refers to the contract of Bernanke, Gertler and Gilchrist (1999), \( MCC \) refers to the myopic contingent contract, and \( Optimal \) refers to the optimal contract with non-myopic entrepreneurs

**B BGG Contract**

In the BGG contract, the lender is guaranteed a fixed rate of return. In this case, the entrepreneur’s Lagrangian will be:

\[ L_{BGG} = (1 - \gamma) \mathbb{E}_t \left\{ N_t \kappa_t R^k_{t+1} g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) + \lambda_{t+1} \left[ \beta \mathbb{E}_t \left\{ U_{C,t+1} \right\} \kappa_t R^k_{t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1) U_{C,t} \right] \right\}. \]
The entrepreneur’s first order conditions with respect to $\kappa_t$ and $\bar{\omega}_{t+1}$ are:

\[
\frac{\partial L_{BGG}}{\partial \kappa_t} = N_t E_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} - E_t \left\{ \lambda_{t+1} \right\} \frac{U_{C,t}}{\kappa_t} = 0 \tag{B.1}
\]

\[
\frac{\partial L_{BGG}}{\partial \bar{\omega}_{t+1}} = N_t \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) + \lambda_{t+1} \beta E_t \left\{ k_{t+1} h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} = 0 \tag{B.2}
\]

If we substitute $\frac{\partial L_{BGG}}{\partial \bar{\omega}_{t+1}}$ into $\frac{\partial L_{BGG}}{\partial \kappa_t}$, we find

\[
N_t E_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} = E_t - \left\{ N_t g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} \frac{U_{C,t}}{\kappa_t}. \tag{B.3}
\]

Rearranging, simplifying and substituting in the stochastic discount factor yields:

\[
\kappa_t E_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} = - \frac{g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t})}{h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t})} \frac{1}{E_t A_{t,t+1}}. \tag{B.4}
\]

### C MCC Contract

In the MCC contract, the entrepreneur’s Lagrangian is:

\[
L^{MCC} = (1 - \gamma) \left\{ N_t \kappa_t E_t \left[ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right] + \lambda_t \left[ E_t \left( \beta U_{C,t+1} k_{t} R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right) - (k_t - 1) U_{C,t} \right] \right\}.
\]

The entrepreneur’s first order conditions with respect to $\kappa_t$ and $\bar{\omega}_{t+1}$ are:

\[
\frac{\partial L^{MCC}}{\partial \kappa_t} = (1 - \gamma) \left[ N_t E_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} + \lambda_t \left( E_t \left\{ \beta U_{C,t+1} R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega, t}) - U_{C,t} \right\} \right) \right] = 0,
\]

\[
\frac{\partial L^{MCC}}{\partial \bar{\omega}_{t+1}} = (1 - \gamma) \left[ N_t \kappa_t R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) + \lambda_t \beta U_{C,t+1} \kappa_t R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right] = 0.
\]

Rearranging these first order conditions, solving in terms of $\lambda_t$ and setting them equal to each other yields:

\[
\kappa_t E_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega, t}) \right\} = - \frac{g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t})}{h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega, t})} \frac{1}{A_{t,t+1}}. \tag{C.1}
\]
D Optimal Contract With Forward Looking Entrepreneurs

Under the optimal contract, the forward looking entrepreneur’s Lagrangian has the following form (if we divide the value function by \((1 - \gamma)N_t(j)\) as a scaling factor):

\[
L^{Optimal} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j)(\Psi_t - 1) + \sum_{s=1}^{\infty} \gamma^s W_{t+s}^r (\Psi_{t+s} - 1) \right. \\
\left. + \sum_{i=0}^{\infty} \lambda_{t+i} \left[ \beta U_{C,t+i+1} \kappa_{t+i} R_{k,t+i+1} h(\bar{\omega}_{t+i+1}, \sigma_{\omega,t+i}) - (\kappa_{t+i} - 1) U_{C,t+i} \right] \right\},
\]

where \(\Psi_t = 1 + \kappa_t \mathbb{E}_t \left\{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \right\}\). The entrepreneur’s first order condition with respect to leverage \(\kappa_t\) is:

\[
\frac{\partial L^{Optimal}_t}{\partial \kappa_t} = (1 - \gamma) \mathbb{E}_t \left\{ N_t(j) g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} + \lambda_t \left[ \beta U_{C,t+1} R_{k,t+1} \beta h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - U_{C,t} \right] \right\} = 0,
\]

where we have used the fact that \(\frac{\partial \Psi_t}{\partial \kappa_t} = \mathbb{E}_t \left\{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \right\}\) and \(\frac{\partial \Psi_{t+1}}{\partial \omega_{t+1}} = 0\) for \(i = 1, 2, ...\)

