Problem Set n° 3	MAT 883: Knot Theory	Due October 1, 2015

(1) Given a connected n-crossing diagram $D \subset S^2$ of a link $L \subset S^3$, resolve each crossing of D any way you like. The result is a collection of k disjoint simple closed curves in the plane. Show how to adapt Seifert's algorithm to produce a spanning surface for L that has Euler characteristic $k - n$. Can you recover the checkerboard surfaces of D in this way?

(2) Suppose that $F \subset S^3$ is a spanning surface for a knot K. Carefully explain how to glue two copies of X_F together to form the unique connected 2-to-1 covering space of X_K.

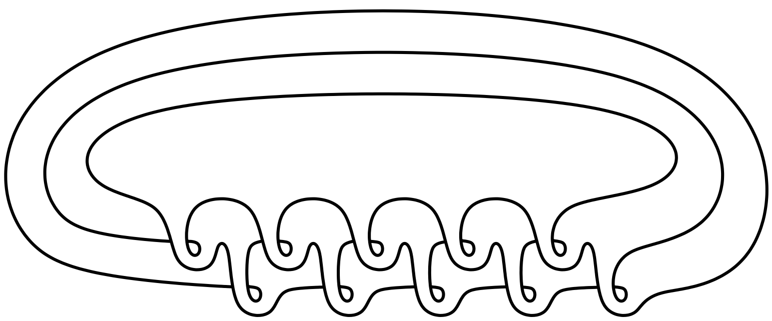
(3) Take a parallel push-off of $T(3,5)$ on the standard torus $T^2 \subset S^3$ on which it sits. What is the linking number between the push-off and $T(3,5)$? Try to determine the answer using the homological description of linking numbers.

(4) Suppose that a is an oriented simple closed curve on a Seifert surface $F \subset S^3$ and $a^+ \subset F^+$ denotes the parallel push-off of both. Prove that the the image of $[a]$ under the composite map

$$H_1(F) \overset{\sim}{\to} H_1(F^+) \overset{i_*}{\to} H_1(S^3 \setminus F) \overset{AD}{\to} H^1(F) \overset{UCT}{\to} H_1(F)^\vee$$

is the element in $H_1(F)^\vee$ defined by $[b] \mapsto \mathrm{lk}(a^+, b)$.

(5) Determine a Seifert matrix V and $\det(tV - V^T)$ for the Seifert surface obtained by applying Seifert's algorithm to the pretzel diagram of $P(p,q,r)$ with all parameters odd and with $(p,q,r) = (-2,3,5)$. Do the same for this Seifert surface for $T(3,5)$:



Do you observe any coincidences?

(6) Suppose that F is a Seifert surface for a knot in S^3 and θ denotes its Seifert form. Define the skew-symmetrization $\theta^- : H_1(F) \times H_1(F) \to \mathbb{Z}$ by $\theta^-(x,y) = \theta(x,y) - \theta(y,x)$, $\forall x, y \in H_1(F)$. Prove that $H_1(F)$ admits a *symplectic basis* for θ^-, meaning a basis $\{x_1, y_1, \ldots, x_g, y_g\}$ such that $\theta^-(x_i, y_j) = \delta_{ij} = -\theta^-(y_j, x_i)$ and $\theta^-(x_i, x_j) = \theta^-(y_i, y_j)$ for all $1 \leq i, j \leq g$.