Day 1: Aug 27
Opening exercise: can 15 students shake hands in such a way that each person shakes the hands of exactly 3 others? Nor can 7, but 8 can... Parity and (double-)counting. Basic terminology about graphs. The Handshake Theorem (a.k.a. the First Theorem of Graph Theory).
Reading: skim the beginning of Chapter 1, pp.1-6.

Day 2: Aug 29
Connectivity.
Reading: pp. 6-8.

Day 3: Aug 31
Connectivity, continued. An extremal argument: a shortest walk between two vertices is a path. Trees and leaves. Another extremal argument: the endpoints of a longest path in a tree are leaves – usually.
Reading: Skim pp. 30-31 and Theorem 1.14 and its proof on pp. 35-36.

Day 4: Sep 5
Resumption of proof that every tree, besides the stump, contains at least two leaves. The no-cycles game, and observation: every tree, including the stump, with \(n\) vertices has \(n-1\) edges. Faulty induction proof.
Reading: p. 30, Theorem 1.10. Note that this is a different proof of the result from the one we are giving in class.

Day 4: Sep 7
Careful proof by induction that a tree with \(n\) vertices has \(n-1\) edges. Subgraphs: induced, spanning, and trees. A graph is connected if and only if it contains a spanning tree (proof to come).
Reading: Section 1.3.2 through the proof of Theorem 1.14. Observe the different order in which the book treats the theorems we did and the alternate methods of argument. For instance, the proof of Theorem 1.10 follows the idea that Jack raised in class.

Day 5: Sep 10
Proof that a graph is connected if and only if it contains a spanning tree.

Day 6: Sep 12
The bridges of Königsberg. Eulerian trails and circuits. Eulerian graphs and even degrees.
Reading: Back to the beginning: pp. 1-2! Section 1.4.1 and the beginning of 1.4.2, pp.52-55.

Day 7: Sep 14
Euler’s theorem: a connected graph has an Eulerian circuit iff all vertex degrees are even iff it is an edge-disjoint union of cycles.
Reading: Section 1.4.2, pp. 56-57.

Day 8: Sep 17
Graph isomorphism. The Petersen graph. Two-colorability.
Reading:

Day 9: Sep 19
A (finite!) graph is bipartite iff it does not contain any odd cycles.
Reading: pp.13-14.

Day 10: Sep 21
Proof using spanning trees of the theorem from last class. Hamiltonian cycles. Statement of Dirac’s theorem.
Reading: Reading: Section 1.4.3 through the statement of Theorem 1.22.
Day 11: Sep 24
Proof of Dirac's theorem.
Reading: Section 1.4.3 through the proof of Theorem 1.22, and the paragraph following it.

Day 12: Sep 26
Tiling and perfect matchings.
Reading: Section 1.7 through 1.7.1.

Day 13: Sep 28
The statement of Hall's theorem.
Reading: statement of Theorem 1.51.

Day 14: Oct 1
Proof of Hall's theorem.
Reading: Class notes; the proof of Theorem 1.51 in the book contains a different (algorithmic) proof.

Day 15: Oct 3
Conclusion of proof of Hall's theorem. Review of major results and their algorithmic consequences (allusion to P vs NP).
Reading: class notes.

Day 16: Oct 5
First midterm.
No class Oct 8

Day 17: Oct 10
Introduction to stable matching.
Reading: Gale and Shapley’s article, pp. 9-12 through the statement of Theorem 1. Section 2.9, pp. 248-250.

Day 18: Oct 12
The Gale-Shapley algorithm.
Reading: Gale and Shapley’s article through its conclusion. Section 2.9.1.

Day 19: Oct 15
Conclusion of the Gale-Shapley algorithm: optimality, pessimality, and applications.
Reading: Review and savor Gale and Shapley’s article. For fun, read the citation for the 2012 Nobel Prize in Economics.

Day 20: Oct 17
Scheduling, matching, and edge-coloring. Bocce tournaments with 4 and with 5 players. Edge-coloring $K_4$, $K_5$, and the Petersen graph.
Reading:

Day 21: Oct 19
Chromatic index. Lower bounds using maximum degree and matching number. $\chi'(K_{2n+1}) = 2n + 1$.
Reading: course notes.

Day 22: Oct 22
Vertex coloring and the chromatic number. Clique and independence numbers. $\chi \geq \omega$.
Reading: course notes; peruse Chapter 1.6.

Day 23: Oct 24
More bounds on chromatic number: $\chi \geq n/\alpha$. Line graphs and graph complements. The relationship between chromatic index and chromatic number.
Reading: peruse Section 1.6.2.

Day 24: Oct 26
$\chi \leq \Delta + 1$. Statements of Brooks’s and Vizing’s theorems (no proofs).


*Reading:* finish Section 1.6.2 (skip the proof of Brooks’s theorem).

**Day 25: Oct 29**

Mycielski’s construction.

*Reading:* course notes. You can look up Mycielski’s construction in a number of sources (regrettably excluding our textbook). I found a nice exposition simply by going online.

**Day 26: Oct 31**


*Reading:* Section 1.8 through the proof of Theorem 1.61.

**Day 27: Nov 2**

The Happy Ending Problem.

*Reading:* Section 2.10.2 up to Theorem 2.27 and after the proof of Theorem 2.30.

**Day 28: Nov 5**

Ramsey numbers. Definition of $R(s, t)$. $R(3, 4) = 9$.

*Reading:* Section 1.8, Theorem 1.62 and its proof.

**Day 29: Nov 7**

Ramsey theory. Establishing the exact value of a Ramsey number. Writing workshop.

*Reading:* Review Section 1.8. Have a look over the writing samples.

**Day 30: Nov 9**

Midterm 2.

**Day 28: Nov 12**

Ramsey’s theorem, part 1.

*Reading:* Continue to read Section 1.8.

**Day 29: Nov 14**

Ramsey’s theorem, part 2.

*Reading:* Finish Section 1.8. Also, have a look around Theorem 2.28 and especially Corollary 2.29 for edification.

**Day 30: Nov 16**

Statement of Schur’s theorem. How not to prove Fermat’s last theorem. AP’s in the primes.

*Reading:* course notes.

**Day 31: Nov 19**

Proof of Schur’s theorem. Potpourri: theorems of van der Waerden and Gallai.

*Reading:* course notes.

**Day 32: Nov 26**

Introduction to extremal graph theory. How many edges can a bipartite graph contain? The AM-GM inequality. Statement of Mantel’s theorem.

*Reading:* course notes.

**Day 33: Nov 28**

The Cauchy-Schwarz inequality and Mantel’s Theorem, Part 1 (inequality).

*Reading:* Lecture notes on course website (1/2).

**Day 34: Nov 30**

Mantel’s Theorem, Part 2 (case of equality). Statement of Turán’s theorem.

*Reading:* Lecture notes on course website (1/2).

**Day 35: Dec 3**

Proof of Turán’s theorem (inequality only).

*Reading:* Lecture notes on course website (1/2).
Day 36: Dec 5
Introduction to $C_4$-free graphs and the theorem of Kővari-Sós-Turán.

*Reading:* Lecture notes on course website (2/2).

Day 37: Dec 7
Comparison of $H$-free graphs, $H \in \{P_4, K_4, C_4\}$. Reiman’s construction and finite planes. Mind-reading.

*Reading:* Lecture notes on course website (2/2).

Day 38: Dec 10
The grand finalé: mind-reading exposed.

*Reading:* magic trick notes.