Price Stability in Small Open Economies*

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First Draft: November 2012 This Version: April 2014

We study the conduct of monetary policy in a continuum of small open economies and obtain a novel closed-form solution that does not restrict the elasticity of substitution between home and foreign goods to one. We give an exact characterization of optimal monetary policy and welfare with and without international policy cooperation, under internationally complete asset markets and financial autarky, for producer currency pricing and local currency pricing. The tractability of our model allows us to parse out the effect of each individual ingredient on optimal policy. Under producer currency pricing we prove that price stability is optimal, while under local currency pricing policy should fix the exchange rate. In the absence of consumption home bias we find that non-unitary elasticity, imperfect risk-sharing, and non-cooperative monetary policy in and of themselves do not lead to deviations from these benchmark policies.

Keywords: Open economy macroeconomics; Optimal monetary policy; Price stability.

JEL Classification Numbers: E50, F41, F42.

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*This is a substantially revised and updated edition of the June 2013 version of the paper. This work grew out of our shared experience in the International Macroeconomics course taught by Fabio Ghironi, to whom we owe a great debt of gratitude. We thank Susanto Basu, Pierpaolo Benigno, Giancarlo Corsetti, Eyal Dvir, Peter Ireland and two anonymous referees for very helpful comments, as well as seminar participants at Boston College, the BC/BU Green Line Macro Meeting, the Eastern Economic Association Meeting and the Canadian Economic Association Meeting. Any errors are our own.

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1 Introduction

Price stability is widely viewed as a benchmark monetary policy for central banks and is optimal in many closed economy studies.\(^1\) However, various ingredients in the open economy drive optimal policy away from price stability. These ingredients include the well-known terms of trade externality that arises from non-cooperative monetary policy (Benigno and Benigno (2003), Corsetti and Pesenti (2001) and Corsetti, Dedola and Leduc (2010)), incomplete international risk-sharing (Corsetti, Dedola and Leduc (2010) and De Paoli (2009a)) and deviations from purchasing power parity resulting from incomplete exchange rate pass-through (Corsetti and Pesenti (2005), Corsetti, Dedola and Leduc (2010), Devereux and Engel (2003), and Engel (2011)).

In this paper, we attempt to disentangle the impact of these key ingredients on the design of optimal monetary policy in small open economies. We employ a novel global closed-form solution that gives an exact characterization of optimal monetary policy and welfare with and without international policy cooperation, under internationally complete asset markets and financial autarky, for producer currency pricing (PCP) and local currency pricing (LCP). Out of the eight cases we examine, six have not previously been studied in the literature: cooperative policy under PCP in both complete markets and financial autarky and cooperative and non-cooperative policy under LCP in both complete markets and financial autarky.

We demonstrate that under PCP, it is optimal for central banks to mimic the flexible price equilibrium through a policy of price stability, while under LCP policy should fix the exchange rate. We show that in the absence of home bias in consumption these results hold irrespective of the degree of monetary policy cooperation or the economy’s asset market structure. Thus, terms of trade externalities or incomplete international risk-sharing are not sufficient in and of themselves to push optimal policy away from price stability under PCP or a fixed exchange rate under LCP. We explain the intuition for each separate case below.

Contributions to Cooperative Policy Under Producer Currency Pricing

We are the first to study cooperative monetary policy under producer currency pricing for small open economies in both complete markets and financial autarky. Under cooperative policy, the monetary authority does not exploit its country level monopoly power to impose a markup on its terms of trade. In the case of internationally complete asset markets, risk-sharing is provided by trade in cross-country contingent securities, so the monetary authority does not need to improve risk-sharing across countries; indeed, it cannot do so. Thus, under cooperative policy in complete markets, we demonstrate that it is optimal for small open economy central banks to employ a policy of price stability, which matches the results for large open economies

\(^1\)In this paper, we refer to price stability as the policy which implements the flexible price allocation. A non-exhaustive list of papers finding price stability to be optimal in closed economies includes Goodfriend and King (1997, 2001), King and Wolman (1998), and Woodford (2000, 2002). See Schmitt-Grohé and Uribe (2011) for a recent survey. A counterexample is provided by multi-sector models such as that in Aoki (2001).
(Benigno and Benigno (2006), Corsetti, Dedola and Leduc (2010)). In the absence of steady state markups (which can be offset via government production subsidies), the flexible price allocation will replicate the Pareto efficient allocation.

In the case of financial autarky, asset markets no longer provide any risk-sharing across countries, so there may be a role for monetary policy to do so. Indeed, this is the case for large open economies, where a global central bank will deviate from price stability when markets are not complete in order to better share risk across countries (Corsetti, Dedola and Leduc (2010)). However, we find that it is not possible for small open economy central banks to improve cross-country risk-sharing, so that even in financial autarky policymakers find it optimal to implement the flexible price allocation through a policy of price stability. This is a novel result that should be explored further in future research: we believe it will prove robust to alternative preference specifications (including home bias in consumption).

To understand the intuition, consider the following example. Suppose the world is composed of two large open economies: one country is hit with a positive technology shock, and one country is hit with a negative technology shock. Under monetary policy cooperation, the central bank in the boom country will deflate its price level to depress demand for its exports, and the central bank in the recession-hit country will inflate its price level to boost demand for its exports. In this way, output (and consumption) will be stabilized even though the two economies are in financial autarky: output will fall in the boom country and rise in the recession-hit country.

On the other hand, small open economies are unable to improve risk-sharing across countries through monetary policy. Suppose we have a continuum of small open economies, half of which are hit with a positive technology shock and half of which are hit with a negative technology shock. The exports of each country depend only on aggregate world demand because each economy is measure zero. If countries hit with a positive shock deflate to decrease demand for their exports and those hit with a negative shock inflate to increase demand for their exports, overall global demand for each country’s exports will remain unaffected (with half of the continuum inflating and half deflating, there will be no discernable impact on demand for an individual country’s exports). Thus, there is no stimulus at the global level, only an inefficient consumption-leisure tradeoff resulting from the monetary authority’s actions in each country.

From a policy perspective, this is quite important as the vast majority of countries in the world are small open economies. Our results demonstrate that, for small open economies, risk-sharing cannot be improved via policy cooperation but only through trade in international financial instruments that provide insurance against downside consumption risk.

**Contributions to Non-Cooperative Policy Under Producer Currency Pricing**

We now move to the non-cooperative equilibria under PCP, where monetary authorities have an incentive to exploit their monopoly power at the country level through the imposition of
terms of trade markups, introducing the aforementioned terms of trade externality. A number of papers examine the role of non-cooperative policy under PCP. Our aim here is to disentangle the effects of individual ingredients on optimal policy and compare the cooperative and non-cooperative allocations.

In the literature on non-cooperative policy under PCP in complete markets, deviations from price stability are optimal (De Paoli (2009b), Faia and Monacelli (2008) and Gali and Monacelli (2005) for small open economy central banks; Benigno and Benigno (2003) and Corsetti, Dedola and Leduc (2010) for large open economy central banks). However, we show that in the absence of home bias, it is optimal for non-cooperative small open economy central banks to mimic the flexible price allocation via a policy of price stability, a result also shown in Faia and Monacelli (2008).

Why does home bias in consumption or large country size cause deviations from the optimality of price stability? Optimal monetary policy should minimize the distance between the desired markup and the actual markup. If the desired markup is fixed, then there is no role for monetary policy to vary over the business cycle, regardless of all other conditions. For small open economies without home bias, the desired markup is fixed because all production is exported. World demand is taken as given so that changes in terms of trade volatility translate into proportional changes in output volatility, which are exactly offsetting in terms of welfare. Then, given constant elasticity of substitution preferences, a constant terms of trade markup is optimal.

On the other hand, for small open economies with home bias (or large open economies) the desired markup is weighted between two goals: charging a terms of trade markup on goods that are exported but no markup on the consumption of domestic goods at home. Naturally, whenever consumption of domestic goods at home increases relative to exports of domestic goods, the desired markup falls and policy should be inflationary. Optimal monetary policy will thus fluctuate over the business cycle when economies are large or when they have home bias, such that deviations from price stability are optimal, and gains from cooperation are present. We demonstrate that it is not simply the terms of trade externality itself, which is still present in a world with no home bias in consumption, but rather a fluctuating optimal markup that leads to deviations from price stability. In practice, central banks should deviate from price stability only when their share of exports in GDP fluctuates substantially.

Our findings also reveal some important points about the effect of asset market structure on optimal monetary policy. De Paoli (2009a) showed that optimal non-cooperative monetary

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2Earlier contributions to the literature, including work by Clarida, Gali and Gertler (2000) and Obstfeld and Rogoff (2000, 2002) emphasized the isomorphism between optimal monetary policy in closed and open economies, finding price stability to be optimal. However, this isomorphism hinges on the assumption of unitary elasticity of substitution between home and foreign products, which implies perfect risk-sharing across countries.

3There is also an additional strategic element present in the large open economy case: policymakers need to internalize their impact on world income.
policy under PCP depends on asset market structure: under financial autarky optimal policy is weighted toward price stability, while under complete markets price stability is less optimal. One might expect that this difference in policy results from the central bank’s incentive to improve risk-sharing through monetary policy. However, we show that in the absence of home bias, it is optimal for small open economy central banks to mimic the flexible price equilibrium even in financial autarky. Therefore, the desire to improve risk-sharing cannot in and of itself explain the difference in optimal polices between complete markets and financial autarky. Rather, it is the variable terms of trade markup arising from non-cooperative policy under home bias which drives deviations from price stability in the small open economy environment. The intuition follows from our previous discussion of optimal cooperative policy under PCP: because small open economy central banks cannot improve risk-sharing across countries, they will not deviate from price stability for that reason alone.

If small open economy central banks cannot improve risk-sharing through monetary policy, why does optimal policy depend on asset market structure in De Paoli’s exercise? The ratio of home consumption of domestic goods relative to exports should behave differently in complete markets than in financial autarky. For example, given a positive technology shock with flexible prices, home consumption of domestically produced goods should increase more strongly in financial autarky than in complete markets, since in the latter case the consumption basket is more stable. Therefore, monetary policy should be more inflationary in financial autarky than in complete markets. The logic for large open economies is similar. In practice, price stability is close to being optimal as long as the ratio of home consumption of domestic goods to exports is fairly stable, regardless of asset market structure. In our paper, the ratio of home consumption of domestic goods to exports is constant and equal to zero because there is no home bias in consumption.

Contributions to Cooperative and Non-Cooperative Policy Under Local Currency Pricing

Finally, we are the first to study optimal monetary policy under LCP in small open economies. We find that it is optimal for small open economy central banks to fix the exchange rate in complete markets and financial autarky, under non-cooperative and cooperative equilibria.

Under LCP, the volatility of the nominal exchange rate causes deviations from the law of one price and therefore calls for at least some degree of nominal exchange rate stabilization. However, fixing the exchange rate stabilizes consumption but not leisure, introducing a tradeoff for policymakers. In the literature on optimal monetary policy under LCP for large open economies, if the utility derived from leisure is linear this tradeoff disappears and policymakers can focus only on stabilizing consumption, such that a fixed exchange rate is optimal (Devereux and Engel (2003) and Corsetti and Pesenti (2005)). If the disutility of labor is non-linear, optimal policy should deviate from a fixed exchange rate (Corsetti, Dedola and Leduc (2010)).
In contrast, our results in the small open economy setting are not dependent on leisure entering the utility function in a linear fashion. Why is this the case? Under LCP, when prices are fixed one period in advance, monetary policy has no effect on exports as they depend only on fixed relative prices and world income. In the absence of consumption home bias, all home produced goods are exported, and thus monetary policy has no effect on domestic labor supply. Moreover monetary policy has no effect on the allocation of goods within the consumption basket. Therefore, in the absence of home bias in consumption, a fixed exchange rate is optimal even when the disutility of labor is non-linear.

For small open economies with home bias, monetary policy will not affect exports but can affect the labor supply via the consumption of home produced goods. In addition, monetary policy can affect the allocation between domestically produced and foreign produced goods in the consumption basket. Therefore, with non-linear disutility of labor, optimal policy should call for partial stabilization of the exchange rate. For large open economies, monetary policy does affect the labor supply through its influence on world income as well as the domestic consumption of home produced goods. Again, if the utility derived from leisure is non-linear, the optimal policy will consist of partial exchange rate stabilization to minimize the losses of deviations from the law of one price and obtain the optimal consumption-leisure tradeoff.

