Collateral Constraints and State-Contingent Contracts

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It is commonly assumed that binding collateral constraints amplify the impact of aggregate shocks on the economy. However, we show that when firms can hedge against aggregate risk with state-contingent lending contracts, binding collateral constraints no longer amplify shocks relative to the basic New Keynesian model. We embed state-contingent lending contracts in a quantitative business cycle model in the spirit of Kiyotaki and Moore (1997) and Iacoviello (2005) and find that in general equilibrium unconstrained lenders sell insurance against aggregate risk to constrained borrowers. The provision of insurance against aggregate risk prevents the usual tightening of collateral constraints during downturns and leads to relatively mild recessions.

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1 Introduction

In a seminal paper, Kiyotaki and Moore (1997) show that collateral constraints can amplify business cycle fluctuations. In their framework, negative shocks lead to a decline in the price of collateral, causing credit constraints to tighten. Tighter credit constraints exacerbate the initial effect of the negative shock on the economy, amplifying the impact of the shock and strengthening the resulting recession.

In spite of the success of the collateral amplification mechanism, there is ongoing debate about its robustness to different types of lending contracts. For example, Krishnamurthy (2003) finds that amplification fails when agents can insure themselves against aggregate shocks in a stripped down two-period version of Kiyotaki and Moore (1997). Does Krishnamurthy’s result hold in a richer, infinite horizon model? We set out to answer this question by examining the robustness of the collateral amplification mechanism in a quantitative business cycle model that closely follows Iacoviello (2005).

We introduce optimal state-contingent lending contracts to the model and find that the collateral amplification mechanism is not robust. Access to state-contingent loans allows borrowers to pledge more resources in good states of the world and fewer resources in bad states of the world, as collateral values covary positively with aggregate macroeconomic conditions. Because they are constrained, entrepreneurs borrow as much as possible and pledge more collateral in good states and less in bad states. The procyclical behavior of loan repayments under the optimal state-contingent contract has a stabilizing influence on business cycle fluctuations and dampens the response of output, consumption and investment to technology and monetary shocks. We thus demonstrate that collateral constraints in and of themselves do not amplify business cycle fluctuations.\footnote{In particular, we find that when mortgage contracts are fully indexed to house prices, amplification disappears. A number of mortgage contracts with varying degrees of insurance are proposed by Shiller and Weiss (1999).} Rather, it is a combination of fixed interest rate contracts and collateral constraints that provides amplification. Relative to the standard New Keynesian model without collateral constraints, the optimal state-contingent contract does not generate amplification.

The debate on the robustness of the collateral amplification mechanism is also relevant for other financial accelerator mechanisms. House (2006) points out that optimal state-contingent lending contracts annihilate the financial accelerator in an overlapping generations model with adverse selection. And recent work by Dmitriev and Hoddenbagh (2013) finds the financial accelerator in the costly state verification framework popularized by Bernanke, Gertler and Gilchrist (1999) is not robust to the introduction of state-contingent lending contracts.\footnote{There is another key element introduced by Dmitriev and Hoddenbagh (2013): forward looking en-} In all of these diverse models of financial frictions, the key assumption driving...
amplification is the absence of insurance against aggregate risk. Relative to the financial accelerator in the costly state verification framework, the advantage of testing the collateral amplification mechanism is the ability to look at a fully dynamic contract between infinitely lived agents instead of a statically optimal contract. Our results also have implications for the literature on costly state enforcement frictions by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), where collateral constraints and non-contingent contracts amplify shocks.

Taken together with the results on the role of state-contingent lending contracts and financial accelerator mechanisms in a number of other models, this paper confirms that financial frictions resulting from moral hazard or aggregate risk cause amplification only in combination with non-contingent lending instruments.

2 Credit Frictions in Partial Equilibrium

We begin by describing the credit friction in partial equilibrium. We have a representative unconstrained lender (the household), and a constrained borrower (the entrepreneur). The household lends money to the entrepreneur for one period and the entrepreneur invests this money in capital, consumption and housing.

Following Kiyotaki and Moore (1997) and Iacoviello (2005), we assume that the borrower can run away with a fraction of housing assets $1 - m$ in each period. In other words, only a fraction of assets $m$ is pledgeable, so that repayments in each state of the world should not exceed the fraction of assets $m$. For the time being we will assume that the entrepreneur has very profitable investment opportunities (we will later prove that this is indeed the case) and would therefore like to borrow as much as possible in each state $s_t$, leading to a binding borrowing constraint in each period. The borrowing constraint can be written as:

$$ b_t r_{t+1} = m q_{t+1} h_t $$

where $b_t$ is the amount borrowed, $r_{t+1} = R_t/P_t$ is the real interest rate, $R_t$ is the nominal interest rate, $P_t$ is the price index, $m$ is a parameter defining the fraction of pledgeable assets, $q_{t+1}$ is the price of housing and $h_t$ is the quantity of housing. The Euler equation for patient households in terms of the real interest rate is:

