Problem Set n° 4 MAT 883: Knot Theory Due October 8, 2015

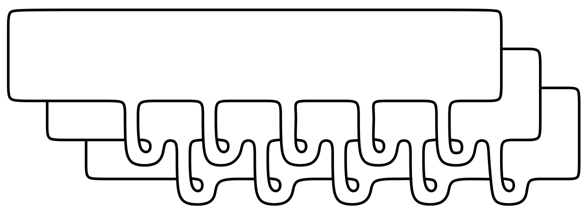
(1) Lickorish Chapter 4, Exercise 1.

(2) Number off as follows:

1. Mustafa 2. Kyle 3. Siddhi 4. Melissa 5. Cristy 6. Tommaso 7. Clayton

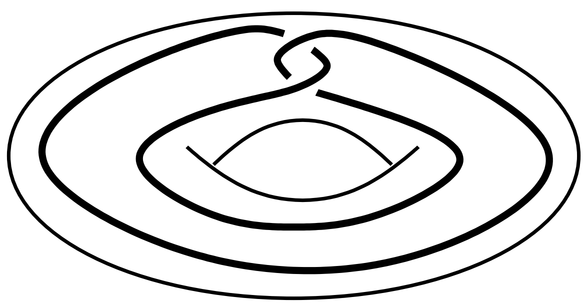
If you are number n, find disk-decomposable Seifert surfaces for the knots 8_k, where $k \equiv n \pmod 7$. Here 8_k is the notation for 8-crossing knots appearing in Rolfsen's table, which is readily available online at `knotinfo`. Appeal to Gabai's theorem to determine $g(8_k)$ in each case. Does it agree with the degree of $\Delta(8_k)$? Incidentally, what are the elementary ideals $E_i(8_k)$? Do you or any of your peers calculate a non-principal elementary ideal in any example?

(3) Show that the familiar surface displayed here is a fiber surface for $T(3,5)$.



Generalize this result to show that the general torus knot $T(p,q)$ is fibered, and determine its genus.

(4) Consider the solid torus V with the embedded simple closed curve P displayed here.



Regarding V as the tubular neighborhood of an unknot U, express $\partial V \approx S^1 \times S^1$, where $S^1 \times \{*\}$ is a meridian for U and $\{*\} \times S^1$ is a Seifert longitude for U. The (untwisted) Whitehead double of a knot K is the knot $Wh(K)$ obtained by choosing a diffeomorphism between $\nu(K)$ and V, matching meridians to meridians and Seifert longitudes to Seifert longitudes, and taking the image of the curve P inside of S^3.

(a) Prove that P does not bound a disk in V. (Hint for a tricky proof: we showed in class that the Stevedore knot 6_1 has a non-trivial Alexander polynomial; so what?)

(b) Prove that $\Delta(Wh(K)) = 1$ for any knot K.

(5) Prove that the exterior of the standard genus-1 Seifert surface for the Whitehead double of the trefoil knot is not diffeomorphic to a handlebody. (Hint: recognize it as a boundary connected sum of two pieces, use that decomposition to calculate its fundamental group, and then distinguish its group from the fundamental group of a handlebody by a group-theoretic argument.)