**Economy Size and the Continuum**

Table 1: Economy Size Measured By Contribution to World Trade

<table>
<thead>
<tr>
<th>Country</th>
<th>Exports*</th>
<th>Imports*</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU†</td>
<td>11.99</td>
<td>13.02</td>
</tr>
<tr>
<td>China</td>
<td>10.68</td>
<td>9.69</td>
</tr>
<tr>
<td>US</td>
<td>8.50</td>
<td>12.86</td>
</tr>
<tr>
<td>Japan</td>
<td>4.50</td>
<td>4.42</td>
</tr>
<tr>
<td>South Korea</td>
<td>3.13</td>
<td>2.92</td>
</tr>
<tr>
<td>Russia</td>
<td>2.93</td>
<td>1.38</td>
</tr>
<tr>
<td>UK</td>
<td>2.79</td>
<td>3.64</td>
</tr>
<tr>
<td>Canada</td>
<td>2.53</td>
<td>2.61</td>
</tr>
</tbody>
</table>

*Exports and imports as a % of world exports and imports (Source: WTO, 2011).
† Excludes internal trade.

Although the continuum enables very sharp predictions, is it a realistic assumption? The data show that it is. For the purposes of this paper, we proxy for an economy’s size by calculating its contribution to world trade – a measure of both size and openness. Table 1 shows the top eight

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4When the disutility of labor is linear, we conjecture that optimal policy will follow the results of Duarte and Obstfeld (2008), where the movement of aggregate quantities replicates the flexible price allocation, even though prices remain sticky in local currency terms.
trading nations measured by exports and imports to the rest of the world as a percentage of total world exports and imports, respectively.\textsuperscript{5} Using this metric, the continuum is a reasonable assumption. The largest economy in the world, the U.S., accounts for only about ten percent of world trade, which demonstrates the importance of openness when evaluating relative country size in the world economy. A country with a large GDP may make up a small percentage of world trade if it is relatively closed to the rest of the world.

The paper proceeds as follows. We begin by laying out the model for PCP in Section 2. In Section 3, we examine optimal monetary policy in the closed economy to develop intuition, and then move to the open economy where we solve for optimal monetary policy under PCP, first in complete markets and then in financial autarky. Finally, we lay out the model for LCP in Section 4, and solve for optimal monetary policy in complete markets and financial autarky. Section 5 concludes.

\section{The Model}

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Gali and Monacelli (2005, 2008).\textsuperscript{6} Each economy consists of a representative household and a representative firm. All countries are identical ex-ante: they have the same preferences, technology, and price-setting. Ex-post, economies will differ depending on the realization of their technology shock. Households are immobile across countries, however goods can move freely across borders. Each economy produces one final good, over which it exercises a degree of monopoly power. This is crucially important: countries are able to manipulate their terms of trade even though they are measure zero. However, because countries are small they will be unable to influence world income and induce expenditure switching to their goods. Policymakers will thus charge a constant terms of trade markup on their exports.

We use one-period-in-advance price setting to introduce nominal rigidities.\textsuperscript{7} Monopolistic firms set next-period’s nominal prices, in terms of domestic currency, prior to next-period’s production and consumption decisions. These firms will charge a constant markup in the flex-price equilibrium, utilizing their monopoly power at the firm level. Given this preset price, firms supply as much output as demanded by households.

We lay out a general framework below, and then hone in on two specific cases: complete markets and financial autarky. To avoid additional notation, we ignore time subindices unless

\textsuperscript{5}EU trade data excludes intra-EU trade because we are trying to capture the influence of a central bank (in this case the ECB) on the global economy. Thus, any trade within a currency union is excluded from our calculation. This is why we look at the EU as a whole, rather than considering Germany, France, Italy, etc. as separate countries. From the perspective of evaluating optimal monetary policy, these countries share the same central bank. We recognize that the euro area is smaller than the EU, but we could not find adequate data at the euro area level that disaggregated internal and external trade, so we were forced to consider the EU as a whole. In this vein, we naturally exclude intra-U.S. trade in our calculation.

\textsuperscript{6}A similar version of this model appears in Dmitriev and Hoddenbagh (2013) where we employ wage rigidity instead of price rigidity and study the optimal design of a fiscal union within a currency union.

\textsuperscript{7}Assuming rigid wages or prices has no impact on the results: they yield identical policy implications.
Households

In each economy $i \in [0, 1]$, there is a representative household with lifetime expected utility

$$\mathbb{E}_{t-1} \left\{ \sum_{k=0}^{\infty} \beta^k \left( C_{it+k}^{1-\sigma} - \frac{N_{it+k}^{1+\varphi}}{1+\varphi} \right) \right\}$$

where $\beta < 1$ is the household discount factor, $C$ is the consumption basket or index, and $N$ is household labor effort (think of this as hours worked). Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_{it} = (1 - \tau_i) \left( \frac{W_{it}}{P_{it}} \right) N_{it} + D_{it} + T_{it}. \quad (2)$$

The distortionary tax rate on household labor income in country $i$ is denoted by $\tau_i$, while $T_{it}$ is a lump-sum tax rebate to households. Net taxes equal zero in the model, as any amount of government revenue is rebated lump-sum to households. The consumer price index corresponds to $P_{it}$, while the nominal wage is $W_{it}$. $D_{it}$ denotes state-contingent portfolio payments expressed in real consumption units, and can be written in more detail as:

$$D_{it}P_{it} = \int_0^1 \mathcal{E}_{ijt} B_{ijt}dj, \quad (3)$$

where $B_{ijt}$ is a state-contingent payment in currency $j$. $\mathcal{E}_{ijt}$ is the exchange rate in units of currency $i$ per one unit of currency $j$; an increase in $\mathcal{E}_{ijt}$ signals a depreciation of currency $i$ relative to currency $j$. When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. The transversality condition simply states that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. Leaving the details in the appendix, we use the following relationship as the transversality condition for complete markets:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t C_{it}^{-\sigma} D_{it} \right\} = 0, \quad (4)$$

while in financial autarky

$$D_{it} = 0.$$
Intuitively, the transversality condition (4) stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Under complete markets, consumers choose a state contingent plan for consumption, labor supply and portfolio holdings in period 0.

**Consumption and Price Indices** Households in each country consume a basket of imported goods. This consumption basket is an aggregate of all of the varieties produced by different countries. The consumption basket for a representative small open economy $i$, which is common across countries, is defined as follows:

$$C_i = \left( \int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{1}{\gamma-1}}$$

(5)

where $c_{ij}$ is the consumption by country $i$ of the final good produced by country $j$, and $\gamma$ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity). Because there is no home bias in consumption, countries will export all of the output of their unique variety, and import varieties from other countries to assemble the consumption basket.

Prices are defined as follows: upper case $P_{ij}$ denotes the price in country $i$ (in currency $i$) of the unique final good produced in country $j$, while $CPI_i$ is the aggregate consumer price index in country $i$. Given the above consumption index, the consumer price index will be:

$$CPI_i = \left( \int_0^1 P_{ij}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}.$$ 

(6)

Consumption by country $i$ of the unique variety produced by country $j$ is:

$$c_{ij} = \left( \frac{P_{ij}}{CPI_i} \right)^{-\gamma} C_i.$$ 

(7)

We assume that producer currency pricing (PCP) holds, and that the law of one price (LOP) holds, so that the price of the same good is equal across countries when converted into a common currency. We define the nominal bilateral exchange rate between countries $i$ and $j$, $E_{ij}$, as units of currency $i$ per one unit of currency $j$. LOP requires that:

$$P_{ij} = E_{ij} P_{jj}.$$ 

(8)

Given LOP and identical preferences across countries, PPP will also hold for all $i, j$ country pairs:

$$CPI_i = E_{ij} CPI_j,$$ 

(9)
The terms of trade for country $j$ will be:

$$TOT_j = \frac{P_{\text{Exports from } j}}{P_{\text{Imports to } j}} = \frac{P_{jj}}{CPI_j}; \quad (10)$$

where $TOT_j$ is defined as the home currency price of exports over the home currency price of imports. Since the domestic consumption basket is entirely composed of foreign inputs, the domestic CPI will be equal to the home currency price of imports. Now we can take (7), and using (8) and (9), solve for demand for country $j$’s unique variety:

$$Y_j = \int_0^1 c_{ij} di = \int_0^1 \left( \frac{P_{ij}}{CPI_i} \right)^\gamma C_i di = \left( \frac{P_{jj}}{CPI_j} \right)^\gamma \int_0^1 C_i di = TOT_j^{-\gamma} C_w. \quad (11)$$

where $C_w$ is defined as the average world consumption across all $i$ economies, $C_w = \int_0^1 C_i di$.

**Production** Each economy $i$ consists of a group of intermediate goods producers, $h \in [0, 1]$, who exercise monopoly power over their unique variety, and a perfectly competitive final goods exporter who aggregates the intermediates in CES fashion into a final export good. For simplicity, we assume that intermediates are non-tradable. Thus, each country will bundle its intermediates into one final export good.$^9$ Figure 1 below illustrates the bundling of intermediates into a final export good, and the bundling of those exports into the household consumption basket. We differentiate between the markup on intermediate domestic goods ($\mu = \frac{\epsilon}{\epsilon - 1}$) and the markup on the final export good ($\mu_\gamma = \frac{\gamma}{\gamma - 1}$), which is imposed only if policymakers decide to manipulate their terms of trade.

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$^9$We assume non-tradable intermediates with a final tradable consumption good that aggregates those intermediates for simplicity. In Gali and Monacelli’s (2005, 2008) setup, intermediate goods are tradable, such that every country’s import consumption basket is made up of an infinite number of varieties imported from an infinite number of countries. This requires integrating over two continuums. While it is straightforward for us to maintain their setup, we prefer the tractable alternative: a final goods exporter bundles the domestically produced intermediates for export. In this way, each country produces only one unique variety, and we only need integrate over one continuum. This assumption does not change the results in any way. In both cases the household consumption basket in each country is made up of imported goods from all $i$ countries, which are themselves made up of intermediates produced domestically.
Production of intermediates requires technology $Z_i$, which is common across firms within a country, and labor $N_i(h)$, which is unique to each firm. We do not need to assume a specific distribution for technology, a luxury afforded by our closed-form solution. We do assume that technology is independent across time and across countries, but is identical across firms within the same country. Given this, the production function of a representative intermediate goods firm $h$ in country $i$ will be:

$$y_i(h) = Z_i N_i(h).$$  \hfill (12)

Because intermediate goods firms produce differentiated varities, they have monopoly power over their output, which leads to a markup in intermediate goods. Perfectly competitive final goods exporters aggregate the intermediate input of each firm, so that production of the representative final exporter in a specific country is:

$$Y_i = \left[ \int_0^1 y_i(h)^{\frac{\varepsilon}{1-\varepsilon}} dh \right]^{\frac{1-\varepsilon}{\varepsilon}}.$$ \hfill (13)

where $\varepsilon$ is the elasticity of substitution between different intermediates, and $\mu = \frac{\varepsilon}{\varepsilon - 1}$ is the markup. The price of the final good in country $i$, $P_{it}$, will be a function of the nominal price for intermediate goods, $p_i(h)$:

$$P_{it} = \left[ \int_0^1 p_i(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}.$$  

Cost minimization by the perfectly competitive final goods exporter leads to the following demand for intermediates:

$$y_i(h) = \left( \frac{p_i(h)}{P_{it}} \right)^{\varepsilon} Y_i.$$ \hfill (14)

In summary, monopoly power may be exercised at the firm and the country level: at the firm level because of the production of differentiated domestic varieties, and at the country level because each economy produces a unique variety for export. Households and final goods exporters have no monopoly power and are perfectly competitive.\footnote{It is entirely plausible to shift the country-level monopoly power from the policymaker to the final goods exporter. All results will be exactly the same: terms of trade markups will be constant — just as in a model of monopolistic competition. As such, price stability will remain optimal under PCP.}

Intermediate goods firms will price their unique good one-period-in-advance according to the following condition, which results from profit maximization:

$$p_{it}(h) = \mu \frac{\mathbb{E}_{t-1} \left\{ C_{it}^{-\sigma} y_i(h) \frac{W_{it}}{Z_{it} CPI_{it}} \right\}}{\mathbb{E}_{t-1} \left\{ C_{it}^{-\sigma} y_i(h) \frac{1}{CPI_{it}} \right\}}.$$ \hfill (15)
Households maximize utility (1) subject to their budget constraint (2). The FOC with respect to labor will give the following household labor supply condition:

$$\frac{W_{it}}{CPI_{it}} = \left( \frac{\chi}{1 - \tau_i} \right) N_i^\sigma C_i^\sigma. \quad (16)$$

Firms are identical, so that in equilibrium $p_i(h) = P_i$ and $y_i(h) = Z_i n_i(h) = Z_i N_i = Y_i$. Using the labor demand condition (15) and the labor supply condition (16), and the fact that prices are preset at time $t - 1$, the labor market clearing condition will be:

$$1 = \left( \frac{\chi\mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1}\{N_i^{1+\sigma}\}}{\mathbb{E}_{t-1}\{C_i^{-\sigma} Y_i^{\frac{1}{\gamma}} (C_{wt})^{\frac{1}{\gamma}}\}}. \quad (17)$$

This is the general labor market clearing condition; it holds in the closed economy and in the open economy for PCP. Under PCP, the demand for the unique variety (11) will give the following labor market clearing condition:

$$1 = \left( \frac{\chi\mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1}\{N_i^{1+\sigma}\}}{\mathbb{E}_{t-1}\{C_i^{-\sigma} Y_i^{\frac{1}{\gamma}} (C_{wt})^{\frac{1}{\gamma}}\}}. \quad (18)$$

Taking the expectations operator out of (18) will give the flexible price equilibrium for PCP.