$$ 1 = E_t \left\{ r_{t+1} \Lambda_{t,t+1} \right\} $$

where the stochastic discount factor is defined as $\Lambda_{t,t+1} = \beta E_{t+1} U_{t+1}/U_{t+1}$. We can multiply the right
and left hand side of equation (2) by $b_t$ and obtain

$$b_t = \mathbb{E}_t \left\{ b_t r_{t+1} \Lambda_{t,t+1} \right\}. \quad (3)$$

Now we simply substitute our expression for $b_t r_{t+1}$ from (1) into (3) and obtain

$$b_t = m h_t \mathbb{E}_t \left\{ q_{t+1} \Lambda_{t,t+1} \right\}. \quad (4)$$

Rearranging, we find the optimal state-contingent real interest rate:

$$r_{t+1} = \frac{b_t r_{t+1}}{b_t} = \frac{m q_{t+1} h_t}{m h_t \mathbb{E}_t \left\{ q_{t+1} \Lambda_{t,t+1} \right\}} = \frac{q_{t+1}}{\mathbb{E}_t \left\{ q_{t+1} \Lambda_{t,t+1} \right\}}. \quad (5)$$

This contract, which we call the optimal state-contingent lending contract, has not previously been studied in the context of the Kiyotaki-Moore credit market friction.

For the sake of comparison, we also consider two standard contracts used in Iacoviello (2005). In the first contract, the interest rate charged on loans is predetermined. That is, the lending rate is non-contingent on the aggregate state of the economy and is set in period $t$ for payment in $t + 1$, such that the real interest rate is fixed:

$$\mathbb{E}_t r_{t+1} = r_{t+1}. \quad (6)$$

We also consider a nominal fixed interest rate, which could be viewed as an indexed contract:

$$r_{t+1} = \frac{R_{t+1}}{\pi_{t+1}}, \quad (7)$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate.

3 Credit Frictions in General Equilibrium

We now embed the three loan contracts in a dynamic New Keynesian model. The model consists of patient households who provide savings (which captures the assumption that households have lower discount rates than firms), entrepreneurs who borrow money from households and invest in projects, retailers who are the source of nominal rigidities, and central banks. Households work for entrepreneurs, who produce intermediate goods that require both labor and housing in the production process. Retailers bundle together the intermediate goods into a final consumption good.
Patient Households

Patient households maximize their lifetime expected utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \left\{ \beta^t \left( \ln c_t' + j \ln h_t' - (L_t')^{\eta/\eta} + \chi \ln (M_t'/P_t) \right) \right\}$$

(8)

where $\beta$ is the household discount factor, $c_t'$ is household consumption, $h_t'$ is housing, $L_t'$ denotes hours worked and $M_t'/P_t$ denotes real money balances. The household budget constraint is as follows:

$$c_t' + q_t \Delta h_t' + R_{t-1} b_{t-1}'/\pi_t = b_t' + w_t' L_t' + F_t + T_t' - \Delta M_t'/P_t$$

(9)

where $r$ is the first difference operator, $q_t = Q_t/P_t$ denotes the real price of housing, $b_t' \equiv B_t'/P_t$ is the real amount of borrowing, $R_{t-1}$ is the nominal interest paid on loans between $t-1$ and $t$, $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, $w_t' \equiv W_t'/P_t$ is the household real wage, $F_t$ are lump-sum profits received from the retailer, and $(T_t' - \Delta M_t'/P_t)$ are central bank transfers to household resulting from money printing. Households maximize their utility (8) subject to the budget constraint (9), choosing consumption, loans, hours worked and housing, leading to the following first order conditions:

$$\frac{1}{c_t'} = \beta R_{t+1} \mathbb{E}_t \left\{ \frac{1}{c_{t+1}' \pi_{t+1}} \right\},$$

(10)

$$w_t' = (L_t')^{\eta-1} c_t'$$

(11)

$$q_t = \frac{j}{h_t'} + \beta \mathbb{E}_t \left\{ \frac{q_{t+1}}{c_{t+1}'} \right\}.$$  

(12)

These three equations yield the household consumption Euler condition, labor supply and housing demand.