The entrepreneur’s first order condition with respect to the productivity cutoff \(\bar{\omega}_{t+1}\) is:

\[
\frac{\partial L^{Optimal}_t}{\partial \bar{\omega}_{t+1}} = (1 - \gamma) \left\{ N_t(j) \kappa_t g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} + \lambda_t \left[ \beta U_{C,t+1} \kappa_t R_{k,t+1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right] \right\} = 0.
\]

(D.1)

where we have used the fact that \(\frac{\partial \Psi_{t+1}}{\partial \omega_{t+1}} = \kappa_t g'(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1}\) and \(\frac{\partial \Psi_{t+1}}{\partial \omega_{t+1}} = 0\) for \(i = 1, 2, ...\)

We then move \(\lambda_t\) to the right hand side of both first order conditions and divide the equations by each other to obtain:

\[
\frac{g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} \right\}}{\beta \mathbb{E}_t \left\{ U_{C,t+1} \kappa_t R_{k,t+1} h(\bar{\omega}_{t+1}) \right\} - \kappa_t U_{C,t}} = \frac{\beta \kappa_t U_{C,t+1} h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{(\kappa_t - 1) U_{C,t} - \kappa_t U_{C,t}}
\]

where we utilized the participation constraint for lenders in the final step. After rearranging and simplifying, we get

\[
\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = \frac{-g_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1}}{h_{\omega}(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \Lambda_{t,t+1}}.
\]

(D.2)
E BGG Contract With Forward Looking Entrepreneurs

For a predetermined lending rate, the non-myopic entrepreneur’s Lagrangian has the following form if we divide the value function by \((1 - \gamma)\gamma^Nt\): 
\[
L = (1 - \gamma)E_t \left\{ N_t(j)(\Psi_t - 1) + \sum_{s=1}^{\infty} \gamma^s W^c_{t+s}(\Psi_{t+s} - 1) \right. \\
+ \sum_{i=0}^{\infty} \lambda_{t+i+1} \left[ \beta E_t \left( U_{C,t+i+1} \right) \kappa_{t+i} R_{k,t+i+1} h(\tilde{\omega}_{t+i+1}, \sigma_{\omega,t+i}) - (\kappa_{t+i} - 1) U_{C,t+i} \right] \}
\]

where \(\Psi_t = 1 + \kappa_t E_t\left\{ g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \right\}\). The entrepreneur’s first order condition with respect to leverage \(\kappa_t\) is:
\[
\frac{\partial L}{\partial \kappa_t} = (1 - \gamma)E_t \left\{ N_t(j)g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} + \lambda_{t+1} \left[ \beta E_t \left( U_{C,t+1} \right) \kappa_{t} R_{k,t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) - U_{C,t} \right] \right\} = 0,
\]
(E.1)

where we have used the fact that \(\frac{\partial \Psi_t}{\partial \kappa_t} = E_t\{ g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \}\) and \(\frac{\partial \Psi_{t+i}}{\partial \kappa_t} = 0\) for \(i = 1, 2, \ldots\)

The entrepreneur’s first order condition with respect to the productivity cutoff \(\tilde{\omega}_{t+1}\) is:
\[
\frac{\partial L}{\partial \tilde{\omega}_{t+1}} = (1 - \gamma) \left\{ N_t(j)\kappa_t g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} + \lambda_{t+1} \left[ \beta E_t \left( U_{C,t+1} \right) \kappa_{t} R_{k,t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right] \right\} = 0.
\]
(E.2)

where we have used the fact that \(\frac{\partial \Psi_t}{\partial \tilde{\omega}_{t+1}} = \kappa_t g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \) and \(\frac{\partial \Psi_{t+i}}{\partial \tilde{\omega}_{t+1}} = 0\) for \(i = 1, 2, \ldots\)