We now turn our attention to the difference between complete markets and financial autarky.

2.1 Complete Markets

In this section, we assume that agents in each economy trade a full set of domestic and foreign state-contingent assets. Households in all countries will maximize their utility (1), choosing consumption, leisure, money holdings, and a complete set of state-contingent nominal bonds, subject to (2).

**Risk-Sharing**  Complete markets and PPP imply the following risk-sharing condition:

$$\frac{C_i^{-\sigma}}{C_i^{-\sigma + 1}} = \frac{C_j^{-\sigma}}{C_j^{-\sigma + 1}} \quad \forall i, j \quad (19)$$

which states that the ratio of the marginal utility of consumption at time $t$ and $t + 1$ must be equal across all countries. Importantly, this condition does not imply that consumption is equal across countries. Consumption in country $i$ will depend on its initial asset position, monetary policy, the distribution of country-specific shocks, the covariance of global and local shocks, and other factors.

When the central bank in economy $i$ changes its policy, the consumption allocation in country $i$ may change as well. For example, monetary policy can affect the covariance between home production and world consumption, and this covariance will influence the level of household
consumption even under complete markets. The risk-sharing condition (19) and the transversality condition (4) are both robust to changes in monetary policy. If we combine the two, the resulting goods market clearing condition will also be robust to changes in monetary policy. When (4), (17), and (19) hold, consumption in country $i$ can be expressed as a function of world consumption:

$$C_{it} = \frac{\mathbb{E}_{t-1} \{ \sum_{t=1}^{\infty} \beta^t \left[ Y_{it}C_{wt}^{1-\sigma}TOT_{it} \right] \}}{\mathbb{E}_{t-1} \{ \sum_{t=1}^{\infty} \beta^t C_{wt}^{1-\sigma} \}} \cdot C_{wt}.$$  

We solve for this expression explicitly in Appendix A.1.

Using the fact that $Z_{it}$ is independent across time and across countries, and prices are preset, (20) is equivalent to

$$C_{it} = \mathbb{E}_{t-1} \{ Y_{it} TOT_{it} \} = C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1} \left\{ Y_{it}^{\frac{1}{\gamma}} \right\}.$$  

2.2 Financial Autarky

The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country (exports) must equal the nominal of consumption in the home country (imports). That is, trade in goods must be balanced. In a model with cross-border lending, bonds would also show up in this condition, but in financial autarky, they are obviously absent. The primary departure from complete markets lies in the household and economy-wide budget constraints.

$$\frac{P_{it} \cdot Y_{i}}{\text{Exports}} = \frac{CPI_{it} \cdot C_{i}}{\text{Imports}}.$$  

Using the fact that (11) holds under both complete markets and financial autarky, and substituting this into (22), one can show that demand for country $i$’s good in financial autarky will be

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{1}{\gamma}}.$$  

Complete markets and autarky differ only by goods market clearing. In complete markets consumption is equal to expected domestic output expressed in consumption baskets; in autarky consumption is equal to realized domestic output expressed in consumption baskets.

3 Optimal Monetary Policy for Producer Currency Pricing

Now that we’ve laid out the model in detail for both complete markets and financial autarky, we consider optimal policy for a variety of scenarios. Without loss of generality, we assume a cashless limiting economy. Central banks will optimize by choosing labor instead of money supply or an interest rate rule, but all three are equivalent in this model: money supply will determine the interest rate, which will in turn determine labor. We prove this in Appendix F.\footnote{Benigno and Benigno (2003) also describe a cashless-limiting economy in detail in their appendix, pp.756-758.}
One can easily write down a money supply rule or interest rate rule that exactly implements
the allocation resulting from optimization over labor. But for the sake of simplicity, we assume
the central bank optimizes over labor.

The timing of the model is described in Figure 2 below. Before any shocks are realized,
national central banks declare their policy for all states of the world. With this knowledge
in hand, households lay out a state-contingent plan for consumption, labor, money and asset
holdings. After that, shocks hit the economy. Note that under financial autarky, no interna-
tional asset trading will occur.

Figure 2

We begin with an analysis of optimal monetary policy in a closed economy version of our
model, and then proceed to the open economy. In all cases, we consider optimal policy under
commitment.

3.1 Closed Economy

The Flexible Price Allocation

To solve for the flexible price allocation in the closed economy, simply take expectations out
of (17), set \( \text{TOT}_t = 1 \), and use goods market clearing \( (Y_t = C_t = N_t Z_t) \). This will give us a
system of two equations in two unknowns, \( N_t \) and \( C_t \):

\[
1 = \left( \frac{\chi \mu}{1 - \tau} \right) \frac{N_t^{1+\varphi}}{C_t^{1-\sigma}}, \\
C_t = N_t Z_t.
\]

The solution to this two equation system is the flexible price allocation for the closed economy:

\[
C_t = \left( \frac{1 - \tau}{\chi \mu} \right)^{\frac{1}{1+\varphi}} Z_t^{\frac{1+\varphi}{1-\sigma}}.
\]

Optimal Monetary Policy

The central bank will choose labor to maximize the expected utility of the representative agent,
given the closed economy labor market and goods market clearing constraints.

\[
\max_{N_t} \mathbb{E}_{t-1} \left\{ \frac{C_t^{1-\sigma}}{1 - \sigma} - \chi \frac{N_t^{1+\varphi}}{(1 + \varphi)} \right\}
\] (24)
Although it is standard practice to use a welfare-loss function for optimal policy evaluation, we can simply use the household utility function because of our global, closed-form solution.\footnote{The only reason to use a welfare-loss function is if the model in question must be approximated around a steady state. Here, no such approximation is required, and thus a welfare-loss function is not needed.}

**Proposition 1** In the closed economy under ex ante commitment, the central bank will maximize (24) subject to (25) and (26). The solution to this problem is: \( C_t = \left( \frac{1 - \tau}{1 - \mu} \right)^{\frac{1}{\sigma + \varphi}} Z_t^{\frac{1 + \varphi}{\sigma + \varphi}} \). The central bank replicates the flexible price allocation via a policy of price stability.

**Proof** See Appendix B. \( \blacksquare \)

**Optimal Policy Under A Social Planner**

**Proposition 2** In the closed economy, the social planner will maximize (24) subject to (26), ignoring the labor condition (25). The solution to this problem is: \( C_t = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma + \varphi}} Z_t^{\frac{1 + \varphi}{\sigma + \varphi}} \).

**Proof** See Appendix B. \( \blacksquare \)

In comparing the solutions described in Proposition 1 and 2, notice that the social planner mimics the flexible price allocation while eliminating the monopolistic markup \( \mu \) (Proposition 2), while the markup and labor tax remain when we only consider optimal monetary policy (Proposition 1). In the case of ex ante commitment in Proposition 1, a fiscal authority may introduce subsidies to exactly offset the markup and replicate the social planner equilibrium. It is straightforward to show that \( \tau = 1 - \mu \) will get rid of the monopolistic distortion on labor inputs and give the Pareto efficient allocation.

We’ve studied optimal policy in the closed economy, and proved the optimality of price stability. The divine coincidence holds, a well known result in the closed economy. One already sees the link between stable monopolistic markups and a desire to mimic the flexible price allocation. We now turn our attention to the open economy, where we prove that optimal monetary policy in closed and small open economies is isomorphic in both complete markets and financial autarky.

**3.2 Global Social Planner**

The global social planner is a benevolent “dictator” that distributes goods across countries in order to maximize aggregate world utility. This scenario is analogous to perfect cooperation among the social planner’s of all \( i \) countries. The global social planner solution defines the
Pareto efficient allocation. Since the economies in our model are identical ex-ante, the global social planner will maximize a weighted utility function over all $i$ countries,

$$\int_0^1 \left[ \frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{(1+\varphi)} \right] \, di,$$

subject to the consumption basket and the aggregate resource constraint:

$$C_i = \left[ \int_0^1 \frac{\gamma}{c_{ij}} \, dj \right]^{\frac{1}{\gamma-1}},$$

$$Y_i = Z_i N_i = \int_0^1 c_{ij} \, dj.$$

**Proposition 3** The global social planner will maximize utility weighted over all $i$ countries (27), subject to (28) and (29). The solution to this optimization problem is:

$$\mathbb{E}\{U_i\} = C_i^{1-\sigma} \left( \frac{1}{1-\sigma} - \frac{1}{1+\varphi} \right),$$

$$C_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_{w}^{1+\varphi},$$

$$N_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_{w}^{(1-\gamma\sigma)(1+\varphi)} Z_i^{\gamma-1},$$

$$Y_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_{w}^{(1-\gamma\sigma)(1+\varphi)} Z_i^{\gamma(1+\varphi)} Z_i^{\gamma(1+\varphi)} Z_{w}^{\gamma(1+\varphi)} Z_i^{\gamma(1+\varphi)} Z_{w}^{\gamma(1+\varphi)} Z_i^{\gamma(1+\varphi)},$$

$$Z_w = \left( \int_0^1 Z_i^{(\gamma-1)(1+\varphi)} \, di \right)^{\frac{1}{1+\gamma\varphi}}.$$

**Proof** See Appendix C. ■

The global social planner allocation above is a traditional benchmark for the evaluation of different policy regimes. Because this is the Pareto efficient allocation, it provides a natural marker with which to compare various policies under commitment. Notice that there are no markups in the Pareto efficient allocation: the benevolent global social planner has eliminated the markup on intermediate goods $\mu$, and resisted the temptation to manipulate the terms of trade. In the next sections we will look closely at optimal monetary policy for central banks and see what conditions are necessary to replicate the global social planner allocation.

### 3.3 Complete Markets

In this section we examine the optimal monetary policy for cooperative and non-cooperative central banks in complete markets. The objective functions for cooperative and non-cooperative central banks are below.
Non-cooperative central banks will choose the optimal amount of labor to maximize their
domestic welfare (30a), while cooperative central banks will choose the optimal amount of
labor in order to maximize the welfare of all $i$ economies (30b). In both cases, policymakers
will maximize the appropriate objective function subject to labor market clearing (31) and
goods market clearing (32) constraints, and production (33) and aggregate world consumption
(34):

$$1 = \left(\frac{\chi \mu}{1 - \tau_i}\right) \frac{\mathbb{E}_{t-1}\{N_{it}^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_{it}^{1-\sigma}Y_{it}^{\gamma} - C_{wt}^{1}\}},$$

(31)

$$C_{it} = C_{wt}^{1}\mathbb{E}_{t-1}\{Y_{it}^{\gamma}\},$$

(32)

$$Y_{it} = Z_{it}N_{it},$$

(33)

$$C_{wt} = \left(\int_{0}^{1} Y_{it}^{\gamma} di\right)^{1/\gamma}.$$  

(34)

**Proposition 4** In complete markets, non-cooperative central banks will maximize (30a) and
cooperative central banks will maximize (30b), subject to (31), (32), (33) and (34). The solution
under commitment for both cooperative and non-cooperative central banks in complete markets
is:

$$\mathbb{E}\{U_i\} = C_i^{1-\sigma}\left(\frac{1}{1 - \sigma} - \frac{1 - \tau_i}{\mu(1 + \varphi)}\right)$$

$$C_i = \left(1 - \tau_i\right)\frac{1}{\chi \mu} Z_{w}^{1+\varphi}$$

$$N_i = \left(1 - \tau_i\right)\frac{1}{\chi \mu} Z_{w}^{(1+\varphi)\gamma(1+\varphi)} Z_{i}^{\gamma-1}$$

$$Y_i = \left(1 - \tau_i\right)\frac{1}{\chi \mu} Z_{w}^{(1+\varphi)\gamma(1+\varphi)} Z_{i}^{\gamma}$$

$$Z_{w} = \left(\int_{0}^{1} Z_{i}^{\gamma}\frac{1}{1+\varphi} di\right)^{1/(1+\varphi)}$$

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking
the flexible price allocation is thus the dominant strategy for non-cooperative central banks.
in complete markets, and is the optimal policy under cooperation. If the government corrects
the distortions due to market power with a non-contingent tax $\tau_i = 1 - \mu$, then the flexible price
allocation in complete markets is identical to the global social planner solution.

