Problem Set n° 5 MAT 883: Knot Theory Due October 16, 2015

(1) Suppose that J is a knot in a solid torus V and J does not bound a disk in V. Let K be a knot in S^3. Form a *satellite* of K by identifying V with $\nu(K)$ by a diffeomorphism that takes a meridian of V to a meridian of K and letting $J \circ K$ denote the image of J in S^3. Prove that if $J \circ K \simeq U$, then $K \simeq U$. (Hint: consider how a disk bounded by $J \circ K$ meets the torus $\partial X_K \subset X_K \subset X_{J\circ K}$.) Deduce the non-cancelability of knots as a corollary.

(2) A braid $\beta \in B_n$ is *homogeneous* if, for each $i = 1, \dots, n-1$, the generator σ_i occurs in β, and the sign of the exponent on each occurrence of it is the same. For example, $\sigma_1^2\sigma_3^{-4}\sigma_2\sigma_1^5$ is a homogeneous braid in B_4, whereas $\sigma_1^2\sigma_3^{-4}\sigma_2\sigma_1^{-5}$ and $\sigma_1^2\sigma_3^{-4}\sigma_1^5$ are not. Prove that the closure of a homogeneous braid is a fibered link.

(3) Suppose that $T \subset S^3$ is a smoothly embedded torus. Must there exist a smoothly embedded solid torus $V \subset S^3$ with $\partial V = T$?

(4) Suppose that $\Sigma \subset S^3$ is a smoothly embedded genus two surface. Must there exist a smoothly embedded genus two handlebody $H \subset S^3$ with $\partial H = \Sigma$?

(5) Prove that if $\pi(K) \approx \mathbb{Z}$, then $K \simeq U$.