One can express \(\lambda_{t+1}\) in the equation \(\frac{\partial L}{\partial \tilde{\omega}_{t+1}} = 0\) as a function of other variables, and substitute the result into \(\frac{\partial L}{\partial \kappa_t} = 0\). Then, using the participation constraint to simplify and after some rearranging, we obtain:
\[
\kappa_t E_t \left\{ \Psi_{t+1} R_{t+1}^k g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -E_t \left\{ g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} \right\}.
\]
(E.3)

It is trivial to show that log-linearization of the BGG contract with myopic or non-myopic agents gives an identical optimality condition. However, this identity does not hold for higher order approximations.
F Complete Log-Linearized Model

In this section we review the whole model in its log-linearized form.

New Keynesian Components

We begin with the set of equations characterizing the standard New Keynesian components of the model. Equation (F.1) gives the Euler equation for state-contingent assets from the FOC for deposits, while (F.2) is the Euler equation for nominal bonds. The labor market clearing condition is given by (F.3), (F.4) is the New Keynesian Phillips curve, (F.5) is the production function, (F.6) gives the evolution of capital, (F.7) refers to the dynamics of the price of capital, (F.8) gives returns to capital, and (F.9) refers to goods market clearing. Shocks to technology, monetary policy, government spending and idiosyncratic risk are defined by (F.10), (F.11), (F.12) and (F.13).

\[ -\sigma \left( E_t \hat{C}_{t+1} - \hat{C}_t \right) + E_t \hat{R}_{t+1} = 0, \]  
\[ \hat{R}_t^n = E_t \hat{R}_{t+1} + E_t \hat{\pi}_{t+1} \]  
\[ \hat{Y}_t - \hat{H}_t - \hat{X}_t - \sigma \hat{C}_t = \eta \hat{H}_t, \]  
\[ \hat{\pi}_t = \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \hat{X}_t + \beta E_t \hat{\pi}_{t+1}. \]  
\[ \hat{Y}_t = \hat{A}_t + \alpha \hat{K}_{t-1} + (1 - \alpha)(1 - \Omega) \hat{H}_t. \]  
\[ \hat{K}_t = \delta \hat{I}_t + (1 - \delta) \hat{K}_{t-1}, \]  
\[ \hat{Q}_t = \delta \phi_K (\hat{I}_t - \hat{K}_{t-1}), \]  
\[ \hat{R}_{t+1}^k = (1 - \epsilon)(\hat{Y}_{t+1} - \hat{K}_t - \hat{X}_{t+1}) + \epsilon \hat{Q}_{t+1} - \hat{Q}_t \]  
\[ Y \hat{Y}_t = C \hat{C}_t + \hat{I}_t + \hat{G}_t + \hat{C}_t^e + \phi_{\mu} \hat{\phi}_{\mu,t}, \]  
\[ \hat{A} = \rho^A \hat{A}_{t-1} + \epsilon^A_t \]  
\[ \hat{R}_t^n = \rho^{R^n} \hat{R}_{t-1}^n + \xi \hat{\pi}_t + \rho^Y \hat{Y}_t + \epsilon^R_t \]  
\[ \hat{G}_t = \rho^G \hat{G}_{t-1} + \epsilon^G_t \]  
\[ \hat{\sigma}_{\omega,t} = \rho^{\sigma_{\omega}} \hat{\sigma}_{\omega,t-1} + \epsilon^{\sigma_{\omega}}_t. \]
Entrepreneurial Consumption and Net Worth

The evolution of entrepreneurial net worth is given by (F.14), where (F.15) defines leverage. Entrepreneurial consumption is defined by (F.16) and the financial premium is given by (F.17).