Proof See Appendix D. ■

There are a few important points to note from this exercise. First of all, note that consumption
in the optimal allocation is not subject to idiosyncratic technology shocks. Because we are
in complete markets, consumption is simply a function of average world technology. Second,
ote note that cooperative and non-cooperative equilibria are identical: both yield the flexible price
allocation.

In complete markets, we’ve shown that small open economy central banks will mimic the
flexible price allocation under both cooperative and non-cooperative regimes, for any elasticity
of substitution between home and foreign goods. In addition, we’ve demonstrated that inter-
national monetary cooperation has no impact on welfare. This is because monetary policy has
no power against non-contingent distortions like markups, and can only address the contingent
price rigidity distortion. In the next section, we conduct the same exercise for financial autarky.

3.4 Financial Autarky

In financial autarky, the objective functions for cooperative and non-cooperative central banks
will be identical to those in complete markets. The only difference in the optimization problem
will be in the goods market constraint. In complete markets home consumption is a function of
expected output (32), while in autarky home consumption is a function of actual output (36).

\[
1 = \left( \frac{x \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\eta} \right\}}{\mathbb{E}_{t-1} \left\{ C_{it}^{-\sigma} Y_{it}^{\frac{\gamma - 1}{\gamma}} C_{w,t}^{\frac{1}{\gamma}} \right\}} \tag{35}
\]

\[
C_{it} = C_{w,t}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma - 1}{\gamma}} \tag{36}
\]

\[
Y_{it} = Z_{it} N_{it} \tag{37}
\]

\[
C_{wt} = \left( \int_{0}^{Y_{it}^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma}} \right)^{\frac{1}{\gamma - 1}}. \tag{38}
\]

Proposition 5 In financial autarky, non-cooperative central banks will maximize (30a) and
cooperative central banks will maximize (30b), subject to (35), (36), (37) and (38). The solution
under commitment for both cooperative and non-cooperative central banks in financial autarky
The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in financial autarky, and is the optimal policy under cooperation.

Proof See Appendix D.

Note that in financial autarky, consumption is a function of idiosyncratic technology $Z_i$, which reflects the lack of international risk-sharing. As in complete markets, the optimal policy for cooperative and non-cooperative central banks is to mimic the flexible price equilibrium. We thus demonstrate the isomorphism between optimal monetary policy in closed and small open economies with no consumption home bias for both complete markets and financial autarky.

As we’ve stated before, the key to this isomorphism stems from the constant optimal terms of trade markup chosen by small open economies with no home bias. Combined with monopolistic firms charging a constant markup, the optimal policy response calls for the removal of the distortive impact of price rigidities and a return to the flexible price allocation. Similarly, when elasticity is unitary and economies are large, as in Clarida, Gali and Gertler (2002) and Obstfeld and Rogoff (2002), the optimal terms of trade markup will be constant and risk-sharing is provided by movements in the terms of trade. As a result, there is no need for monetary policy to fluctuate over the business cycle or to improve risk-sharing across countries; mimicking the flexible price allocation is optimal.

4 Optimal Monetary Policy for Local Currency Pricing

We now turn our attention to the case of LCP. Modeling LCP in a continuum of small open economies is quite difficult because the law of one price and purchasing power parity no longer hold. In two economy models, keeping track of exchange rate policy is trivial, but in the continuum this becomes challenging. Although we will gloss over many of the methodological nuances necessary to deal with LCP in the continuum, all modeling details can be found in
Appendix E. To simplify expressions, we assume log utility ($\sigma = 1$), but allow $\varphi$ to vary as before.

Under LCP, firms price their export good one-period-in-advance in the currency of the importing country. As such, there is zero exchange rate pass-through into import prices. Each country’s consumer price index will thus be fixed one-period-in-advance, as exchange rate movements will have no impact on import prices. When firms price in this way, the labor market and goods market clearing constraints will differ from those under PCP. In complete markets and financial autarky, goods and labor market clearing conditions for LCP are:

$$1 = \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \{ N_{it}^{1+\varphi} \}}{\mathbb{E}_{t-1} \{ C_{it}^{-1} Y_{it} TOT_{it} \}} \tag{39}$$

$$C_{it} = \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{0it} Y_{it} \tag{40}$$

To keep track of exchange rate policy in the continuum, we introduce the concept of a numeraire currency, which we assume is the currency of country 0. Thus, $\mathcal{E}_{0it}$ is the exchange rate between country $i$ and the numeraire country, and $P_{0it}$ is the import price of the numeraire country’s good in currency $i$.

**Proposition 6** Under LCP, non-cooperative central banks will maximize (30a) subject to (39) and (40). A fixed exchange rate will be the Nash equilibrium policy for a non-cooperative central bank in both complete markets and financial autarky under LCP.

**Proof** See Appendix E. ■

$$\mathbb{E}_{t-1} \{ C_{it}^{-\sigma} Y_{it} \mathcal{E}_{0it} \} = \frac{\chi \mu}{1 - \tau_i} \frac{\mathbb{E}_{t-1} \{ N_{it}^{1+\varphi} \}}{\mathbb{E}_{t-1} \{ \mathcal{E}_{0it} C_{jt} \}} \tag{41}$$

$$C_{it} = \mathcal{E}_{0it} \int_0^1 C_{jt} \mathcal{E}_{0jt} dj \tag{42}$$

**Proposition 7** Under LCP, cooperative central banks will maximize (30b) subject to (41) and (42). The optimal policy for cooperative central banks in both complete markets and financial autarky will be a fixed exchange rate.

**Proof** See Appendix E. ■

Price stability is not optimal under LCP. Instead, central banks should fix the exchange rate. Why is a fixed exchange rate optimal under LCP? As we discussed in the introduction, LCP insulates household consumption baskets in each country from exchange rate fluctuations. Under LCP, a constant proportion of world income is spent on each unique import good because exchange rate fluctuations do not pass-through into imported goods prices. When there is no home bias in consumption, policymakers in small open economies cannot affect domestic labor
supply through monetary policy. In this environment, where domestic output and labor are unaffected by monetary policy, stabilizing consumption becomes the central bank's only goal and is accomplished via a fixed exchange rate. With a fixed exchange rate, labor fluctuates with productivity shocks but consumption remains constant. In contrast, when economies are large monetary policy can influence domestic labor under LCP. In such cases, a fixed exchange rate is no longer optimal (Corsetti, Dedola and Leduc (2010)). We believe that optimal policy will also deviate from a fixed exchange rate for small open economies with home bias in consumption because monetary policy can influence domestic labor supply.

5 Conclusion

We derive the first closed-form solution for an open economy model that does not restrict substitutability between home and foreign goods to one. We model the world as a continuum of small open economies interacting in general equilibrium, rather than two large open economies. The tractability of our framework requires simplifying assumptions along other dimensions. Prices are set one period in advance, and each country exports all of its production and imports varieties from all other countries to aggregate into a final consumption basket.

Using this setup, we answer the following question: what are the theoretical conditions under which it is optimal for a central bank to mimic the flexible price allocation? We study this question in eight different cases, six of which have not previously been examined in the literature (cooperative policy under PCP in complete markets and financial autarky, and cooperative and non-cooperative policy under LCP in complete markets and financial autarky). We prove that under PCP, small open economy central banks should mimic the flexible price allocation through a policy of price stability, while under LCP policy should fix the exchange rate. Our results demonstrate that, in the absence of home bias in consumption, optimal policy should not deviate from these benchmark policies irrespective of the economy’s asset market structure or the degree of monetary policy cooperation. Our analysis clarifies when and why optimal monetary policy deviates from price stability in the small open economy environment.
References


Technical Appendix

A Risk-Sharing

The household in country $i$ will maximize lifetime utility (1), subject to the following budget constraint and transversality condition:

$$C_i(s_t)P_t(s_t) = W_i(s_t)N_i(s_t) + \int_0^1 \mathcal{E}_{ij}(s_t)B_{ij}(s_t) dj,$$

$$\sum_{t=1}^{\infty} \sum_{s_t} \int_0^1 q_j(s_t)B_{ij}(s_t) dj = 0.$$  (44)

$B_{ij}(s_t)$ denotes the state-contingent bond that pays in currency $j$ in state $s_t$; $q_j(s_t)$ is the price of that bond in period 0 (when all trading occurs). $q_j(s_t)$ is arbitrary up to a constant. Household in period 0 cares about relative price of claims across states and currencies. The transversality condition stipulates that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. The household Lagrangian is:

$$\mathcal{L}_i = \sum_{t=1}^{\infty} \beta^t Pr(s_t) \left\{ U_i(C(s_t)) - V_i(N(s_t)) + \frac{\lambda_i(s_t)}{P_t(s_t)} \left[ W_i(s_t)N_i(s_t) + \int_0^1 \mathcal{E}_{ij}B_{ij}(s_t(s_t)) dj - C_i(s_t)P_t(s_t) \right] \right\}$$

$$- \lambda_{i0} \sum_{t=1}^{\infty} \sum_{s_t} \int_0^1 q_j(s_t)B_{ij}(s_t) dj,$$  (45)

Now take the FOC with respect to state contingent bonds $B_{ij}(s_t)$:

$$\frac{\partial \mathcal{L}_i}{\partial B_{ij}(s_t)} = \lambda_{i0} q_j(s_t) + \frac{\beta^t \lambda_i(s_t) Pr(s_t) \mathcal{E}_{ij}(s_t)}{P_t(s_t)} = 0,$$  (46)

which gives price of the state-contingent bond,

$$q_j(s_t) = \frac{\beta^t \lambda_i(s_t) Pr(s_t) \mathcal{E}_{ij}(s_t)}{\lambda_{i0} P_t(s_t)}.$$  (47)

The analogous FOC for country $j$: $\frac{\partial \mathcal{L}_j}{\partial B_{jij}(s_t)} = 0$ will yield:

$$q_j(s_t) = \frac{\beta^t \lambda_j(s_t) Pr(s_t) \mathcal{E}_{jj}(s_t)}{\lambda_{j0} P_j(s_t)}.$$  (48)

Using $\mathcal{E}_{jj}(s_t) = 1$ and setting (47) equal to (48), we get the risk-sharing condition

$$\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \frac{\lambda_{i0}}{\lambda_{j0}} \frac{P_i(s_t)}{P_j(s_t) \mathcal{E}_{ij}(s_t)}.$$  (49)

When PPP holds, $\frac{P_i(s_t)}{P_j(s_t) \mathcal{E}_{ij}(s_t)} = 1$, and the risk-sharing condition simplifies to $\frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \left( \frac{C_i(s_t)}{C_j(s_t)} \right)^{-\sigma} = \frac{\lambda_{i0}}{\lambda_{j0}}$. When the consumption ratio is constant across countries, $C_{it} = A_i C_{wt}$.
In order to solve for $A_i$, we substitute (47) into the transversality condition.

\[
\sum_{t=1}^{\infty} \sum_{s_t} J_{0}^{1} q_j(s_t) B_{ij}(s_t) dj = \sum_{t=1}^{\infty} \sum_{s_t} J_{0}^{1} \beta^t \frac{\lambda_i(s_t) P \text{r}(s_t) E_{ij}(s_t)}{\lambda_0 P_i(s_t)} B_{ij}(s_t) dj \\
= \sum_{t=1}^{\infty} \sum_{s_t} \beta^t \frac{\lambda_i(s_t) P \text{r}(s_t)}{\lambda_0 P_i(s_t)} \int_{0}^{1} E_{ij}(s_t) B_{ij}(s_t) dj \\
= \sum_{t=1}^{\infty} \sum_{s_t} \beta^t \frac{\lambda_i(s_t) P \text{r}(s_t)}{\lambda_0 P_i(s_t)} \left( P_i(s_t) C_i(s_t) - W_i(s_t) N_i(s_t) \right) \\
= 0
\]

We substitute $C_i(s_t) = A_i C_w(s_t)$ into the above equation, and solve for $A_i$.

\[
A_i = \frac{\sum_{t=1}^{\infty} \sum_{s_t} \beta^t W(s_t) N(s_t) \lambda_i(s_t) P \text{r}(s_t)}{\sum_{t=1}^{\infty} \sum_{s_t} \beta^t C_w(s_t) \lambda_i(s_t) P \text{r}(s_t)} \\
= \frac{\sum_{t=1}^{\infty} \beta^t E_{t-1} \left\{ \frac{W_{t} \lambda_i(s_t)}{P_i} \right\}}{\sum_{t=1}^{\infty} \beta^t E_{t-1} \{ C_{wt} \lambda_i(s_t) \}} \\
= \frac{\sum_{t=1}^{\infty} \beta^t E_{t-1} \left\{ Y_{it}^{\gamma} C_{wt}^{-\gamma} \lambda_i(s_t) \right\}}{\sum_{t=1}^{\infty} \beta^t E_{t-1} \{ C_{wt} \lambda_i(s_t) \}} \\
= \frac{\sum_{t=1}^{\infty} \beta^t E_{t-1} \left\{ Y_{it}^{\gamma} C_{wt}^{-\gamma} C_{wt}^{-\sigma} \right\}}{\sum_{t=1}^{\infty} \beta^t E_{t-1} \{ C_{wt} C_{wt}^{-\sigma} \}}
\]

where we used $\lambda_i(s_t) = A_i^{-\sigma} C_w^{-\sigma}(s_t)$. This gives us the definition of complete markets from the text, equation (20).
B Closed Economy: Propositions and Proofs

Proposition 1 In the closed economy under ex ante commitment, the central bank will maximize (24) subject to (25) and (26). The solution to this problem is: $C_t = \left(\frac{1-\tau}{\mu\chi}\right)\frac{1}{\sigma+\nu}Z_t^{1/\sigma+\nu}$. The central bank replicates the flexible price allocation via a policy of price stability.

