Tiling.
(1) Is it possible to tile the following damaged checkerboard by dominoes? Why or why not? (Notice that it has a square removed from its interior.)

Matching.
(2) Suppose that \( k \geq 1 \) and \( G = (V, E) \) is a \( k \)-regular bipartite graph with bipartition \( V = A \sqcup B \).
   (a) Prove that \( |A| = |B| \).
   (b) Prove that \( G \) contains a perfect matching.
(3) Suppose that \( G = (V, E) \) is a bipartite graph with bipartition \( V = A \sqcup B \) and that Hall’s condition holds:

\[
(*) \text{ for every subset } S \subseteq A, |N(S)| \geq |S|.
\]

Prove that \( G \) contains a matching that uses every vertex of \( A \). (Hint: enlarge \( G \) to a graph to which Hall’s theorem applies, and use it to derive the desired conclusion about \( G \).)

Stable Matching.
(4) From the textbook, Section 2.9, Problem 2: Suppose \( M_1 \) and \( M_2 \) are two stable matchings between \( n \) men and \( n \) women, and we allow each woman to choose between the man she is paired with in \( M_1 \) and the partner she receives in \( M_2 \). Each woman always chooses the man she prefers. Show that the result is a stable matching between the men and the women. (Note that you must prove that the new assignment is a matching before you can prove that it is stable.)