\[
\hat{N}_{t+1} = \epsilon_N (\hat{N}_t + \hat{R}_{t+1} + \kappa (\hat{R}_{t+1}^k - \hat{R}_{t+1}) + \nu \hat{\sigma}_{\omega,t}) + (1 - \epsilon_N) (\hat{Y}_t - \hat{X}_t), \tag{F.14}
\]

\[
\hat{\kappa}_t = \hat{K}_t + \hat{Q}_t - \hat{N}_t \tag{F.15}
\]

\[
\hat{C}_{e,t+1} = \hat{N}_t + \hat{R}_{t+1} + \kappa (\hat{R}_{t+1}^k - \hat{R}_{t+1}) + \nu \hat{\sigma}_{\omega,t} \tag{F.16}
\]

\[
\mathbb{E}_t \hat{R}_{t+1}^k - \mathbb{E}_t \hat{R}_{t+1} = \nu \hat{\kappa}_t + \nu \sigma \hat{\sigma}_{\omega,t} \tag{F.17}
\]

Dynamics of the Lending Rate

The BGG lending rate is defined as (F.18a), the MCC lending rate is defined as (F.18b) and the optimal lending rate is defined as (F.18c). These log-linear expressions are derived in Appendix G.

\[
\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \begin{cases} 0 & \text{(F.18a)} \\ \hat{R}_{t+1}^k - \mathbb{E}_t \hat{R}_{t+1}^k - \alpha \sigma (\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) & \text{(F.18b)} \\ \hat{R}_{t+1}^k - \mathbb{E}_t \hat{R}_{t+1}^k - \alpha \left[ \sigma (\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \right] & \text{(F.18c)} \end{cases}
\]

\[
\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1} \left\{ (\kappa - 1)(\hat{R}_{t+2}^k - \hat{R}_{t+2}) + \hat{R}_{t+2}^k + \nu \hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2} \right\} \tag{F.19}
\]

Monitoring Costs

\[
\hat{\phi}_{\mu,t} = \hat{C}_{e,t+1} + \nu \mu \left( \frac{1}{\kappa - 1} \hat{\kappa}_t - (\hat{R}_{k,t+1} - \hat{R}_{t+1}) \right) + \nu \sigma_{\mu} \hat{\sigma}_{\omega,t} \tag{F.20}
\]

G Log-linearization of the Lending Contracts

Log-linearization of the Common Optimality Condition

We begin by log-linearizing the common optimality condition for each contract. The non-linear participation constraint and FOC are, respectively:

\[
\beta \mathbb{E}_t \left\{ U_{C,t+1} \right\} k_t \hat{R}_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - (k_t - 1) U_{C,t} = 0, \tag{G.1}
\]

\[
\kappa_t \mathbb{E}_t \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\mathbb{E}_t \left\{ \frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}. \tag{G.2}
\]
In their linearized form, these become:

\[-\sigma \left( E_t \hat{C}_{t+1} - \hat{C}_t \right) + k^k_t + h_{\omega} \tilde{\omega} \hat{\omega}_{t+1} + h_{\sigma} \omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{G.3} \]

\[\hat{\kappa}_t + E_t \hat{R}^k_{t+1} + \frac{g_{\omega}}{g} E_t \hat{\omega}_{t+1} + \frac{g_{\sigma}}{g} \omega \hat{\sigma}_{\omega,t} = \left( g_{\omega} \frac{h_{\omega}}{g} - h_{\omega} \right) \tilde{\omega} E_t \hat{\omega}_{t+1} + \frac{h_{\omega}}{h_{\omega}} \hat{\omega} E_t \hat{\omega}_{t+1} + \left( \frac{g_{\omega}}{g} - \frac{h_{\omega}}{h_{\omega}} \right) \sigma_{\omega,\omega,\hat{t+1}}. \tag{G.4} \]

Now we take the expected value of the participation constraint (G.3) and obtain:

\[E_t \hat{\Lambda}_{t+1} + E_t \hat{R}^k_{t+1} + h_{\omega} \tilde{\omega} E_t \hat{\omega}_{t+1} + h_{\sigma} \omega \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t. \tag{G.5} \]