Proof: The flexible price allocation is obtained from solving this two equation system in two unknowns $(C,N)$:

\begin{align*}
1 &= \mu\chi \frac{N_t^{1+\varphi}}{1-\tau}C_t^{1-\sigma}, \\
C_t &= Z_tN_t.
\end{align*}

The solution is $C_t = \left(\frac{1-\tau}{\mu\chi}\right)\frac{1}{\sigma+\nu}Z_t^{1/\sigma+\nu}$. Now, let’s reformulate the central bank’s problem by substituting the labor market clearing condition (25) and the goods market clearing condition (26) into the objective function.

\begin{align*}
\max_{C_t} & \mathbb{E}_{t-1} \left\{ C_t^{1-\sigma} - \frac{\chi C_t^{1+\varphi}Z_t^{1-\varphi}}{1+\varphi} \right\} \\
\text{s.t.} & \quad 1 = \mu\chi \frac{N_t^{1+\varphi}}{1-\tau} \frac{C_t^{1-\sigma}}{\mathbb{E}_{t-1}C_t^{1-\sigma}}.
\end{align*}

The Lagrangian for this constrained optimization problem is

\begin{align*}
\mathcal{L} &= \mathbb{E}_{t-1} \left\{ C_t^{1-\sigma} - \frac{\chi C_t^{1+\varphi}Z_t^{1-\varphi}}{1+\varphi} \right\} + \lambda \left( \mathbb{E}_{t-1}C_t^{1-\sigma} - \frac{\mu\chi}{1-\tau} \mathbb{E}_{t-1} \left\{ C_t^{1+\varphi}Z_t^{-(1+\varphi)} \right\} \right). \tag{50}
\end{align*}

The first order condition with respect to consumption is\(^{13}\)

\begin{align*}
\frac{\partial \mathcal{L}}{\partial C_t} &= (1 + \lambda(1-\sigma)) C_t^{1-\sigma} - \chi \left( 1 + \frac{\lambda\mu}{1-\tau} \right) C_t^{\varphi}Z_t^{-1-\varphi} = 0. \tag{51}
\end{align*}

which is equivalent to

\begin{align*}
C_t^{1-\sigma} = \chi \left( 1 + \frac{\lambda\mu}{1-\tau} \right) \frac{1}{\lambda(1-\sigma)} N_t^{1+\varphi}. \tag{52}
\end{align*}

Given that $\lambda$ is a constant and not a variable, the first order condition and the budget constraint are satisfied only under the flexible price equilibrium. Thus, the central bank’s optimal policy is to mimic the flexible price equilibrium in the closed economy. ■

Proposition 2 In the closed economy, the social planner will maximize (24) subject to (26), ignoring the labor condition (25). The solution to this problem is: $C_t = \left(\frac{1}{\tau}\right)^{\frac{1}{\sigma+\nu}} Z_t^{\frac{1}{\sigma+\nu}}$.

\(^{13}\)One can easily carry out the same exercise by optimizing with respect to labor.
Proof: Insert the aggregate goods market clearing constraint (26) directly into the objective function, replacing $N_{t+1}$, and maximize this objective function.

$$\max_{C_{t+1}} \left[ \frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \left( \frac{C_{t+1}}{Z_{t+1}} \right)^{1+\varphi} \right]$$

The solution to this optimization problem is $C_t = \left( \frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}$. ■
C Global Social Planner: Proposition and Proof

**Proposition 3** The global social planner will maximize utility weighted over all $i$ countries (27), subject to (28) and (29). The solution to this optimization problem is:

$$E\{U_i\} = C_i^{1-\sigma} \left( \frac{1}{1-\sigma} - \frac{1}{1+\varphi} \right),$$

$$C_i = \left( \frac{1}{\chi} \right)^{1+\varphi} Z_i^{1+\varphi},$$

$$N_i = \left( \frac{1}{\chi} \right)^{1+\varphi} Z_i^{1+\varphi} Z_i^{1+\gamma},$$

$$Y_i = \left( \frac{1}{\chi} \right)^{1+\varphi} Z_i^{1+\varphi} Z_i^{1+\gamma},$$

$$Z_w = \left( \int_0^1 Z_i \frac{(\gamma-1)(1+\varphi)}{(\gamma-1)(1+\varphi)} \, di \right)^{\frac{\gamma}{1+\varphi}}.$$

**Proof:** If we substitute (28) and (29) directly into the objective function (27), then we can reformulate the problem as follows:

$$\max_{c_{ij}} \int_0^1 \left[ \left( \int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{\gamma(1-\sigma)}{\gamma-1}} - \frac{\chi}{1+\varphi} \left( \int_0^1 c_{ji} dj \right)^{1+\varphi} \right] \, di. \quad \text{ (53)}$$

Rearranging, we have

$$\max_{c_{ij}} \frac{1}{1-\sigma} \int_0^1 \left( \int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{\gamma(1-\sigma)}{\gamma-1}} \, di - \frac{\chi}{1+\varphi} \int_0^1 \left( \int_0^1 c_{ji} dj \right)^{1+\varphi} \, di. \quad \text{ (54)}$$

The FOC with respect to $c_{ij}$ is

$$0 = \left( \int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{\gamma(1-\sigma)}{\gamma-1}} c_{ij} - \frac{1}{\gamma} \left( \int_0^1 c_{ji} dj \right)^{\frac{\varphi}{Z_j}}. \quad \text{ (55)}$$

This is equivalent to

$$0 = \left( \int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{1-\sigma \gamma}{\gamma-1}} c_{ij} - \frac{1}{\gamma} \left( \int_0^1 c_{ji} dj \right)^{\varphi} \frac{1}{Z_j},$$

$$\Rightarrow 0 = C_i^{\frac{1-\sigma \gamma}{\gamma}} c_{ij}^{\frac{1}{\gamma}} - \frac{N_j^\varphi}{Z_j},$$

28
and solving for $c_{ij}$ we have:

$$c_{ij} = {Z_j C_i^{1-\gamma \sigma} \over \chi^\gamma N_j^{\gamma \varphi}}.$$  \hfill (56)

The consumption basket in country $i$ ($C_i$) can then be expressed as:

$$C_i = \left( \int_0^1 c_{ij}^{\gamma - 1} \, dj \right)^{\frac{1}{\gamma - 1}},$$

$$= \left[ \int_0^1 \left( {Z_j C_i^{1-\gamma \sigma} \over \chi^\gamma N_j^{\gamma \varphi}} \right)^{\frac{1}{\gamma - 1}} \, dj \right]^{\frac{1}{\gamma - 1}},$$

$$= \left( {1 \over \chi} \right)^{\frac{1}{\gamma - 1}} \left[ \int_0^1 \left( {Z_j \over N_j^{\gamma \varphi}} \right)^{(\gamma - 1)} \, dj \right]^{\frac{1}{\gamma - 1}}.$$  \hfill (57)

So $C_i$ does not depend on its own technology $Z_i$. Now, let’s solve for labor ($N_i$) and output ($Y_i$).

$$N_i = \frac{Y_i}{Z_i} \quad \text{from goods market clearing}$$

$$= \frac{\int_0^1 c_{ij} \, dj}{Z_i} \quad \text{from (29)}$$

$$= \frac{\int_0^1 \left( {Z_j C_j^{1-\gamma \sigma} \over \chi^\gamma N_j^{\gamma \varphi}} \right) \, dj}{Z_i} \quad \text{from (56)}$$

$$= \frac{Z_i^{\gamma - 1}}{\chi^\gamma N_i^{\gamma \varphi}} \int_0^1 C_j^{1-\gamma \sigma} \, dj,$$  \hfill (58)

From (57), we know that $C_i = C_j = C$ for all $i, j$. So we can take $C_j$ outside of the integral in (58) and solve for $N_i$:

$$N_i = \frac{Z_i^{\gamma - 1} C_j^{1-\gamma \sigma} \chi^\gamma N_i^{\gamma \varphi}}{\chi^\gamma N_i^{\gamma \varphi}}$$

$$\Rightarrow N_i = \left( {Z_i^{\gamma - 1} C_j^{1-\gamma \sigma} \over \chi^\gamma} \right)^{\frac{1}{1 + \gamma \varphi}}.$$  \hfill (59)

Similarly, output will be:

$$Y_i = \left( {Z_i^{\gamma (1+\varphi)} C_j^{1-\gamma \sigma} \chi^\gamma} \right)^{\frac{1}{1 + \gamma \varphi}}.$$  \hfill (60)

Substitute (59) and (60) back into the definition of the consumption basket (57), and solve for
the consumption basket $C$ in each country, which will be identical:

$$
C = \left( \frac{1}{\chi} \right)^{\frac{1}{\frac{1}{\sigma} + \varphi}} \left\{ \int_{0}^{1} \left( \frac{Z_j^{\gamma-1} C^{1-\gamma} \sigma}{\chi^\gamma} \right)^{\frac{1}{1+\gamma\varphi}} Z_j^{\gamma-1} dj \right\}^{-\frac{1}{\frac{1}{\sigma(\gamma-1)}}},
$$

$$
C_i^{\frac{1}{\frac{1}{\sigma} + \varphi}} = \left( \frac{1}{\chi} \right)^{\frac{1}{\frac{1}{\sigma} + \varphi}} \left[ \int_{0}^{1} \left( \frac{Z_j^{\gamma-1}}{\chi^\gamma} \right)^{\frac{\gamma-1}{1+\gamma\varphi}} Z_j^{\gamma-1} dj \right]^{\frac{1}{\frac{1}{\sigma(\gamma-1)}}},
$$

$$
\Rightarrow C = C_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\frac{1}{\sigma} + \varphi}} \left( \int_{0}^{1} Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} dj \right)^{\frac{1}{\frac{1}{\sigma(\gamma-1)}}}(\gamma-1) \cdot (61)
$$

Solve for labor and output by substituting (61) into (59) and (60) respectively:

$$
N_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\frac{1}{\sigma} + \varphi}} \left( \int_{0}^{1} Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} \sigma \right)^{\frac{1-\gamma}{1+\gamma\varphi}} Z_i^{\frac{1}{\frac{1}{\sigma} + \varphi}}(\gamma-1),
$$

$$
Y_i = \left( \frac{1}{\chi} \right)^{\frac{1}{\frac{1}{\sigma} + \varphi}} \left( \int_{0}^{1} Z_j^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} \sigma \right)^{\frac{1-\gamma}{1+\gamma\varphi}} Z_i^{\frac{1}{\frac{1}{\sigma} + \varphi}}(\gamma-1),
$$

(62)

(63)

This is the Pareto efficient allocation. When $\gamma \to \infty$, the flexible price allocation and the global social planner allocation become identical. Consumption is identical to the first order between social planner and flexible price allocation. However, it is not true for labor. ■
D Producer Currency Pricing: Propositions and Proofs

Proposition 4 In complete markets, non-cooperative central banks will maximize (30a) and cooperative central banks will maximize (30b), subject to (31), (32), (33) and (34). The solution under commitment for both cooperative and non-cooperative central banks in complete markets is:

\[
\mathbb{E}\{U_i\} = C_{i}^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)}\right),
\]

\[
C_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{1+\varphi}{\sigma+\varphi}},
\]

\[
N_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{(1-\gamma)(1+\varphi)}{1+\gamma(\sigma+\varphi)}} Z_{i}^{\frac{1-\gamma}{1+\gamma(\sigma+\varphi)}},
\]

\[
Y_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{1-\gamma(1+\varphi)}{1+\gamma(\sigma+\varphi)}} Z_{i}^{\frac{1+\varphi}{1+\gamma(\sigma+\varphi)}},
\]

\[
Z_{w} = \left(\int_{0}^{1} Z_{i}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma(\sigma+\varphi)}}\frac{1}{1+\gamma(\sigma+\varphi)}\right)\frac{1+\varphi}{1+\gamma(\sigma+\varphi)}.
\]

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in complete markets, and is the optimal policy under cooperation. If the government corrects the distortions due to market power with a non-contingent tax \(\tau_i = 1 - \mu\), then the flexible price allocation in complete markets is identical to the global social planner solution.

