Scheduling.
(1) A league consists of an even number of teams. We wish to design a schedule in which every team plays every other team exactly once, and so that no team plays more than one match on any given day. Design such a schedule that uses the fewest number of days. (Hint: think about the schedule we devised in class for an odd number of teams. Is there a simple way to add a new team to this schedule?)

Edge coloring.
(2) The double $2G$ of a graph $G$ is obtained by replacing every edge of $G$ by a pair of parallel edges.
   (a) Prove that $\chi'(2G) \leq 2\chi'(G)$.
   (b) Prove that $\chi'(2G) = 2\chi'(G)$ if $G$ is a complete graph with an odd number of vertices.
   (c) Let $P$ denote the Petersen graph. Show that $\chi'(2P) = 6$ and thereby conclude that $\chi'(2P) < 2\chi'(P)$.
   (♠) Let $kG$ denote the graph obtained by replacing every edge of $G$ by $k$ parallel edges. What is $\chi'(kP)$?

Vertex coloring.
(3) Draw a graph $G$ with degree sequence $(3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5)$ that has no triangle (that is, a $K_3$ subgraph) and such that $\chi(G) \geq 4$, which, yes, you must prove. (Hint: its vertices are shown below.)

(4) (a) Suppose that a simple graph contains an odd cycle. Prove that it contains an induced odd cycle.
   (b) Suppose that $G$ is a graph in which any two odd cycles share at least one vertex in common. Prove that $\chi(G) \leq 5$. (Hint: what happens when you remove an induced odd cycle from $G$?)

Line graphs.
(5) Let $L(G)$ denote the line graph of a graph $G$.
   (a) What, with proof, is $\alpha(L(G))$ in terms of another invariant that we have studied?
   (b) Prove that if $G$ is a graph, then $L(G)$ does not contain an induced subgraph isomorphic to $K_{1,3}$.