Define \( \hat{\Delta}_t = \hat{R}^k_t - \hat{R}_t \), and rewrite the system as:

\[\frac{1}{\kappa - 1} \hat{\kappa}_t - h_{\sigma} \omega \hat{\sigma}_{\omega,t} - E_t \hat{\Delta}_{t+1} = \frac{h_{\omega}}{h} \tilde{\omega} E_t \hat{\omega}_{t+1}, \]

\[\hat{\kappa}_t + E_t \hat{\Delta}_{t+1} = \left( \frac{g_{\omega}}{g} - \frac{h_{\omega}}{h} \right) \sigma_{\omega,\omega,\hat{t+1}} = \left( g_{\omega} \frac{h_{\omega}}{g} - h_{\omega} \right) E_t \hat{\omega}_{t+1} + \left( g_{\omega} \frac{h_{\omega}}{g} - h_{\omega} \right) \omega E_t \hat{\omega}_{t+1}. \tag{G.6} \]

Now we can set these two equations equal to each other and eliminate \( \omega \):

\[E_t \hat{\Delta}_{t+1} = \frac{\left( g_{\omega} \frac{h_{\omega}}{g} - h_{\omega} \right) \sigma_{\omega,\omega,\hat{t+1}}}{\left( g_{\omega} \frac{h_{\omega}}{g} - h_{\omega} \right)} \left( \frac{1}{\kappa - 1} \hat{\kappa}_t - \frac{h_{\sigma} \omega}{h} \hat{\sigma}_{\omega,t} - E_t \hat{\Delta}_{t+1} \right). \tag{G.7} \]

We can rearrange this to obtain:

\[E_t \hat{\Delta}_{t+1} = \frac{1}{\kappa - 1} \hat{\kappa}_t + \left[ \frac{g_{\omega}}{g} - \frac{h_{\omega}}{h} \right] \frac{h_{\omega}}{h} \hat{\omega} E_t \hat{\omega}_{t+1} + \left[ \frac{g_{\omega}}{g} - \frac{h_{\omega}}{h} \right] \omega E_t \hat{\omega}_{t+1}. \tag{G.8} \]

This can be simplified to give the log-linear optimality condition, which is identical for all three lending contracts:

\[E_t \hat{\Delta}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \sigma_{\omega,\omega,\hat{t+1}} \tag{G.9} \]
where

\[
\nu_{\kappa} = \left( \frac{g \omega_{\omega} - h \omega_{\omega}}{g \omega - h \omega} \right) \frac{1}{\kappa - 1},
\]

\[
\nu_{\sigma} = \left[ - \frac{h \omega_{\omega}}{h} \left( \frac{g \omega_{\omega} - h \omega_{\omega} - g \omega}{g} \right) + \frac{h}{h} \left( \frac{g \omega_{\omega} - h \omega_{\omega} - g \omega}{g} \right) \right] \left( \frac{g \omega_{\omega} - h \omega_{\omega} - g \omega}{g} \right).
\]

(G.10)

(G.11)

Log-linearization of the BGG Lending Rate

The log-linear lending rate in BGG is given by:

\[
\hat{R}_{t+1} - E_t \hat{R}_{t+1} = 0 \quad (G.12)
\]

Log-linearization of the MCC Contract

The non-linear participation constraint and FOC are, respectively:

\[
\beta E \left\{ U_{C,t+1} k_t R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k_t - 1) U_{C,t} = 0,
\]

\[
\kappa_t E \left\{ R_{t+1}^k g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = - \left( \frac{g'(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{h'(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right) \Lambda_{t,t+1}.
\]

(G.13)

(G.14)

In their linearized form, these become:

\[
E_t \hat{R}_{t+1}^k - E_t \hat{R}_{t+1} + \frac{h \omega_{\omega}}{h} \bar{\omega} E_t \hat{\omega}_{t+1} + \frac{h}{h} \sigma_{\omega} \hat{\sigma}_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t,
\]

\[
\hat{\kappa}_t + E_t \hat{R}_{t+1}^k + \frac{g \omega}{g} \bar{\omega} E_t \hat{\omega}_{t+1} + \frac{g \sigma_{\omega}}{g} \sigma_{\omega} \hat{\sigma}_{\omega,t} = - \hat{\Lambda}_{t,t+1} + \left( \frac{g \omega_{\omega}}{g \omega} - \frac{h \omega_{\omega}}{h \omega} \right) \bar{\omega}_{t+1} + \left( \frac{g \sigma_{\omega}}{g \omega} - \frac{h \omega_{\omega}}{h \omega} \right) \sigma_{\omega} \hat{\sigma}_{\omega,t}.
\]