Proposition 5 In financial autarky, non-cooperative central banks will maximize (30a) and cooperative central banks will maximize (30b), subject to (35), (36), (37) and (38). The solution under commitment for both cooperative and non-cooperative central banks in financial autarky is:

\[
\mathbb{E}\{U_i\} = C_{i}^{1-\sigma}\left(\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu(1+\varphi)}\right),
\]

\[
C_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{1+\varphi}{\sigma+\varphi}},
\]

\[
N_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{(1-\gamma)(1+\varphi)}{1+\gamma(\sigma+\varphi)}} Z_{i}^{\frac{1-\gamma}{1+\gamma(\sigma+\varphi)}},
\]

\[
Y_{i} = \left(\frac{1-\tau_i}{\chi\mu}\right)\frac{1}{\sigma+\varphi} Z_{w}^{\frac{1-\gamma(1+\varphi)}{1+\gamma(\sigma+\varphi)}} Z_{i}^{\frac{1+\varphi}{1+\gamma(\sigma+\varphi)}},
\]

\[
Z_{w} = \left(\int_{0}^{1} Z_{i}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma(\sigma+\varphi)}}\frac{1+\varphi}{1+\gamma(\sigma+\varphi)}\right)\frac{1+\varphi}{1+\gamma(\sigma+\varphi)}.
\]

The resulting equilibrium allocation exactly coincides with the flexible price allocation. Mimicking the flexible price allocation is thus the dominant strategy for non-cooperative central banks in financial autarky, and is the optimal policy under cooperation.
**Proof:** The objective function for non-cooperative and cooperative central banks will be, respectively:

\[
\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\}, \tag{64a}
\]

\[
\max_{\forall N_{it}} \int_0^1 \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \, di. \tag{64b}
\]

Policymakers in each scenario will maximize their objective function subject to the following constraints:

\[
1 = \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{it}^{1-\sigma} Y_{it}^{\gamma-1} C_{wt}^{\gamma}} \tag{65}
\]

\[
Y_{it} = Z_{it} N_{it}, \tag{66}
\]

\[
C_{wt} = \left( \int_0^1 Y_{it}^{\gamma-1} \right)^{\frac{1}{\gamma}}. \tag{67}
\]

\[
C_{it} = C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1} \left\{ Y_{it}^{\frac{1}{\gamma}} \right\} \tag{68a}
\]

\[
C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{1}{\gamma}} \tag{68b}
\]

where (67) is a constant, (68a) refers to goods market clearing and risk-sharing under complete markets, while (68b) refers to goods market clearing and risk-sharing under financial autarky.

Notice that under complete markets we can substitute (68a) into (65) so that:

\[
1 = \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{it}^{1-\sigma} Y_{it}^{\gamma-1} C_{wt}^{\gamma}} \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{it}^{1-\sigma} Y_{it}^{\gamma-1} C_{wt}^{\gamma}} \tag{69}
\]

Plug in $C_{it} = \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{it}^{1-\sigma} Y_{it}^{\gamma-1} C_{wt}^{\gamma}}$:

$C_{wt}$ is constant:

\[
= \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{wt}^{1-\sigma} \mathbb{E}_{t-1} \left\{ \left[ \mathbb{E}_{t-1} \left\{ Y_{it}^{\gamma-1} \right\} \right]^{-\sigma} Y_{it}^{\gamma-1} C_{wt}^{\gamma} \right\}} \tag{69}
\]

LIE = $\left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{ wt}^{1-\sigma} \left[ \mathbb{E}_{t-1} \left\{ Y_{it}^{\gamma-1} \right\} \right]^{-\sigma} \mathbb{E}_{t-1} \left\{ Y_{it}^{\gamma-1} \right\}}$.

\[
= \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{\mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}}{C_{wt}^{1-\sigma} \left[ \mathbb{E}_{t-1} \left\{ Y_{it}^{\gamma-1} \right\} \right]^{1-\sigma}} \tag{69}
\]
where LIE denotes the law of iterated expectations. The denominator of (69) is constant, so its expectation will also be constant. We now take the expectation of the entire denominator so that this expression will be identical in complete markets and financial autarky. Continuing on, we can rewrite (69) as

\[
\left( \frac{\chi_{\mu}}{1 - \tau_i} \right) \mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\} = \left( \frac{\chi_{\mu}}{1 - \tau_i} \right) \mathbb{E}_{t-1} \left\{ N_{it}^{1+\varphi} \right\}
\]

Continuing its expectation will also be constant. We now take the expectation of the entire denominator where LIE denotes the law of iterated expectations. The denominator of (69) is constant, so its expectation in both complete markets and financial autarky will be identical. It is trivial to verify that under financial autarky the price-setting condition equal to (70) by substituting (68b) into (65).

Using (70) as our constraint in complete markets and in financial autarky, we can formulate a Lagrangian for the non-cooperative and cooperative cases:

\[
\mathcal{L} = \mathbb{E}_{t-1} \left\{ C_{it}^{1-\sigma} \right\} - \frac{\chi_{\mu}}{1 - \tau_i} N_{it}^{1+\varphi} + \lambda_i \left( \mathbb{E}_{t-1} \left\{ C_{it}^{1-\sigma} \right\} - \frac{\chi_{\mu}}{1 - \tau_i} N_{it}^{1+\varphi} \right)
\]

\[
\mathcal{L} = \int_0^1 \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1 - \sigma} \right\} - \frac{\chi_{\mu}}{1 - \tau_i} N_{it}^{1+\varphi} + \lambda_i \left( \mathbb{E}_{t-1} \left\{ C_{it}^{1-\sigma} \right\} - \frac{\chi_{\mu}}{1 - \tau_i} N_{it}^{1+\varphi} \right) \, di
\]

Using $C_{it} = C_{it}^{1-\sigma} N_{it}^{\gamma-1} Z_{it}^{1-\gamma}$ for complete markets, or $C_{it} = C_{it}^{1-\sigma} N_{it}^{\gamma-1} Z_{it}^{1-\gamma}$ for financial autarky, we can take the first order condition with respect to $N_{it}$.

\[
\frac{\partial \mathcal{L}}{\partial N_{it}} = C_{it}^{-\sigma} \left[ 1 + \lambda_i(1 - \sigma) \right] \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{Y_{it}^{\gamma-1} C_{it}^{1}}{N_{it}} \right) - \chi \left[ 1 + \lambda_i \frac{\mu}{1 - \tau_i}(1 + \varphi) \right] \left( \frac{1}{N_{it}} \right) N_{it}^{1+\varphi} = 0
\]

In equilibrium, this equals:

\[
1 = \chi \left( \frac{1 + \lambda_i \mu(1 + \varphi)}{1 - \tau_i} \right) \left( \frac{N_{it}^{1+\varphi}}{C_{it}^{-\sigma} Y_{it}^{\gamma-1} C_{it}^{1}} \right)
\]

This equation holds in both complete markets and financial autarky, and differs from the flexible price equilibrium only by the constant term. However, subject to labor market clearing, this

\[\text{[14] Remember that we are optimizing given the fact that state } s_t \text{ is realized. Expectations in our context thus refer to a summation over all possible states multiplied by the probability of each state occurring. For example, } \mathbb{E}_{t-1} \{ C_{it}^{1-\sigma} \} = \sum_{s_t} C_{it}^{1-\sigma}(s_t) \Pr(s_t). \]

33
constant will coincide with the flexible price equilibrium. The flexible price equilibrium in complete markets and financial autarky is found by taking expectations out of the labor market clearing condition (65) and substituting in goods market clearing (66):

\[ 1 = \left( \frac{\chi \mu}{1 - \tau_i} \right) \frac{Y_{it}}{C_{it}^{\frac{1+\gamma}{\sigma}} C_{wt}^{\frac{\gamma}{1+\gamma}}}. \]  

(74)

For complete markets, we can express output as a function of technology and a constant term by substituting (68a) into (74):

\[ Y_{it} = A_i Z_{it}^{(\gamma - 1)(1 + \phi) \gamma}. \]  

We can do the same for autarky by substituting (68b) into (74), but leave that to the reader. Using this expression for output, consumption in complete markets in country \( i \) can be expressed as

\[ C_{it} = A_i^{\gamma} C_{wt}^{\frac{1}{\gamma}} \left\{ Z_{it}^{(\gamma - 1)(1 + \phi) \gamma} \right\}. \]  

(75)

Now substitute (75) back into the flexible price equilibrium (74)

\[ 1 = \left( \frac{\chi \mu}{1 - \tau_i} \right) C_{wt}^{\gamma} A_i^{\gamma} C_{it}^{\frac{1+\gamma}{\sigma}} \left\{ Z_{it}^{(\gamma - 1)(1 + \phi) \gamma} \right\} \cdot \]  

(76)

and rearrange and solve for \( A_i \):

\[ A_i = \left[ \left( \frac{1 - \tau_i}{\chi \mu} \right)^{\gamma} C_{it}^{\gamma \sigma - \frac{1}{\gamma}} E_{t-1} \left\{ Z_{it}^{(\gamma - 1)(1 + \phi) \gamma} \right\} \right]^{\frac{1}{\gamma \sigma + \gamma \phi}}. \]  

(77)

Now, substitute the solution for \( A_i \), (77), into (75):

\[ C_{it} = \left[ \left( \frac{\chi \mu}{1 - \tau_i} \right)^{\gamma} C_{wt}^{\gamma \sigma + \gamma \phi} E_{t-1} \left\{ Z_{it}^{(\gamma - 1)(1 + \phi) \gamma} \right\} \right]^{\frac{1}{\gamma \sigma + \gamma \phi}}. \]  

(78)

Using the fact that \( C_{wt} = \int_0^1 C_{it} di \) and setting \( Z_w = \left( \int_0^1 Z_i^{(\gamma - 1)(1 + \phi) \gamma} di \right)^{\frac{1+\gamma}{\gamma - 1}(1 + \phi)} \), consumption for country \( i \) in complete markets is:

\[ C_{it} = \left( \frac{1 - \tau_i}{\chi \mu} \right)^{\frac{1}{\gamma \sigma + \gamma \phi}} Z_w^{\frac{1+\gamma}{\gamma - 1}(1 + \phi)}. \]  

(79)

Solving for labor and output using (79) is a straightforward exercise. The solution to the central bank’s problem in complete markets and financial autarky for cooperative and non-cooperative equilibria, coincides exactly with the flexible price allocation. Here we’ve explicitly outlined the proof for complete markets. The proof for financial autarky is identical up to (74). We simply substitute (68b) into (74) to get the optimal allocation under financial autarky. ■
E Local Currency Pricing: Propositions and Proofs

Proposition 6 Under LCP, non-cooperative central banks will maximize (30a) subject to (39) and (40). A fixed exchange rate will be the Nash equilibrium policy for a non-cooperative central bank in both complete markets and financial autarky under LCP.

Proof: In this section we explain the case of LCP in the continuum framework. First, let another country \(j\) import goods from firm \(h\) in country \(i\).

