Problem Set n$^{\text{o}}$ 6 MAT 883: Knot Theory Due November 11, 2015

(1) Find a compact, connected, properly embedded surface in the solid torus that is incompressible but not π_1-injective.
(2) Suppose that $K \subset S^3$ is a fibered knot and that $p_1, p_2 : X_K \to S^1$ are two different fiberings. Prove that there exists an orientation-preserving diffeomorphism $f : X_K \to X_K$ such that $p_2 = p_1 \circ f$. (Cerf's theorem about diffeomorphisms of S^3 might be applicable but inessential.)
(3) Prove the bound $|\sigma(K)| \leq 2g_4(K)$, where $g_4(K)$ denotes the smooth slice genus.
(4) Prove that the pretzel knot $P(3, 3, -3)$ is smoothly slice.
(5) Prove that if a knot is smoothly slice, then so is its untwisted Whitehead double.
(6) Fix a knot K and regard the signature function $\sigma_\omega(K)$ as a function of the complex number $\omega \in S^1 - \{1\}$. Show that the discontinuities of $\sigma_\omega(K)$ occur at the roots of Δ_K on the unit circle.
(7) Freedman's theorem asserts that if $\Delta_K = 1$, then K is topologically slice. We also asserted without proof that if K is topologically slice, then $\sigma(K) = 0$. Prove directly that $\Delta_K = 1 \implies \sigma(K) = 0$.