(G.15)

(G.16)

Now we plug the participation constraint (G.15) into the FOC (G.16) and obtain

\[
\hat{\kappa}_t + E_t \hat{R}_{t+1}^k + \frac{g \omega}{h} \left( \frac{1}{\kappa - 1} \hat{\kappa}_t - E_t \hat{R}_{t+1}^k + E_t \hat{R}_{t+1} - \frac{h \sigma_{\omega}}{h} \sigma_{\omega} \hat{\sigma}_{\omega,t} \right) + \frac{g \sigma_{\omega}}{g} \sigma_{\omega} \hat{\sigma}_{\omega,t} =
\]

\[
- \hat{\Lambda}_{t,t+1} + \left( \frac{g \omega_{\omega}}{g \omega} - \frac{h \omega_{\omega}}{h \omega} \right) \bar{\omega}_{t+1} + \left( \frac{g \sigma_{\omega}}{g \omega} - \frac{h \omega_{\omega}}{h \omega} \right) \sigma_{\omega} \hat{\sigma}_{\omega,t}.
\]

(G.17)

Using \( \frac{\omega}{h} = (k - 1) \), we can simplify the last expression to:

\[
k(E_t \hat{R}_{t+1}^k - E_t \hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - E_t \hat{\Lambda}_{t,t+1} + \left( - \frac{g \sigma_{\omega}}{g h \omega} + \frac{g \sigma_{\omega}}{g \omega} - \frac{g \sigma_{\omega}}{g \omega} + \frac{h \omega}{h \omega} \right) \sigma_{\omega} \hat{\sigma}_{\omega,t} = \left( \frac{g \omega_{\omega}}{g \omega} - \frac{h \omega_{\omega}}{h \omega} \right) \bar{\omega}_{t+1}.
\]

(G.18)
Define lender’s returns by log-linearizing $R_{t+1} = \frac{\kappa \sigma_t}{\kappa t} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) R^k_{t+1}$:

$$\hat{R}_{t+1} = -\frac{1}{k - 1} \hat{k}_t + \frac{h_{\omega}}{h} \tilde{\omega}_{t+1} + \frac{h_{\sigma_{\omega}}}{h} \sigma_{\omega,t} + \hat{R}^k_{t+1}. \quad (G.19)$$

Now substitute in the expression for the cutoff and obtain

$$\hat{R}_{t+1} = \frac{h_{\omega}}{g_{\omega}} - \frac{h_{\omega}}{h_{\omega}} \left[ k(\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \left( -\frac{g_{\omega} h_{\sigma_{\omega}}}{g h_{\omega}} + \frac{g_{\sigma_{\omega}}}{g} - \frac{g_{\omega}}{g} + \frac{h_{\omega}}{h_{\omega}} \right) \sigma_{\omega} \hat{\sigma}_{\omega,t} \right]$$

$$- \frac{1}{k - 1} \hat{k}_t + \frac{h_{\sigma_{\omega}}}{h} \sigma_{\omega} \hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \quad (G.20)$$

Finally, substitute in the expression for leverage:

$$\hat{R}_{t+1} = \frac{h_{\omega}}{g_{\omega}} - \frac{h_{\omega}}{h_{\omega}} \left[ \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \left( -\frac{g_{\omega} h_{\sigma_{\omega}}}{g h_{\omega}} + \frac{g_{\sigma_{\omega}}}{g} - \frac{g_{\omega}}{g} + \frac{h_{\omega}}{h_{\omega}} \right) \sigma_{\omega} \hat{\sigma}_{\omega,t} \right]$$

$$+ \left[ -\frac{h_{\omega}}{h} \left( \frac{g_{\omega}}{g_{\omega}} - \frac{h_{\omega}}{h_{\omega}} - \frac{g_{\omega}}{g} \right) + \frac{h_{\omega}}{h} \left( \frac{g_{\omega}}{g_{\omega}} - \frac{h_{\omega}}{h_{\omega}} - \frac{g_{\omega}}{g} \right) \right] \sigma_{\omega} \hat{\sigma}_{\omega,t}$$

$$+ \frac{h_{\sigma_{\omega}}}{h} \sigma_{\omega} \hat{\sigma}_{\omega,t} + (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{R}^k_{t+1}. \quad (G.21)$$