Price Setting

Profits from the exports of firm \(h\) in country \(i\) to country \(j\) in domestic currency will be:

\[ P_{jit}(h)\xi_{ijt}C_{jit}(h) - W_{it}N_{jit}(h) \] (B.1)

where \(P_{jit}(h)\) denotes the price charged by firm \(h\) in country \(j\) which is located in country \(i\). This price will be denominated in currency \(j\). \(N_{jit}(h)\) is the amount of labor used by firm \(h\) (which is manufacturing in country \(i\)) in the production of its exports to country \(j\). Given this, firm \(h\) will choose the price that maximizes its expected profit:

\[ \mathbb{E}_{t-1}\left\{ C_{ijt}^{-\sigma} \left( \frac{P_{jit}(h)\xi_{ijt}C_{jit}(h) - W_{it}N_{jit}(h)}{CPI_{it}} \right) \right\} \] (B.2)

We also have demand for firm \(h\)'s good:

\[ C_{jit}(h) = \left( \frac{P_{jit}(h)}{P_{jit}} \right)^{-\varepsilon} C_{jit}. \] (B.3)

In order to solve the maximization problem, we substitute the demand for firm \(h\)'s good (B.3) into the expected profit function (B.2) and take the FOC with respect to \(P_{jit}(h)\). The FOC will be:

\[ \mathbb{E}_{t-1}\left\{ C_{ijt}^{-\sigma} \left( \frac{P_{jit}(h)\xi_{ijt}C_{jit}(h)}{P_{jit}(h)CPI_{it}} \right) \right\} = \frac{\varepsilon}{\varepsilon - 1} \mathbb{E}_{t-1}\left\{ \frac{W_{it}N_{jit}(h)}{P_{jit}(h)CPI_{it}} \right\} \] (B.4)

\(P_{jit}(h)\) on the LHS will cancel out, and we will be left with:

\[ \mathbb{E}_{t-1}\left\{ C_{ijt}^{-\sigma} \xi_{ijt}C_{jit}(h) \right\} = \mu \mathbb{E}_{t-1}\left\{ \frac{W_{it}N_{jit}(h)}{P_{jit}(h)CPI_{it}} \right\} \] (B.5)

Labor Market Clearing

We can take out the \(h\) index for firms because they are all identical. Now, substituting labor supply \(\frac{\chi_i}{1-\tau_i} \frac{W_{it}}{CPI_{it}} = N_{it}^{\sigma}C_{it}^{\sigma}\) into the above equation, we get

\[ \mathbb{E}_{t-1}\left\{ C_{ijt}^{-\sigma} \xi_{ijt}C_{jit} \right\} = \chi_i \mu \frac{1}{1 - \tau_i} \mathbb{E}_{t-1}\left\{ \frac{N_{jit}^{\sigma}N_{jit}}{P_{jit}} \right\}. \] (B.6)
Substituting the identities $N_{jit} = C_{jit}Z_{it}^{-1}$ and $C_{jit} = \left( \frac{E_{it}}{CPI_{it}} \right)^{-\gamma} C_{jt}$ into the above equation gives:

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma E_{jit} P_{jit}^{-\gamma} CPI_{jt} C_{jit} \right\} = \frac{\chi \mu}{1 - \tau_i} \mathbb{E}_{t-1} \left\{ \frac{N_{it}^2 P_{jit}^{-\gamma} CPI_{jt} C_{jit}}{Z_{it} P_{jt}} \right\}.
$$

(B.7)

Now we use the fact that $CPI_{it}, P_{jit}$ and $N_{it}$ are predetermined:

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma E_{jit} Y_{it} P_{jit} C_{jt} \right\} = \frac{\chi \mu}{1 - \tau_i} \mathbb{E}_{t-1} \left\{ N_{it}^{1+\sigma} C_{jt} \right\}.
$$

(B.8)

The next step in our derivation requires that we assume there is a common numeraire country, call it 0, and all countries fix their currency with respect to this numeraire currency. This means that $E_{ijt} = E_{0it}E_{0jt}$. When combined with predetermined prices and output, the following condition emerges:

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma E_{ijt} Y_{it} P_{jit} C_{jt} \right\} = \frac{\chi \mu}{1 - \tau_i} \mathbb{E}_{t-1} \left\{ N_{it}^{1+\sigma} C_{jt} \right\}.
$$

(B.9)

E.1 Non-Cooperative Equilibrium

Below we describe the non-cooperative Nash equilibrium under LCP. Here we assume that all countries, with the exception of country $i$, have a fixed exchange rate with the numeraire currency, such that $E_{0it} = 1$. Given this assumption, the labor clearing condition becomes

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma Y_{it} P_{jit} E_{0it} \right\} = \frac{\chi \mu}{1 - \tau_i} \mathbb{E}_{t-1} \left\{ N_{it}^{1+\sigma} \right\}.
$$

(B.10)

Let us now write out the same equation for the numeraire country exporting to $i$ and $j$ respectively:

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma Y_{it} P_{0it} E_{0it} \right\} = \frac{\chi \mu}{1 - \tau_0} \mathbb{E}_{t-1} \left\{ N_{0it}^{1+\sigma} \right\} \frac{\mathbb{E}_{t-1} C_{it}}{\mathbb{E}_{t-1} E_{0it} C_{it}},
$$

(B.11)

$$
\mathbb{E}_{t-1} \left\{ C_{it} \sigma Y_{it} P_{0it} E_{0it} \right\} = \frac{\chi \mu}{1 - \tau_0} \mathbb{E}_{t-1} \left\{ N_{0it}^{1+\sigma} \right\} \frac{\mathbb{E}_{t-1} C_{jt}}{\mathbb{E}_{t-1} E_{0it} C_{jt}}.
$$

(B.12)

If we divide (B.11) by (B.12), and use the fact that prices are predetermined and $E_{0it} = 1$, we get

$$
P_{0it} = \frac{CPI_{it}}{CPI_{jt}} = \frac{CPI_{it}}{CPI_{0it}} = \frac{\mathbb{E}_{t-1} C_{it}}{\mathbb{E}_{t-1} [E_{0it} C_{it}]}.
$$

(B.13)

Financial Autarky

In autarky, we assume all economies in the rest of the world ($-i$) are ex-ante identical and that the actions of country $i$ will not influence their policy decisions. Therefore, prices and price indices will be equalized across all $-i$ countries, so that $P_{0it} = P_{jit}$ and $CPI_{0it} = CPI_{jt}$ respectively. This will lead to the following set of identities:

$$
C_{it} = \frac{\int_0^1 C_{jit} P_{0it} E_{ijt} dj}{CPI_{it}} = \frac{P_{0it}}{CPI_{it}} \int_0^1 C_{jit} E_{ijt} dj = \frac{P_{0it}}{CPI_{it}} \mathbb{E}_{0it} \int_0^1 C_{jit} E_{ijt} dj = \frac{P_{0it}}{CPI_{it}} E_{0it} Y_{it}.
$$

(B.14)
Output is standard:

\[ Y_{it} = \int_0^1 C_{jit} dj = \int_0^1 \left( \frac{P_{jit}}{CPI_{it}} \right)^{-\gamma} C_{jit} dj = \left( \frac{P_{0it}}{CPI_{0it}} \right)^{-\gamma} C_{wt}, \]

(B.15)

\[ \frac{P_{0it}}{CPI_{0it}} = Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}}. \]

(B.16)

Now we can plug these equations into the labor market clearing equation (39).

\[ E_{t-1} C_{it}^{1-\sigma} = \frac{\chi \mu}{1 - \tau_i} E_{t-1} N_{it}^{1+\phi} \]

(B.17)

Goods market clearing will be the following

\[ C_{it} = \frac{P_{0it}}{CPI_{0it}} \mathcal{E}_{0it} Y_{it} = \frac{P_{0it}}{CPI_{0it}} CPI_{0it} \mathcal{E}_{0it} Y_{it} = \frac{P_{0it}}{CPI_{0it}} Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}}. \]

(B.18)

Using (B.13), this can be rewritten as:

\[ C_{it} = \frac{E_{t-1} \{ \mathcal{E}_{0it} C_{it} \}}{E_{0it} E_{t-1} \{ C_{it} \}} Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}}. \]

(B.19)

The optimization problem of the non-cooperative central bank in country \( i \) will then be:

\[ \mathcal{L} = E_{t-1} \left\{ C_{it}^{1-\sigma} \right\} - \chi Y_{it}^{1+\phi} E_{t-1} \left\{ Z_{it}^{1-\phi} \right\} + \lambda_1 \left[ E_{t-1} C_{it}^{1-\sigma} - \frac{\chi \mu}{1 - \tau_i} E_{t-1} N_{it}^{1+\phi} \right] + \lambda_2 \left[ C_{it} - \frac{E_{t-1} \{ \mathcal{E}_{0it} C_{it} \}}{E_{0it} E_{t-1} \{ C_{it} \}} Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}} \right] \]

(B.20)

Maximization with respect to \( \mathcal{E}_{0it} \) will yield the following FOC:

\[ \frac{\partial \mathcal{L}}{\partial \mathcal{E}_{0it}} = - \frac{1}{\mathcal{E}_{0it}} Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}} + \frac{E_{t-1} \{ \mathcal{E}_{0it} C_{it} \}}{E_{0it} E_{t-1} \{ C_{it} \}} Y_{it}^{-\frac{1}{\gamma}} C_{wt}^{-\frac{1}{\gamma}} = 0 \]

\[ \Rightarrow E_{t-1} \{ \mathcal{E}_{0it} C_{it} \} = \mathcal{E}_{0it} E_{t-1} \{ C_{it} \} \]

(B.21)

This proves that the optimal exchange rate chosen by the central bank must not be state-contingent. In other words, the central bank will choose to fix its exchange rate.

**Complete Markets**

In this section, we assume that the degree of exchange rate pass-through is governed by parameter \( \eta \in [0, 1] \), where \( \eta = 1 \) is perfect pass-through (PCP) and \( \eta = 0 \) is zero pass-through (LCP). Maximization of (45) with respect to state contingent bonds yields the following FOC:

\[ \frac{\lambda_i(s_t)}{\lambda_j(s_t)} = \frac{\lambda_{0i}}{\lambda_{0j}} \frac{P_i(s_t)}{P_j(s_t) \mathcal{E}_{ij}(s_t)} = \frac{\lambda_{0i}}{\lambda_{0j}} \frac{\mathcal{E}_{0it}(s_t)}{\mathcal{E}_{0jt}(s_t) \mathcal{E}_{ij}(s_t)} = \frac{\lambda_{0i}}{\lambda_{0j}} \mathcal{E}_{ijt}^{\eta-1} \]

(B.22)

Using marginal utility, this will become:

\[ \frac{C_{it}^{1-\sigma}}{C_{jt}^{1-\sigma}} = \mathcal{E}_{ijt}^{\eta-1}. \]

(B.23)
So we can express consumption in the following way: \( C_{it} = A_i \mathcal{E}_{i0t}^{1-\eta} C_{jt} \). Using the fact that \( C_{jt} = C_{wt} \), this will become \( C_{it} = A_i \mathcal{E}_{i0t}^{1-\eta} C_{wt} \). To find \( A_i \) we plug this expression into the transversality condition.

\[
\sum_{t=1}^{\infty} \sum_{s_t} \int_0^1 q_j(s_t) B_{ij}(s_t) dj = \sum_{t=1}^{\infty} \sum_{s_t} \int_0^1 \beta t \lambda_i(s_t) P_t(s_t) \mathcal{E}_{ij}(s_t) \frac{B_{ij}(s_t)}{\lambda_0 P_i(s_t)} dj
\]

\[
= \sum_{t=1}^{\infty} \sum_{s_t} \beta t \lambda_i(s_t) P_t(s_t) \mathcal{E}_{ij}(s_t) \int_0^1 B_{ij}(s_t) dj
\]

\[
\equiv \sum_{t=1}^{\infty} \sum_{s_t} \beta t \lambda_i(s_t) P_t(s_t) \mathcal{E}_{ij}(s_t) \left( P_t(s_t) C_i(s_t) - W_t(s_t) N_i(s_t) - \Pi_t(s_t) \right)
\]

\[
= 0
\]

\[
A_i = \frac{\sum_{t=0}^{\infty} \sum_{s_t} \beta t W(s_t) N(s_t) \lambda_i(s_t) P_t(s_t)}{\sum_{t=0}^{\infty} \sum_{s_t} \beta t \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) P_t(s_t)}
\]

\[
= \frac{\sum_{t=0}^{\infty} \beta t \mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}}{\sum_{t=0}^{\infty} \beta t \mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}}
\]

\[
= \sum_{t=0}^{\infty} \beta t \mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}
\]

\[
= \sum_{t=0}^{\infty} \beta t \mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}
\]

\[
= \sum_{t=0}^{\infty} \beta t \mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}
\]

Given the solution for \( A_i \), it follows that:

\[
C_{it} = \frac{\sum_{t=0}^{\infty} \beta t \mathcal{E}_{0t} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}}{\sum_{t=0}^{\infty} \beta t \mathcal{E}_{0t} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}} \mathcal{E}_{i0t}^{1-\eta} C_{wt}.
\]

(B.24)

If technology shocks are independent across time, then \( C_{it} = \frac{\mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}}{\mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}} \mathcal{E}_{i0t}^{1-\eta} C_{wt} \). Assuming independence of shocks across countries as well, we obtain \( C_{it} = \frac{\mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}}{\mathcal{E}_{t-1} \left\{ \mathcal{E}_{i0t}^{1-\eta} C_{wt} \lambda_i(s_t) \right\}} \mathcal{E}_{i0t}^{1-\eta} C_{wt} \).
Under LCP, if all countries except $i$ have a fixed exchange rate, we have

$$C_{it} = \mathbb{E}_{t-1} \left\{ \int_0^1 C_{jit} P_{0it} \mathcal{E}_{ijt} dj \right\} \frac{\mathcal{E}_{0it}^{\frac{1}{2}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{0it}^{\frac{1}{2}} \right\}}$$

$$= \mathbb{E}_{t-1} \left\{ \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{0it} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj \right\} \frac{\mathcal{E}_{0it}^{\frac{1}{2}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{0it}^{\frac{1}{2}} \right\}}$$

$$= \mathbb{E}_{t-1} \left\{ \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{0it} \mathcal{E}_{0it} Y_{it} \right\} \frac{\mathcal{E}_{0it}^{\frac{1}{2}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{0it}^{\frac{1}{2}} \right\}}$$

Now we assume $\sigma = 1$, so that we have log utility. The goods market clearing constraint in complete markets will then be:

$$C_{it} = \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{0it} Y_{it} \quad \text{(B.25)}$$

Notice that this expression is identical to (40), the goods market clearing constraint in financial autarky. Since the labor equilibrium condition is also identical to the labor equilibrium condition in autarky, and the objective function does not change, the solution to the central bank’s optimization problem in complete markets will be identical to that in autarky.

