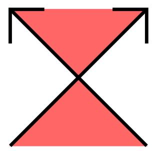
Problem Set n^o 7	MAT 883: Knot Theory	Due November 20, 2015

(1) Which twist knots have a Seifert surface whose Seifert form has a metabolizer?

(2) Prove that if F is a spanning surface for a link $L \subset S^3$ and $\langle x, x\rangle_F > 0$ for all $x \in H_1(F)$, $x \neq 0$, then $F \cap X_L$ is incompressible in X_L.

(3) Suppose that D is a diagram of an oriented knot K and R is the associated red spanning surface coming from a checkerboard coloring. Prove that $e(R)/2$ equals the sum of the values $\zeta(c)$ over all crossings c where the local coloration and orientation appear as shown.



Derive a formula for the signature of K. What happens in this formula if there are no crossings of the above type? How does this relate to the output of Seifert's algorithm?

(4) A *torsion curve* in a non-orientable surface is a curve whose complement in the surface is orientable. Generalize the example of the trefoil done in class for by showing that if F is a non-orientable spanning surface for a knot $K \subset S^3$ and $\gamma \subset F$ is a torsion curve, then $\langle[\gamma],[\gamma]\rangle_F = e(F)/2$.

(5) Suppose that F is a spanning surface for a link L in a closed, orientable 3-manifold. Show that $F \cap X_L$ is π_1-injective in X_L if and only $F^{\pm}$ is incompressible in X_F.