After some rearranging and canceling out like terms, we obtain the log-linear MCC lending rate:

$$\hat{R}_{t+1} = \frac{h_{\omega}}{g_{\omega}} - \frac{h_{\omega}}{h_{\omega}} \left[ \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} \right] - (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}) + \hat{R}^k_{t+1}. \quad (G.22)$$

This can be rewritten in the same form as Corollary 2 in the text:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_{t+1} \hat{R}^k_{t+1} - \tilde{\alpha} \sigma \hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1} \quad (G.23)$$

where $\tilde{\alpha} = \frac{h_{\omega}}{h_{\omega} - \frac{g_{\omega}}{k - 1}}$.

Log-linearization of the Optimal Contract

The non-linear participation constraint and FOC are, respectively:

$$\beta \mathbb{E}_t \left\{ U_{C,t+1} k_t \hat{R}^k_{t+1} h(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right\} - (k - 1) U_{C,t} = 0 \quad (G.24)$$

$$\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R^k_{t+1} g(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = -\frac{g'(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \Psi_{t+1} h'(\tilde{\omega}_{t+1}, \sigma_{\omega,t}) \Lambda_{t,t+1}}{h(\tilde{\omega}_{t+1}, \sigma_{\omega,t})} \hat{\Lambda}_{t,t+1} \quad (G.25)$$


where \( \Psi_t \) is defined as:

\[
\Psi_t = 1 + \kappa_t \mathbb{E}_t \{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{k,t+1} \Psi_{t+1} \}. 
\]

In their linearized form, the participation constraint and FOC become:

\[
- \sigma \left( \mathbb{E}_t \hat{C}_{t+1} - \hat{C}_t \right) + \mathbb{E}_t \hat{R}_t^k = \frac{h_\omega}{h} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{h_\sigma}{h} \sigma_{\omega,t} = \frac{1}{\kappa - 1} \hat{\kappa}_t, \tag{G.27}
\]

\[
\mathbb{E}_t \Psi_{t+1} + \hat{\kappa}_t + \mathbb{E}_t R_{t+1}^k + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_\sigma}{g} \sigma_{\omega,t} = \hat{\Psi}_{t+1} - \hat{\Lambda}_{t+1} + \left( \frac{g_\omega}{g} - \frac{h_\omega}{h} \right) \bar{\omega} \hat{\omega}_{t+1}
\]

\[
+ \left( \frac{g_\sigma}{g} - \frac{h_\sigma}{h} \right) \sigma_{\omega,t}. \tag{G.28}
\]

where

\[
\hat{\Psi}_t = \epsilon_N \left( \hat{\kappa}_t + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_\sigma}{g} \sigma_{\omega,t} + \mathbb{E}_t \hat{R}_t^k + \mathbb{E}_t \hat{\Psi}_{t+1} \right). \tag{G.29}
\]

Now we substitute the participation constraint into the optimality condition and obtain:

\[
\hat{\kappa}_t + \mathbb{E}_t R_{t+1}^k + \frac{g_\omega}{g} \bar{\omega} \mathbb{E}_t \hat{\omega}_{t+1} + \frac{g_\sigma}{g} \sigma_{\omega,t} = \hat{\Psi}_{t+1} - \hat{\Lambda}_{t+1} + \left( \frac{g_\omega}{g} - \frac{h_\omega}{h} \right) \bar{\omega} \hat{\omega}_{t+1}
\]

\[
+ \left( \frac{g_\sigma}{g} - \frac{h_\sigma}{h} \right) \sigma_{\omega,t} + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}. \tag{G.30}
\]