Entrepreneurs

Entrepreneurs produce intermediate goods using a Cobb-Douglas production function composed of technology $A$, labor $L$ and real estate $h$:

$$Y_t = Ah_t^\nu L_t^{1-\nu}.$$  

(13)

As in Bernanke, Gertler and Gilchrist (1999), we assume that retailers buy intermediate goods at the wholesale price $P_t^w$ and bundle them together in constant elasticity of substitution fashion into a final consumption good with price index $P_t$. We follow Iacoviello (2005) and define $X_t \equiv P_t/P_t^w$ as the markup of final over intermediate goods. The entrepreneur’s
expected lifetime utility is given by

$$E_t \sum_{t=0}^{\infty} \gamma^t \ln c_t$$

(14)

where $c_t$ is entrepreneurial consumption and $\gamma > \beta$ so that entrepreneurs borrow money from patient households because they have a higher discount factor. The budget constraint for entrepreneurs is as follows:

$$Y_t / X_t + b_t = c_t + q_t \Delta h_t + R_t b_{t-1} / \pi_t + w'_t L_t.$$  

(15)

Entrepreneurs maximize their expected utility (14) subject to the budget constraint (15). The first order conditions to this optimization problem with respect to loans, housing and labor are given by:

$$\frac{1}{c_t} = \gamma R_{t+1} E_t \left\{ \frac{1}{c_{t+1} \pi_{t+1}} \right\} + \lambda_t R_{t+1}$$

(16)

$$\frac{q_t}{c_t} = E_t \left\{ \frac{\gamma}{c_{t+1}} \left[ \nu \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1} \right] + m \lambda_t q_{t+1} \pi_{t+1} \right\}$$

(17)

$$w'_t = (1 - \nu) \frac{Y_t}{X_t L_t}$$

(18)

These three equations yield the entrepreneur’s consumption Euler condition (16), housing demand (17), and labor demand (18).

**Retailers**

Sticky prices are introduced via the inclusion of a retail sector. Each retailer $z$ purchases intermediate goods $Y_t$ from entrepreneurs at the wholesale price $P^w_t$ and then costlessly differentiates these goods into $Y_t(z)$ and sell $Y_t(z)$ at the price of $P_t(z)$. Final goods are then aggregated in CES fashion according to the function $Y^f_t = \left( \int_0^1 Y_t(z)^{\frac{\varepsilon}{\varepsilon-1}} dz \right)$ where $\varepsilon > 1$. The price index will be $P_t = \left( P_t^{1-\varepsilon} (z) dz \right)^{\frac{1}{1-\varepsilon}}$. The optimal reset price $P^*_t(z)$ is given by the solution to the following equation:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \left[ \frac{P^*_t(z)}{P_{t+k}} - \frac{X}{X_{t+k}} \right] Y^*_{t+k}(z) \right\} = 0$$

(19)
where $X_t$ is the markup, and $X = (\varepsilon - 1)/\varepsilon$ is the steady state markup. The aggregate price level is defined by

$$P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta)(P^*_t)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{20}$$

where $1 - \theta$ is the probability that firms are able to reset their prices in a given period.

**Monetary Policy**

The central bank conducts monetary policy through a Taylor Rule of the following type

$$\log(R_t) - \log(R) = \rho^R \left( \log(R_{t-1}) - \log(R) \right) + \rho^\pi \pi_t + \rho^Y \left( \log(Y_t) - \log(Y_{t-1}) \right) + \epsilon_t^R. \tag{21}$$

where $R$ is the steady state nominal interest rate, $\rho^R$ and $\rho^\pi$ determine the relative importance of the past interest rate and past inflation in the central bank’s interest rate rule. Shocks to the nominal interest rate are given by $\epsilon^R$.

**Real Interest Rate**

The Euler equation for patient households in real terms is

$$\frac{1}{c_t'} = \beta \mathbb{E}_t \left\{ r_t^{t+1} \frac{c_{t+1}'}{c_{t+1}} \right\}. \tag{22}$$

Using this Euler equation, we solve for three different Fisher equations for the real interest rate in Section 2: a state-contingent real interest rate (Equation 5), a fixed real interest rate (Equation (6)) and a fixed nominal interest rate (Equation 7). We then embed these three different contracts into the general equilibrium setting to study the collateral constraint amplification mechanism in a quantitative business cycle framework.

**Market Clearing**

Finally, we have market clearing in the goods, labor, housing and loan markets:

$$Y_t = c_t + c'_t, \tag{23}$$

$$L_t = L'_t, \tag{24}$$

$$H_t = h_t + h'_t, \tag{25}$$

$$b_t + b'_t = 0. \tag{26}$$
Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) process for technology is given by

\[ \log(A_t) = \rho A \log(A_{t-1}) + \epsilon_t^A, \tag{27} \]

where \( \epsilon^A \) denotes an exogenous shock to technology. Nominal interest rate shocks are defined by the Taylor Rule in (21).