E.2 Cooperative Equilibrium

Below we describe the cooperative equilibrium under LCP.

**Proposition 7** Under LCP, cooperative central banks will maximize (30b) subject to (41) and (42). The optimal policy for cooperative central banks in both complete markets and financial autarky will be a fixed exchange rate.

**Proof:**

**Financial Autarky**

$$C_{it} = \frac{\int_0^1 C_{jit} P_{0it} \mathcal{E}_{ijt} dj}{CPI_{it}} \quad \text{(B.26)}$$

Now we assume $\sigma = 1$, and $P_{jit} = P_{0it} = CPI_{0it} = CPI_{it}$ because of ex-ante symmetry, so the above equation becomes:

$$C_{it} = \mathcal{E}_{0it} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj. \quad \text{(B.27)}$$

**Complete Markets**

$$C_{it} = \mathbb{E}_{t-1} \left\{ \int_0^1 C_{jit} P_{0it} \mathcal{E}_{ijt} dj \right\} \frac{\mathcal{E}_{0it}^{\frac{1}{2}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{0it}^{\frac{1}{2}} \right\}} = \mathbb{E}_{t-1} \left\{ \frac{P_{0it}}{CPI_{it}} \mathcal{E}_{0it} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj \right\} \frac{\mathcal{E}_{0it}^{\frac{1}{2}}}{\mathbb{E}_{t-1} \left\{ \mathcal{E}_{0it}^{\frac{1}{2}} \right\}}$$

Again, we assume $\sigma = 1$, and $P_{jit} = P_{0it} = CPI_{0it} = CPI_{it}$ because of ex-ante symmetry, so the above equation becomes:

$$C_{it} = \mathcal{E}_{0it} \int_0^1 C_{jit} \mathcal{E}_{0jt} dj. \quad \text{(B.29)}$$
Notice that this expression is identical to autarky. Since the labor equilibrium condition is also identical to autarky, the optimization problem for cooperative central banks under LCP will be identical in complete markets and autarky.

We know that output will be predetermined because prices are predetermined. As a result, exchange rate policy can affect only consumption, but not labor. Therefore the maximization problem faced by cooperative central banks will be as follows:

$$\mathcal{L} = \int_0^1 C_{it}^{1-\sigma} di = \int_0^1 C_{it}^{1-\sigma} di + \int_0^1 \lambda_{it} \left( C_{it} - \mathcal{E}_{i0t} \int_0^1 C_{jt} \mathcal{E}_{j0t}^{-1} dj \right) di. \quad (B.30)$$

The FOC with respect to $\mathcal{E}_{i0t}$ is

$$\frac{\partial \mathcal{L}}{\partial \mathcal{E}_{i0t}} = -\lambda_{it} \frac{C_{it}}{\mathcal{E}_{i0t}} + \int_0^1 \lambda_{jt} \mathcal{E}_{j0t} C_{it} \mathcal{E}_{i0t}^{-2} dj = 0, \quad (B.31)$$

which yields the following condition:

$$\lambda_{it} \mathcal{E}_{i0t} = \int_0^1 \lambda_{jt} \mathcal{E}_{j0t} dj. \quad (B.32)$$

The above equation holds for all $i, j$ pairs, so that $\lambda_{it} \mathcal{E}_{i0t} = \lambda_{jt} \mathcal{E}_{j0t}$. This equation holds when exchange rates are fixed. ■
F Cashless Economy

In this section we demonstrate the equivalence of the central bank optimizing over labor or the money supply. We begin with the closed economy and then proceed to the open economy. We will follow the notation employed in the rest of the paper.

F.1 Closed Economy

When we have money in the model, the household will maximize the following utility function:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left( \frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\}$$

where \(M/P\) denotes the household’s real money balances. The household budget constraint is:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = (1-\tau) \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1}) \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t.$$  \(\text{(B.33)}\)

The FOC with respect to \(M_t\) is:

$$C_t^{1-\sigma} = \rho \left( \frac{M_t}{P_t} \right)^{-b} + \frac{P_t}{P_{t+1}} \mathbb{E}_t \{ C_t^{1-\sigma} \}.$$  \(\text{(B.34)}\)

Labor market clearing implies

$$1 = \left( \frac{\chi \mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}},$$  \(\text{(B.35)}\)

while goods market clearing gives

$$C_t = N_t Z_t.$$  \(\text{(B.36)}\)

We can now formulate the full optimization problem by the central bank, which maximizes the utility of the household by choosing a state contingent plan for the money supply.

$$\max_{M_t, \ldots} \lim_{\rho \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left( \frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\}$$  \(\text{(B.37)}\)

subject to

$$C_t = N_t Z_t,$$  \(\text{(B.38)}\)

$$1 = \frac{\chi \mu}{1-\tau} \frac{\mathbb{E}_{t-1}\{N_t^{1+\varphi}\}}{\mathbb{E}_{t-1}\{C_t^{1-\sigma}\}},$$  \(\text{(B.39)}\)

$$C_t^{1-\sigma} = \rho \left( \frac{M_t}{P_t} \right)^{-b} + \frac{P_t}{P_{t+1}} \mathbb{E}_t \{ C_t^{1-\sigma} \}.$$  \(\text{(B.40)}\)

Notice that the price level \(P\) is undefined in these equations because prices are set one-period-in-advance.

The solution to (B.37)-(B.40) will be inferior to that of the following optimization problem,
where the central bank chooses labor.

\[
\max \lim_{N_t \to 0} \sum_{k=0}^{\infty} \left\{ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + \frac{\rho}{1-b} \left( \frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N_{t+k}^{1+\eta}}{1+\eta} \right\} \tag{B.41}
\]

subject to

\[
C_t = N_t Z_t \tag{B.42}
\]

\[
1 = \left( \frac{\mu}{1-\tau} \right) \frac{E_{t-1}\{N_t^{1+\varphi}\}}{E_{t-1}\{C_t^{1-\alpha}\}}. \tag{B.43}
\]

Why is the solution to the problem characterized by (B.37)-(B.40) inferior to the problem characterized by (B.41)-(B.43)? In the first problem money supply and inflation define the dynamics of consumption, and labor and goods market clearing constraints have to be satisfied. On the other hand, in the second problem the consumption path is arbitrary as long as it does not violate labor and goods market clearing.

As we demonstrated in Appendix B, the solution to (B.41)-(B.43) is

\[
C_t = \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{B.44}
\]

We can construct a money supply rule that replicates this allocation by substituting (B.44) into (B.40) and solving for \( M_t \).

The following money supply rule will implement the consumption allocation given by (B.44):

\[
\left[ \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}} \right]^{-\sigma} - \frac{P_t}{P_{t+1}} E_t \{ C_{t+1}^{-\alpha} \} = \rho \left( \frac{M_t}{P_t} \right)^{-b} \tag{B.45}
\]

Thus, in equilibrium money supply would follow

\[
\left[ \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}} \right]^{-\sigma} - \frac{P_t}{P_{t+1}} E_t \left\{ \left[ \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1}{\sigma+\varphi}} Z_t^{\frac{1+\varphi}{\sigma+\varphi}} \right]^{-\sigma} \right\} = \rho \left( \frac{M_t}{P_t} \right)^{-b} \tag{B.46}
\]

Notice, that the money supply rule (B.45) is different from (B.46) in the sense that it ensures only one equilibrium consumption path.

**F.2 Open Economy**

In the open economy, the central bank’s money supply path is again subject to goods and labor market clearing constraints. For simplicity we will concentrate on the case of financial autarky, but the derivation for complete markets is quite similar. Also we ignore \( i \) subindices denoting individual countries to eliminate unnecessary notation. A central bank optimizing over money supply \( M \) in the open economy will face the following problem:
\[
\max \lim_{\beta \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C^{1-\sigma}_{t+k}}{1-\sigma} + \frac{\rho}{1-b} \left( \frac{M_{t+k}}{P_{t+k}} \right)^{1-b} - \chi \frac{N^{1+\eta}_{t+k}}{1+\eta} \right\},
\]  
subject to
\[
C_t = C_1^\gamma w_t N_t^{\frac{1-\gamma}{\gamma}} Z_t^{\frac{1-\gamma}{1-\gamma}},
\]  
\[
1 = \left( \frac{\chi \mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1} \{ N_{t+1}^{1+\varphi} \}}{\mathbb{E}_{t-1} \{ C_{t-1}^{1-\sigma} \}},
\]  
\[
C_t = \left[ \rho \left( \frac{M_t}{CPI_t} \right)^{-b} + \mathbb{E}_t \left\{ \frac{CPI_t}{CPI_{t+1}} C_{t+1}^{1-\sigma} \right\} \right]^\sigma.
\]  
The optimization problem faced by a central bank in the open economy choosing labor \( N \) is given by:

\[
\max \lim_{\beta \to 0} \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ \frac{C^{1-\sigma}_{t+k}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+k}}{CPI_{t+k}} \right)^{1-b} - \chi \frac{N^{1+\eta}_{t+k}}{1+\eta} \right\},
\]  
subject to
\[
C_t = C_1^\gamma w_t N_t^{\frac{1-\gamma}{\gamma}} Z_t^{\frac{1-\gamma}{1-\gamma}},
\]  
\[
1 = \left( \frac{\chi \mu}{1-\tau} \right) \frac{\mathbb{E}_{t-1} \{ N_{t+1}^{1+\varphi} \}}{\mathbb{E}_{t-1} \{ C_{t-1}^{1-\sigma} \}},
\]  
\[
C_t = \left[ \rho \left( \frac{M_t}{CPI_t} \right)^{-b} + \mathbb{E}_t \left\{ \frac{CPI_t}{CPI_{t+1}} C_{t+1}^{1-\sigma} \right\} \right]^\sigma.
\]  

As in the closed economy, when central banks optimize over money (B.47)-(B.51), the solution will be inferior to that when central banks optimize over labor (B.52)-(B.55). As we showed in Appendix D, the solution to (B.52)-(B.55) is:

\[
C_t = \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1-\gamma}{\gamma}} \left( Z_t^{\frac{1-\gamma}{1-\gamma}} Z_{wt}^{\frac{1+\varphi}{1+\varphi}} \right)^{\frac{1-\gamma}{1-\gamma+\varphi}}.
\]  
The money supply rule that implements the same allocation is given by:

\[
\left[ \left( \frac{1-\tau}{\chi \mu} \right)^{\frac{1-\gamma}{\gamma}} \left( Z_t^{\frac{1-\gamma}{1-\gamma}} Z_{wt}^{\frac{1+\varphi}{1+\varphi}} \right)^{\frac{1-\gamma}{1-\gamma+\varphi}} \right]^{-\sigma} = \rho \left( \frac{M_t}{CPI_t} \right)^{-b} + \mathbb{E}_t \left\{ \frac{CPI_t}{CPI_{t+1}} C_{t+1}^{1-\sigma} \right\}
\]  

In financial autarky, we know that \( CPI_t C_t = P_t Y_t \). Assuming that the producer price path \( P \) is equal to 1, we can plug in the expression \( CPI_t = \frac{Y_t}{C_t} \) and (B.56) to determine the dynamics of real money supply and formulate a money supply rule that exactly replicates the allocation given by B.56).