Using \( - \frac{g_\sigma}{g} = (k - 1) \), we can simplify the previous expression to

\[
\left( \frac{g_\omega}{g} - \frac{h_\omega}{h} \right) \bar{\omega} \hat{\omega}_{t+1} = k (\mathbb{E}_t \hat{R}_t^k - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t+1} - \mathbb{E}_t \hat{\Lambda}_{t+1}
\]

\[
+ \left( - \frac{g_\omega h_{\sigma}}{gh_{\omega}} + \frac{g_\sigma}{g} - \frac{h_\sigma}{h} \right) \sigma_{\omega,t} \sigma_{\omega,t} - (\hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}). \tag{G.31}
\]

Define the lender’s returns by log-linearizing \( R_{t+1} = \frac{\kappa_t}{\kappa_t - 1} h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{t+1}^k \):

\[
\hat{R}_{t+1} = - \frac{1}{k - 1} \hat{\kappa}_t + \frac{h_\omega}{h} \bar{\omega} \hat{\omega}_{t+1} + \frac{h_\sigma}{h} \sigma_{\omega,t} \sigma_{\omega,t} + \hat{R}_{t+1}^k. \tag{G.32}
\]

Now substitute in the expression for the cutoff and obtain:

\[
\hat{R}_{t+1} = \left( \frac{h_\omega}{g_\omega} - \frac{h_\omega}{h} \right) \left[ k (\mathbb{E}_t \hat{R}_t^k - \mathbb{E}_t \hat{R}_{t+1}) + \hat{\Lambda}_{t+1} - \mathbb{E}_t \hat{\Lambda}_{t+1} + \left( - \frac{g_\omega h_{\sigma}}{gh_{\omega}} + \frac{g_\sigma}{g} - \frac{h_\sigma}{h} \right) \sigma_{\omega,t} \sigma_{\omega,t}
\]

\[
- (\hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}) \right] - \frac{1}{k - 1} \hat{\kappa}_t + \frac{h_\omega}{h} \sigma_{\omega,t} \sigma_{\omega,t} + \hat{R}_{t+1}^k. \tag{G.33}
\]
Rearranging and canceling out like terms yields:

\[
\hat{R}_{t+1} = \frac{h_\omega}{g_\omega} \left[ \hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1} - (\hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}) \right] + \hat{R}^k_{t+1} - (\mathbb{E}_t \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}_{t+1}). \tag{G.34}
\]

This can be rewritten in the same form as Corollary 3 in the text:

\[
\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}^k_{t+1} - \mathbb{E}_t \hat{R}^k_{t+1} - \tilde{\alpha} \left[ \sigma (\hat{C}_{t+1} - \mathbb{E}_t \hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \right]. \tag{G.35}
\]

where \(\tilde{\alpha} = \frac{h_\omega}{g_\omega - h_\omega}\). Finally, we eliminate \(\omega\) in the expression for \(\hat{\Psi}_t\):

\[
\hat{\Psi}_t = \epsilon_N \left[ \frac{h_\omega}{h_\omega} \left( \frac{1}{\kappa - 1} \hat{\kappa}_t - \mathbb{E}_t \hat{R}^k_{t+1} + \mathbb{E}_t \hat{R}_{t+1} - \frac{h_\sigma}{h_\omega} \sigma_\omega \hat{\sigma}_{\omega,t} \right) + \frac{g_\sigma}{g_\omega} \sigma_\omega \hat{\sigma}_{\omega,t} + \mathbb{E}_t \hat{R}^k_{t+1} \right] \tag{G.36}
\]

and we can rearrange this expression to match Corollary 3 in the text:

\[
\hat{\Psi}_{t+1} = \epsilon_N \mathbb{E}_{t+1} \left\{ \frac{h_\omega}{h_\omega} \left[ \left( \hat{R}^k_{t+2} - \hat{R}_{t+2} \right) + \hat{R}^k_{t+2} + \nu_\Psi \sigma_{\omega,t+1} + \hat{\Psi}_{t+2} \right] \right\} \tag{G.37}
\]

where \(\nu_\Psi = \frac{g_\sigma - h_\omega}{g_\omega} \frac{\sigma_{\omega,t}}{\sigma_\omega}\).