Equilibrium

The model has 15 endogenous variables and 15 equations. The endogenous variables are: \( c, c', L, L', Y, Y', b, b', w', R, P, P^*, X, \lambda, \) and \( q \). The equations defining these endogenous variables are (10), (11), (12), (13), (16), (17), (18), (19), (20), (21), (23), (24), (25) and (26), and the equation defining the real interest rate is given by (5), (6) or (7). The exogenous process for technology follows (27). Nominal interest rate shocks are defined by the Taylor rule in (21).

4 Simulations

Our baseline calibration largely follows Iacoviello (2005) and can be found in Table 1. We set the discount factor for households \( \beta = 0.99 \), the risk aversion parameter \( \sigma = 1 \) so that utility is logarithmic in consumption and the elasticity of labor \( \eta = 1.01 \). Housing enters the utility function with weight \( j = 0.1 \). Entrepreneurs have a discount factor \( \gamma = 0.98 \). Again, note that \( \beta > \gamma \) to ensure a binding collateral constraint. The share of housing in the Cobb-Douglas production function is \( \nu = 0.03 \), while the loan-to-value ratio for entrepreneurs is 0.89.

For price-setting, we assume the Calvo parameter \( \theta = 0.75 \), so that only 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the Iacoviello (2005) monetary policy rule and set the autoregressive parameter on the nominal interest rate to \( \rho^R = 0.73 \) and the parameter on current inflation to \( \rho^\pi = 0.27 \). We set the persistence of shocks to technology \( \rho^A = 0.95 \). In our quantitative analysis we compare two allocations: the competitive equilibrium under the debt contract with a predetermined lending rate and the competitive equilibrium under the state contingent lending rate.

Figure 1 shows impulse responses for a one percent technology shock. Under the fixed rate contract (labeled as “Predetermined Real Rate”) real returns do not react to the shock. Borrowers can pledge more collateral during a boom as their collateral is now more valuable. The wealth of borrowers also increases substantially, and being impatient they expend as
many resources as possible to buy more housing and consume while times are good. Therefore, consumption and investment climb dramatically under the fixed rate contract. On the other hand, under the optimal contract (labeled as “Optimal Indexation”) borrowers are only slightly better off, as higher returns from production are offset by the higher interest rate they must pay to lenders. This redistribution of resources dampens the response of output, consumption and investment to the initial shock. Under the optimal state-contingent contract a small welfare redistribution leads to lower inflation, lower output, less lending and lower entrepreneurial housing than the response of these variables under the fixed rate contract.

Figure 2 plots impulse responses for a 25 basis point (annualized) decline in the nominal interest rate. The positive monetary shock increases the inflation rate, along with output, lending and the housing price. The response of these variables is much stronger under the fixed rate contract than the optimal state-contingent contract. As in the case of a technology shock, the entrepreneurial share of the housing market does not react to a monetary shock. Again, the key difference between the two lending contracts is illustrated through the real returns on the loan. In the case of the fixed rate contract, real returns do not react to the shock on impact, so that housing prices and quantities increase substantially. As households feel much wealthier, they borrow much more to finance further consumption and investment, leading to a large positive impact on output. This effect is significantly dampened under the optimal state-contingent contract, where real returns increase on impact, forcing borrowers to repay higher amounts and preventing a large increase in house prices and borrowing.

Figure 3 compares the output response of the fixed rate contract and the optimal state-contingent contract against a frictionless benchmark. As our frictionless benchmark, we take the share of entrepreneurs in the economy to zero and set the collateral constraint coefficient $m$ to infinity, so that the model converges to the standard New Keynesian framework with only patient households. Relative to the frictionless benchmark, the fixed rate contract generates much larger amplification for both technology and monetary shocks. In contrast, the optimal state-contingent contract generates a similar amount of amplification to the frictionless benchmark, and amplification is even smaller in some periods following the shock. The stabilizing influence of the optimal state-contingent lending contract completely eliminates the collateral amplification mechanism.

5 Conclusion

This paper contributes to the literature on financial frictions in macroeconomics by introducing state-contingent lending contracts to the collateral constraint framework of Kiyotaki and Moore (1997). In their framework, a negative shock leads to a feedback loop of deteriorating collateral values and tighter credit constraints. We show that two key assumptions drive this collateral amplification mechanism. First, the interest rate on loans is non-contingent
and does not vary with the aggregate state of the economy. Second, borrowers do not have access to secondary financial markets where they can trade aggregate risk. We relax both assumptions and find no amplification relative to the basic New Keynesian model, even in the presence of collateral constraints. Our results confirm Krishnamurthy’s (2003) findings in a fully dynamic, quantitative business cycle framework, and also match the ability of state-contingent lending contracts to remove the amplifying property of financial accelerator mechanisms in models of adverse selection (House (2006)) and costly state verification (Dmitriev and Hoddenbagh (2013)).
References


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Figure 1: Technology Shock
Figure 2: Monetary Shock
Figure 3: Amplification Relative to a Frictionless Benchmark